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Quantum magnetotransport in a highly correlated two-dimensional electron liquid on a superfluid helium surface

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The theoretical concept of the inelastic quantum magnetotransport of highly correlated surface electrons on superfluid helium is presented. The low-temperature magnetoconductivity data are obtained from the damping of the edge magnetoplasmons. It is shown that the temperature and magnetic field dependences of the magnetoconductivity can be perfectly described by the inelastic many-electron theory as the interplay of two kinds of Landau level broadening produced by scatterers and by mutual Coulomb interaction.

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Introduction

In the presence of a strong magnetic field B oriented normally the surface electrons (SE) on superfluid helium provide a unique possibility for studying quantum transport phenomena in a highly correlated, two-dimensional ($2D$) electron liquid. At typical electron densities $n \sim 10^8$ cm^{-2} and temperatures $T < 0.5$ K the Coulomb coupling parameter (the mean Coulomb energy over the mean kinetic energy) $G = e^2\sqrt{\pi n}/(k_B T)$ attains unusually large values ($G \sim 100$). Since the Fermi energy of SE is much less than $k_B T$, the ultra-quantum limit ($\hbar\omega_c >> k_B T$ where ω_c is the cyclotron frequency) is easily achieved and at typical helium temperatures nearly all electrons populate the ground Landau level.

A $2D$ electron gas in a magnetic field is a singular system – the electron energy spectrum becomes discrete; therefore, the usual Born approximation fails to describe the quantum magnetotransport. The conventional way to treat this system is the self-consistent Born approximation (SCBA) [1], in which the effects of level broadening caused by scatterers are taken into account. For SE on the superfluid helium the level broadening is the smallest energy parameter: $\Gamma << k_B T$. Therefore, at $\hbar\omega_c >> k_B T$ the SE are confined to a very narrow energy space of the ground level. In the limit $\Gamma \to 0$, the elastic and inelastic scattering processes result in the different analytical behavior of the magnetoconductivity: $\sigma_{xx} \propto 1/\Gamma \to \infty$ for the elastic scattering from impurities, whereas $\sigma_{xx} \to 0$ for the inelastic scattering within the ground level. It means that the correct result for σ_{xx} should be crucially dependent on the relationship between the two small parameters, the energy exchanged at a collision $\Delta\omega$, and the Landau level broadening Γ. It may be far away from the result of the usually used elastic approximation (the nonlinear breakage of the elastic approximation was recently discussed in Ref. 2).

The first zero-field conductivity measurements [3–5], have shown that in spite of the high Coulomb correlations, in the low temperature (LT) range down to the Wigner solid transition the SE

mobility behaves mostly like the mobility of an ideal 2D electron gas. The cause of this behavior is that the wavelength of thermal electrons is much shorter than the Coulomb correlation length [6]. In a magnetic field the energy scale of the SE density of states (Γ) is usually much less than thermal energy and the many-electron effect can be more pronounced.

The SE scattering arises from capillary wave quanta (ripplons) and helium vapor atoms. In the vapor atom scattering regime ($T > 1$ K) the quantization of the electron motion caused by the magnetic field leads to unexpected Hall effect: the Hall angle decreases with the magnetic field B in the ultra quantum limit, since the effective collision frequency $\nu(B)$ increases faster with B than the cyclotron frequency [7,8]. It was shown that the effect and the experimental data can be perfectly described by the extended SCBA theory [7] up to 20 T. In this regime the many-electron effect is usually small with the exception of the narrow temperature range around 1 K, where the magnetoconductivity becomes density dependent if the magnetic field is weak [9,10]. It should be pointed out that under such conditions the energy exchanged at a collision is of the same order of magnitude as Γ and the inelastic effect of the electron-atom scattering which is neglected in Refs. 9 and 10, should be additionally analyzed.

In the ripplon scattering regime ($T < 0.7$ K) the experimental and theoretical situations are much more complicated. The experimental magnetoconductivity data performed by different experimental groups contradict each other. According to Ref. 11 and 12, the SE magnetoconductivity σ_{xx} has a minimum at $T \approx 1$ K and increases slowly down to $T \approx 0.5$ K, while in Ref. 13 the SE resistivity ρ_{xx} and, consequently, σ_{xx} decrease with decreasing temperature to 0.4 K. Therefore, any alternative experimental method of studying the quantum magnetotransport at LT is welcome.

At LT small deviations from the axial symmetry of the experimental cell spoils, in our view, the conventional analysis of the data due to the excitation of low frequency edge magnetoplasmons (EMP). At the same time, the EMP excitation technique can be a tool for studying the quantum magnetotransport [14]. As was shown in a rather general way [15,16], at strong magnetic fields the damping of the EMP is proportional to the longitudinal conductivity. Therefore, the EMP damping measurement can be an alternative way of determining the SE magnetoconductivity.

The electron-ripplon scattering is analogous to the electron scattering by acoustic phonons in solids. There is, however, a substantial difference in the SCBA treatment of SE due to the unusual ripplon dispersion, $\omega_q = (\alpha/\rho)^{1/2} q^{3/2}$ (here α is the surface tension and ρ is the liquid helium mass density). In the semiconductor 2D electron systems the acoustic phonon scattering is usually treated as quasi-elastic [17] since the typical phonon energies are much smaller than Γ. Previous theories of the ripplon-induced quantum magnetotransport, the single-electron theory (Saitoh [18]), and the many-electron theory (Dykman and Khazan (DK) [19]), which were organized in a much more complex way than the conventional SCBA theory, were based on the quasi-elastic approximation. It can be shown that for the electron-ripplon scattering the inelastic parameter $\delta = \hbar\omega_q / \Gamma$ increases with the magnetic field due to $\omega_q \propto q^{3/2} \propto B^{3/4}$ (usually $\Gamma \propto \sqrt{B}$). Additionally, δ increases with decreasing T and soon becomes larger than unity. Therefore, the theory of the LT quantum magnetotranspot of SE should be initially formulated as an inelastic quantum transport theory.

In this paper we report the theoretical concept of the inelastic quantum magnetotransport of the SE on superfluid helium for electron-atom and electron-ripplon scattering, and the LT magnetoconductivity data obtained from the damping coefficient of the EMP. The many-electron effect which is important at rather weak magnetic fields is taken into account by means of the Coulomb correction to the broadening of the single-electron density of states, Γ_C . The theory based on the extended SCBA reproduces the results of the previous approaches as the opposite limiting cases, if the inelastic effect is ignored and if the Landau level broadening is successively reduced to the ripplon-induced broadening $\Gamma \to \Gamma_r$ ($\Gamma_C = 0$; the single electron Saitoh's theory) and to the Coulomb broadening $\Gamma \to \Gamma_C$ ($\Gamma_r = 0$; the DK theory). Under real experimental conditions, Γ transforms continuously from Γ_C to Γ_r with the increase of the magnetic field B and the inelastic effect substantially affects the quantum magnetotransport of SE, reducing both Γ and σ_{xx} . The new LT data of the SE magnetoconductivity as a function of T and B are in good agreement (without any adjusting parameter) with the presented theory of inelastic quantum magnetotransport. The same approach applied as a test to the electron-atom scattering describes previously measured σ_{xx} data, including the effects caused by the electron-electron interaction.

Theoretical concept

Our intention is to describe the quantum magnetotransport of an electron liquid of which the mean potential energy is approximately one hundred times larger than the mean kinetic energy. Under such condition the conventional approaches developed for electrons with weak mutual interaction can scarcely be used. As was shown in Ref. 7, it is natural to assume that the highly correlated electron liquid is in equilibrium in the center-of-mass frame moving along the surface with a drift velocity $\mathbf{u}_d$ in crossed magnetic **B** and electric **E** fields. In this case the substantial simplification of the mathematical formalism appears to be possible, since the conductivity of SE can be expressed in terms of the equilibrium dynamic structure factor of the 2*D* electron liquid [7,8].

Effective collision frequency

We start with the interaction Hamiltonian which allows one to describe the inelastic magnetotransport induced by vapor atoms and ripplons in a similar way:

$$H_{\text{int}} = \sum_{j=a,r} \sum_{\mathbf{q}} U_j \, n^{(\text{el})}_{-\mathbf{q}} \, A_{j,\mathbf{q}} \,, \qquad n^{(\text{el})}_{\mathbf{q}} = \sum_{e} \exp(-i\mathbf{q}\mathbf{r}_e) \,. \tag{1}$$

Here we use j to distinguish the electron-atom interaction ($j = a$) from the electron-ripplon interaction ($j = r$) and introduce the notation

$$A_{r,\mathbf{q}} = b_{\mathbf{q}} + b^{+}_{-\mathbf{q}} \,, \quad A_{a,\mathbf{q}} = \sum_{k} \eta_k \sum_{\mathbf{K}'} a^{+}_{\mathbf{K}'-\mathbf{K}} \, a_{\mathbf{K}} \,, \tag{2}$$

$$U_r = V_q \sqrt{\frac{\hbar q}{2\rho\omega_q}} \,, \quad U_a = \frac{2\pi\hbar^2 s}{m} \,, \quad \eta_k = \langle 1 | e^{ikz} | 1 \rangle \,,$$

$n^{(\text{el})}_{\mathbf{q}}$ is a 2*D* Fourier transform of the electron density; $b^{+}_{\mathbf{q}}$ and $a^{+}_{\mathbf{q}}$ are the creation operators of ripplons and ^{4}He atoms; $\langle 1 \| 1 \rangle$ means the average over the ground surface level; $\mathbf{R} = \{\mathbf{r}, z\}$; $\mathbf{K} = \{\mathbf{q}, k\}$; V_q is the electron-ripplon coupling [20]; s is the electron-atom scattering length, and m is the free electron mass.

According to Refs. 7 and 8, the quantum magnetotransport can be described by the elementary expressions for the conductivity tensor with the field- and density-dependent effective collision frequency $\nu(B, n)$. For a highly correlated 2*D* electron liquid we have

$$\nu = \frac{1}{2m k_B T} \sum_{\mathbf{q}} q^2 \left\{ U_r^2 N^{(r)}_q S_0(q, \omega_q) + \frac{1}{2} U_a^2 \sum_{k} |\eta_k|^2 \sum_{\mathbf{K}'} N^{(a)}_{\mathbf{K}'} \, S_0(q, \Delta\omega_a) \right\} , \tag{3}$$

where $N^{(r)}_q$ and $N^{(a)}_{\mathbf{K}'}$ are the distribution functions of ripplons and vapor atoms;

$$S_0(q, \omega) = N_e^{-1} \int e^{i\omega t} \langle n^{(\text{el})}_{\mathbf{q}}(t) \, n^{(\text{el})}_{-\mathbf{q}}(0) \rangle \, dt$$

is the equilibrium dynamic structure factor; $\hbar\Delta\omega_a = \varepsilon^{(a)}_{\mathbf{K}'} - \varepsilon^{(a)}_{\mathbf{K}'-\mathbf{K}}$ is the energy exchanged as a result of the electron-atom collisions, and N_e is the total number of electrons. In this treatment, the main problem is to find the appropriate approximation for $S_0(q, \omega)$.

It should be pointed out that Eq. (3) would still contain the many-electron effect even if we would use the single-electron approximation for the dynamic structure factor

$$S_0(q, \omega) = \frac{2\hbar}{\pi^2 N_e l^2} \int dE f(E)[1 - f(E + \hbar\omega)] \times \sum_{N,N'} J_{N,N'} \, \text{Im}\, G_N(E) \, \text{Im}\, G_{N'}(E + \hbar\omega) \,. \tag{4}$$

Here l is the magnetic length ($= \sqrt{\hbar c/eB}$); $f(E)$ is the Fermi-distribution function; $J_{N,N'} = |\langle N, X| \times \exp(i\mathbf{q}\mathbf{r}_e)|N', X - q_y l^2\rangle|^2$, and $G_N(E)$ is the single-electron Green's function. In this approximation the theory is the quantum analog of the semi-classical treatment of highly correlated electrons by means of the drift-velocity-shifted distribution function $f_{\mathbf{k}} = f(E - \hbar\mathbf{k}\mathbf{u}_d)$ [6,21].

Following the general idea of Ref. 22, we take additionally into account the many-electron effect as a Coulomb correction to broadening of the single-electron density of states Γ_C . In this picture an electron feels the fluctuation field of others electrons as a random potential, since the density fluctuation has spectral intensities at very low frequencies. Im $G_N(E)$ is therefore assumed to have a semielliptic shape:

$$\text{Im}\, G_N(E) = -\frac{2}{\Gamma_N} \sqrt{1 - [(E - E^*_N)/\Gamma_N]^2} \,, \tag{5}$$

where E^*_N is the central position of the Landau level. The level broadening Γ_N is formed by all the present interactions, including the mutual interaction of the SE. Thus, here the many-electron effect is finally taken into account in two major respects:

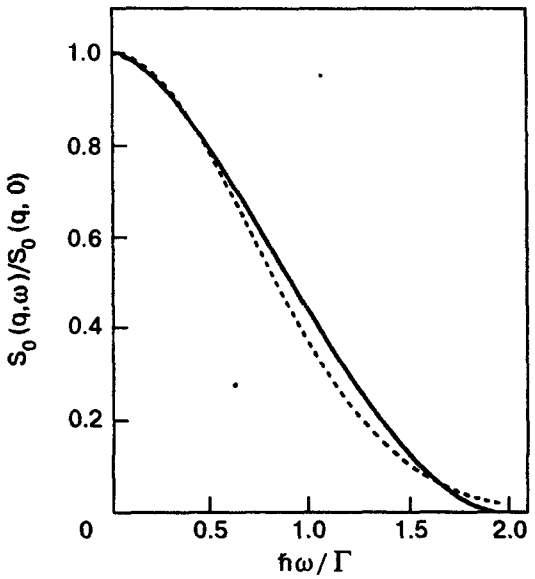

Fig. 1. The dynamic structure factor vs. the inelastic parameter for the semielliptic (solid) and Gaussian (dashed) shapes of the density of states in the ultraquantum limit.

first, high Coulomb correlations form the equilibrium dynamic structure factor included in Eq. (3); secondly, the mutual interaction affects the single electron density of states.

Equation (3)–(5) establish the relationship between the effective collision frequency and the level broadening. In the ultraquantum limit ($N = 0$, $\Gamma_0 \equiv \Gamma$) we have

$$S_0(q,\omega) \simeq \frac{32\hbar}{3\pi\Gamma}\exp\left(-\frac{q^2l^2}{2}\right)\chi\left(\frac{\hbar\omega}{\Gamma}\right), \qquad (6)$$

$$\chi(\delta) = \frac{3}{4}\int_{-1}^{1-\delta}\sqrt{1-x^2}\,\sqrt{1-(x+\delta)^2}\,dx\,.$$

The function $\chi(\delta)$ describes the inelastic effect. For small values of the energy exchanged as a result of a collision, $\hbar\omega << \Gamma$, the quantum magnetotransport can be considered as quasielastic, $\chi \to 1$. At $\hbar\omega \sim \Gamma$, as it is shown in Fig. 1, the inelastic effect substantially reduces $S_0(q, \omega)$ and consequently $\nu(B, n)$.

In general, the inelastic effect is difficult to describe for the electron-vapor atom scattering because of additional integrations over k and $\mathbf{K}'$ which appear in Eq. (3) if $\Delta\omega_a \neq 0$. In this case we can therefore substantially simplify the problem as follows. Usually, $q', k' >> q, k$, and $\Delta\omega_a \simeq \hbar(\mathbf{qq}' + kk')/M$ (here M is the helium atom mass). First, we disregard the term proportional to $\mathbf{q}$, since it is important only in the range of strong magnetic fields where the inelastic effect can be ignored. Taking into account that at $B < 10$ T there is nearly no difference between σ_{xx} calculated for the Gaussian and semielliptic shapes of the density of states [8], we use the more simple Gaussian shape for describing the inelastic effect of the electron-atom scattering:

$$\nu^{(a)} = \nu_0^{(a)}\frac{16}{3\pi}\int_0^\infty \frac{dy}{(1+y^2)^3\sqrt{1+\delta_a^2y^2/2}}\,, \qquad (7)$$

$$\delta_a = \frac{4\hbar\gamma}{\Gamma}\sqrt{\frac{k_BT}{M}}\,.$$

Here δ_a is the inelastic parameter for the electron-atom scattering,

$$\nu_0^{(a)} = \frac{\sqrt{\pi}\,\omega_c\Gamma_{a,0}^2}{4\Gamma k_BT}\exp\left[-\left(\frac{\Gamma}{4k_BT}\right)^2\right]\coth\left(\frac{\hbar\omega_c}{2k_BT}\right), \qquad (8)$$

γ is the parameter of the SE wave function $\langle 1|z\rangle \propto z\exp(-\gamma z)$, which increases with the holding electric field $E_\perp$. As usual, we combined the interaction parameters into $\Gamma_{a,0}$, which is equal to the Landau level broadening caused by the electron-atom interactions only, $\Gamma_{a,0} = \hbar\sqrt{(2/\pi)\omega_c\nu(0)}$. It should be pointed out, however, that here $\Gamma_{a,0}$ does not originate from the density of states, and differs from Γ due to other interactions. In this form, the effective collision frequency has the required analytical behavior at $\Gamma \to 0$ ($\nu \propto 1/\Gamma \to \infty$), which proves additionally the importance of the self-consistent broadening of the Landau levels.

In the case of the electron-ripplon scattering, the inelastic parameter is of a very simple form, $\delta_r = \hbar\omega_0/\Gamma$ [here $\omega_0^2 = 2^{3/2}\alpha/(\rho l^3)$] and $\nu^{(r)}$ has no additional integrations typical of the electron-atom scattering. Therefore, Eq. (6) can be directly insert into

$$\nu^{(r)}(B, n) = \frac{1}{8\pi m\alpha}\int_0^\infty qV_q^2S_0(q, \omega_q)dq\,. \qquad (9)$$

Landau level broadening

According to general rules of the many-particle physics [23], if the Coulomb interaction is neglected, the SE self-energy can be written as

$$\Sigma_N(E) = i \sum_{\mathbf{q}} \sum_{N'} J_{N,N'} \int \frac{d\omega}{2\pi} G_{N'}(E - \hbar\omega) \times$$

$$\times \sum_{j=a,r} U_j^2 D_j(q, \omega) . \qquad (10)$$

Here we introduced the correlators

$$D_j(q, t - t') = -i \langle T[A_{j,\mathbf{q}}(t) A_{j,-\mathbf{q}}(t')] \rangle ,$$

which are similar for the two kinds of electron scattering ($j = a, r$).

It is easy to see that in the limit $\hbar\omega << E$, Eq. (10) reduces to $\Sigma_N = \frac{1}{4} \Gamma_N^2 G_N(E)$ and the broadening can be found self-consistently by using Dyson's equation. In this case Im $G_N(E)$ has a semielliptic shape, with the level broadening $\Gamma = (\Gamma_{a,0}^2 + \Gamma_{r,0}^2)^{1/2}$. Here $\Gamma_{a,0}$ is the usual vapor-atom-induced broadening, and $\Gamma_{r,0}$ is the ripplon-induced, quasielastic broadening, which is twice as large as a result of the qualitative analysis of Ref. 24:

$$\Gamma_{r,0}^2 = \frac{\Lambda_0^2 k_B T}{\pi \alpha l^4} I^* , \quad I^* = \int_0^\infty W^2(x) e^{-x} dx/x . \qquad (11)$$

Here we use the notation

$$W(x) = x w\left(\frac{x}{2\gamma^2 l^2}\right) + \frac{eE_\perp l^2}{\Lambda_0} ; \quad \Lambda_0 = \frac{e^2(\varepsilon - 1)}{4(\varepsilon + 1)} ;$$

$$w(y) = -\frac{1}{1-y} + \frac{1}{(1-y)^{3/2}} \ln\left[\frac{(1 + \sqrt{1-y}}{\sqrt{y}}\right] ;$$

ε is the dielectric constant. Since the elastic broadening is caused by ripplons $\Gamma_{r,0} \propto \sqrt{T}$, the effective collision frequency $\nu \propto 1/\Gamma_{r,0}$ reproduces the result of the previous single-electron theory [18]: $\sigma_{xx} \propto 1/\sqrt{T}$.

In Eq. (8) the term proportional to $E_\perp^2$ has a logarithmic divergence for small q. We should cut it off at wave vectors, for which the approximations made above fails (for instance, it can be $q \approx \sqrt{\pi n}$). Still, in the LT limit which we are considering the only low electron densities, $n < 5 \cdot 10^7$ cm^{-2}, are important and this term can be neglected at $B > 1$ T.

In the inelastic theory the situation is much more complicated. Im $G_N(E)$ is nonetheless assumed to be of a sharp semielliptic shape with the level broadening defined as $\Gamma_N = -2\text{Im}\, \Sigma_N(E_N^*)$. Usually, $N_q^{(r)} >> 1$; in this case the ripplon Green's function $D_r(q, \omega)$ and $D_a(q, \omega)$ have similar structures:

$$D_r(q, \omega) = -2\pi i N_q^{(r)} [\delta(\omega - \omega_q) + \delta(\omega + \omega_q)] , \qquad (12)$$

$$D_a(q, \omega) = -2\pi i \sum_k |\eta_k|^2 \sum_{\mathbf{K}'} N_{\mathbf{K}}^{(a)} \delta(\omega + \Delta\omega_a) .$$

Therefore, the self-consistent equation for the level broadening can be written as

$$\Gamma_N = 2 \sum_{\mathbf{q}} \sum_{N'} J_{N,N'} \int \frac{d\omega}{2\pi} \text{Im}\, G_{N'}(E - \hbar\omega) \times$$

$$\times \sum_{j=a,r} U_j^2 \text{Im}\, D_j(q, \omega) . \qquad (13)$$

From this equation it can be seen that the mixing of the Landau levels can be ignored ($N' = N$) if $\Gamma_N << \hbar\omega_c$ (which we assume). At the same time, it follows that the inelastic effect changes the level broadening if $\hbar\omega \sim \Gamma_N$.

In the presence of the Coulomb correction to the broadening of the single-electron density of states Γ_C , the self-consistent equation for the total level broadening Γ can be written as

$$\Gamma^2 = \Gamma_C^2 + \Gamma_{a,0}^2 Y_a(\delta_a) + \Gamma_{r,0}^2 Y_r(\delta_r) . \qquad (14)$$

Here δ_a and δ_r are the inelastic parameters for the two kinds of electron scattering;

$$Y_a(\delta) = \frac{16}{3\pi} \int_0^\infty \frac{dy}{[1 + y^2]^3 \sqrt{1 + \delta^2 y^2}} ;$$

$$Y_r(\delta) = \frac{1}{I^*} \int_0^\infty e^{-x\sqrt{1-\delta^2 x^{3/2}}} W^2(x) \theta(1 - \delta^2 x^{3/2}) \frac{dx}{x} ;$$

and $\theta(x)$ is the unit step function. Since the parameters δ_r and δ_a depend on Γ, Eq. (14) is a transcendental equation whose solution as a function of basic parameters T, B, and n should be found numerically.

Regarding the Coulomb broadening, we will use the results of Refs. 9 and 19 ($\Gamma_C = \hbar/\tau_e = eE_f l$), where the fluctuating electric fields were calculated

$$E_f \approx 0.84 \left(\frac{4\pi k_B T n^{3/2}}{\varepsilon}\right)^{1/2} .$$

Since the ripplon or vapor-atom-induced broadening increases with the magnetic field (which is approximately proportional to $\sqrt{B}$) while $\Gamma_C \propto$

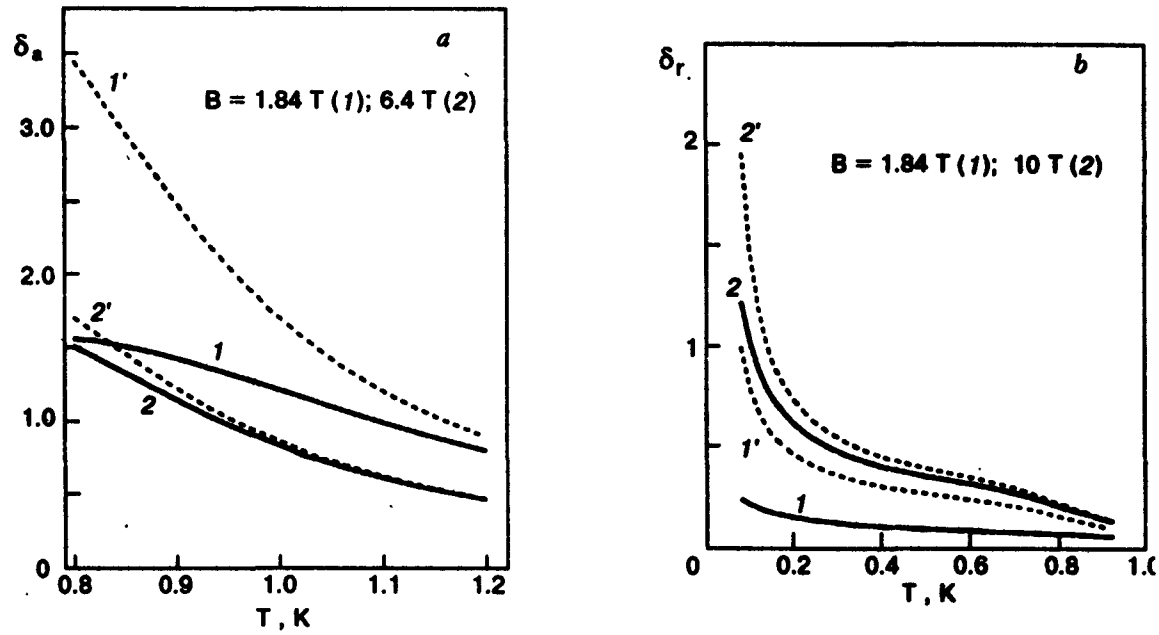

Fig. 2. The inelastic parameters δ_a (*a*) and δ_r (*b*) vs. temperature for $n = 3.5 \cdot 10^7$ cm^{-2} and two values of the magnetic field: the many-electron theory (solid); the single-electron theory (dashed).

$\propto 1/\sqrt{B}$, the Coulomb correlations affect the quantum magnetotransport only at weak magnetic fields and high electron densities.

It is instructive to plot the inelastic parameters as functions of T for different values of the magnetic field, as it is done in Fig. 2. For the electron-atom scattering it follows (Fig. 2,*a*) that at temperatures of about 1 K, where the many-electron effect is usually studied [9,10], the Coulomb correlations substantially reduce the inelastic parameter. Still, the single-electron approximation (dashed curves) and many-electron theory (solid curves) give $\delta_a \sim 1$. According to Fig. 1, at such δ_a we could expect a large decrease in Γ and $\nu^{(a)}$. Regarding the level broadening and effective collision frequency induced by the electron-atom interactions, the inelastic effect nonetheless turns out to be less important due to the additional integrations over k and $\mathbf{K}'$ in Eqs. (3) and (12). As it is shown in Figs. 3 and 4, the many-electron effect and the contribution of ripplons to the level broadening prevent a strong decrease in Γ and $\nu^{(a)}$ at weak magnetic fields and $T < 1$ K.

The situation is different for the electron-ripplon scattering where Γ_C and $\Gamma_{r,0}$ have the same T-dependences: Γ_C, $\Gamma_{r,0} \propto \sqrt{T}$. The inelastic parameter δ_r increases with decreasing temperature and the many-electron effect cannot suppress the increase at strong magnetic fields, as is clearly seen in Fig. 2,*b*. In this case there is no additional integration except for the integration over q [Eq. (9)]. Therefore, the inelastic effect should crucially affect the quantum

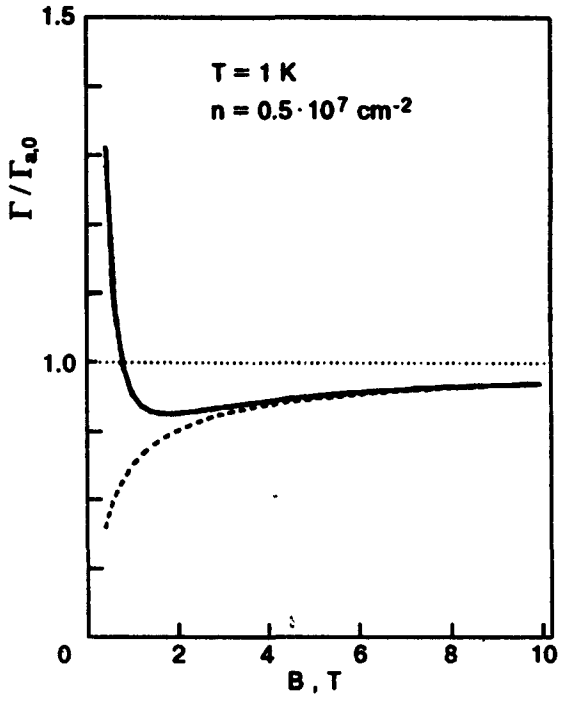

Fig. 3. The Landau level broadening Γ as a function of the magnetic field: the many-electron theory (solid); the single-electron theory (dashed).

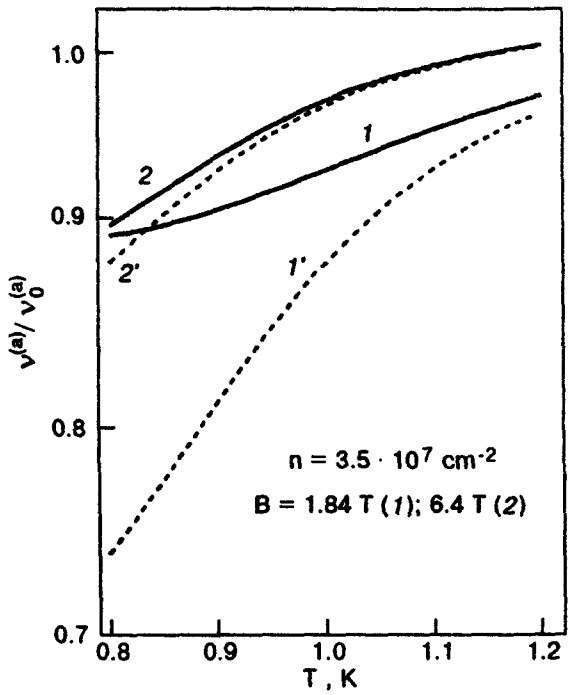

Fig. 4. The temperature dependence of the inelastic factor $\nu^{(a)}/\nu_0^{(a)}$ for two values of the magnetic field: the many-electron theory (solid); the single-electron theory (dashed).

cially affect the quantum magnetotransport of SE, which changes the T-dependences of Γ and σ_{xx} at LT.

Experimental method

As was stated in the introduction, our experimental method of determining σ_{xx} is based on the general theoretical conclusions of Refs. 15 and 16 that at strong enough magnetic fields the damping coefficient of EMP is proportional to the longitudinal conductivity (usually it is valid at $B > 0.5$ T). In this case the proportionality constant can be considered as a geometrical factor which is independent of B and T. Therefore, at fixed n the EMP damping can be used for determining temperature and magnetic field dependences of the SE magnetoconductivity. The edge phenomena for SE are smoothed over large distances, which assures that the data correspond to the real SE conductivity.

To study EMP damping we will use the conventional experimental technique which is similar to the one described in Ref. 25. The electron sheet employed in the present work has a circular geometry with a diameter of 30 mm, which is shaped by an electrode assembly. The electrode assembly consists of a circular disk of 10 mm in diameter and four surrounding arc-shaped outer electrodes. The total diameter of the assembly is 30 mm. The electrodes were immersed in liquid helium, 1.0 mm under the surface. The width of the edge of the electron disk was estimated to be 0.3 mm. The resonance curve was obtained by sweeping the frequency of an ac excitation voltage, which was applied to one of the four surrounding electrodes. The output signal from the opposite electrode was analyzed by a two-phase lock-in. The magnetic field and temperature were held constant during the measurement of each resonance curve. The electron density was fixed at $n = 3.5 \cdot 10^7$ cm^{-2}.

Since the first mode was contaminated with a low-frequency tail and unfavorable noise, the second mode with a higher wave number was used to obtain the damping coefficient by fitting to the Lorentzian. Above 1 K the magnetic field dependence of the damping coefficient was found to be the same as the B-dependence of the previously studied longitudinal conductivity of SE. Therefore, we have determined the geometrical factor which gives the relationship between the damping coefficient and σ_{xx} at $T = 1.1$ K, where the magnetoconductivity is well understood, both theoretically and experimentally, while the ripplon contribution and inelastic effect (according to Fig. 4) can be disregarded. This factor was then used in the ripplon scattering regime.

It should be noted that the driving amplitude had to be kept low in order to avoid the nonlinear distortion of the EMP line shape.

Results and discussion

To check that the many-electron effect is taken into account correctly in the theory presented above, we must first apply the theory to the high-temperature regime, $T = 1.2$ K. The many-electron theory (solid curve) and the single-electron theory (dashed curve) results are shown in Fig. 5 for two electron densities. It can be seen that the many-electron effect can be ignored at low densities ($n < 0.5 \cdot 10^8$ cm^{-2}). At substantially higher density $n = 3.2 \cdot 10^8$ cm^{-2} and $B \leq 5$ T, the Coulomb correction to the Landau level broadening affects the field dependence of σ_{xx} . The experimental data of Ref. 26 for the SE conductance measured for $n = 3.2 \cdot 10^8$ cm^{-2} at $T = 1.2$ K are plotted in Fig. 5. The conductance is proportional to σ_{xx} with the numerical factor of the order of unity unknown for the experiment. We have chosen the proportionality constant to fit the experimental data at extremely high magnetic fields ($B = 20$ T), where the many-electron effect can be neglected. The data and the theory will then still in good agreement in the magnetic field range where the many-electron effect

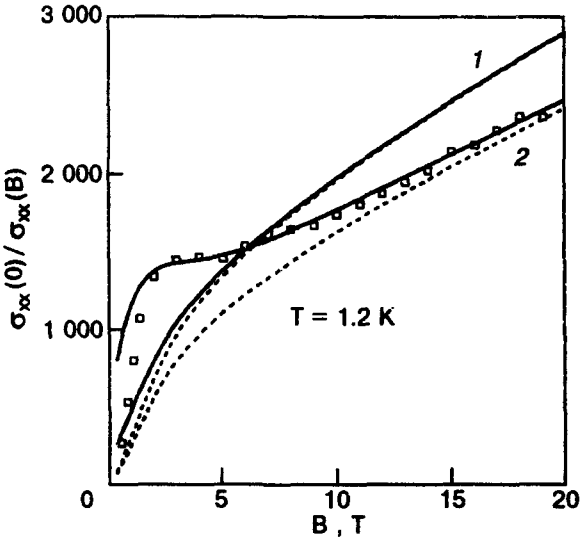

Fig. 5. $\sigma_{xx}(0)/\sigma_{xx}(B)$ vs. B for $n = 0.5 \cdot 10^8$ cm^{-2} (*1*) and $n = 3.2 \cdot 10^8$ cm^{-2} (*2*) at $T = 1.2$ K. The lines show the many-electron theory (solid) and single-electron theory (dashed). Data (open squares) are taken from Ref. 26.

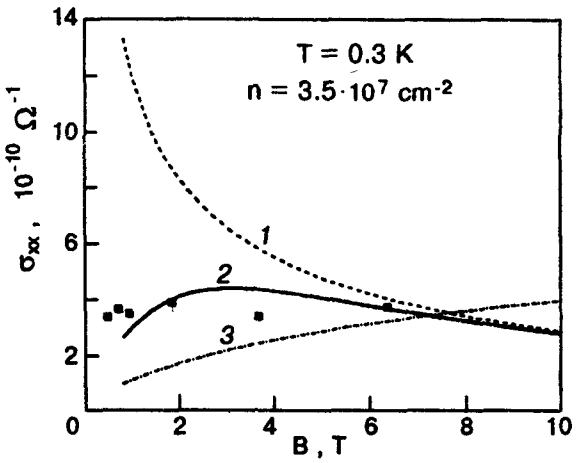

Fig. 6. The magnetoconductivity of SE vs. B at $T = 0.3$ K: the single-electron approximation (*1*); the many-electron theory reported (*2*); the DK many-electron theory (*3*). Solid squares are the experimental data found from the EMP damping.

validity of the approximation made for the dynamic structure factor $S_0(q, \omega)$ of a highly correlated $2D$ electron liquid. It is instructive to note that in the high temperature range and $B \geq 1$ T, the quantum magnetotransport of SE can be described by using the simple many-electron correction to the level broadening $\Gamma = (\Gamma_{a,0}^2 + \Gamma_C^2)^{1/2}$ in a single-electron expression for $\nu(B)$ presented in the form of Eq. (8).

In the ripplon scattering regime, the magnetic field dependence of σ_{xx} is shown in Fig. 6 for different approaches. It follows that the single-electron approximation ($\Gamma_C = 0$) fails to describe the field dependence of σ_{xx} obtained from the damping coefficient of EMP (solid squares). At the same time, the many-electron DK theory results (curve *3*) cannot be fitted to the data. The theory presented here reproduces the B-dependence of the DK theory, if the ripplon contribution to the level broadening and inelastic effect are ignored: $\Gamma \to \Gamma_C$, $\delta_r \to 0$. Still, in this case, our theory gives σ_{xx} values which are approximately 2.6 times higher than the result of the DK theory. This is attributable to another way of treatment of the magnetoconductivity of the highly correlated electron liquid used here. It should be pointed out that the factor 2.6 is very important for describing the experimental data. Indeed, it is impossible to fit the data and the DK theory by just replacing $\Gamma_C \equiv \equiv \hbar/\tau_e$ by $\Gamma = (\Gamma_{r,0}^2 + \Gamma_C^2)^{1/2}$ in the final conductivity equation, since it would reduce $\sigma_{xx} \propto 1/\Gamma_C$ from the data. Therefore, we conclude that the method proposed by us is a more adequate way of treating the conductivity of highly correlated $2D$ electron liquid.

The solid curve in Fig. 6 calculated on the basis of theory of this paper without a fitting parameter is in good agreement with the new experimental data. In the wide range of magnetic fields σ_{xx} has a very weak B-dependence because of the interplay of the different contributions (Γ_C and $\Gamma_{r,0}$) to the level broadening. The deviation from the data seen at $B \leq 1$ T is beyond the validity of the approximations used by us (at such fields the mixing of Landau levels becomes important). The curve approaches the single-electron curve (curve *1*) in the limit of strong magnetic field, since $\Gamma_C \propto \propto 1/\sqrt{B} \to 0$. This is reasonable despite the large value of the Coulomb coupling parameter $G \approx 60$, since the typical wave vectors in the dynamic structure factor $q \propto \sqrt{B}$ are much larger than $\sqrt{\pi n}$ [according to Ref. 27, in this limit $S_0(q, \omega)$ is nearly the same as that of an ideal electron gas].

The agreement between the theory and the σ_{xx} data looks even more convincing in Fig. 7, where the results are plotted as a function of temperature for three different values of the magnetic field used in the experiment. The increase of σ_{xx} with decreasing temperature to 0.2 K is consistent with the prediction of the quasielastic theory (Saitoh [18]): $\sigma_{xx} \propto 1/\sqrt{T}$. At lower temperatures $T \leq 0.2$ K, the many-electron theory curves (solid) and the data deviate from the result of the quasielastic approximation — the stronger the magnetic field, the more SE magnetoconductivity deviates due to the inelastic effect. With the increase of the magnetic field,

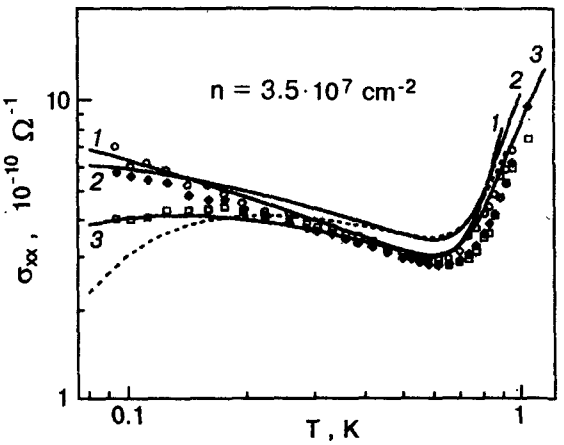

Fig. 7. σ_{xx} vs. T for three values of the magnetic field: $B = 1.84$ T [curve *1*, ○]; $B = 3.67$ T [curve *2*, ◆]; $B = 6.4$ T [curve *3*, □]. The many-lectron theory (solid) and the single electron theory (dashed; $B = 6.4$ T) have no fitting parameters.

the curves and the data gradually approach the single-electron curve (dashed) plotted for the strongest magnetic field used in the experiment. The high-temperature deviation of the weak-field curves from the data might be caused by the increase of the population of higher Landau levels, which was disregarded in the electron-ripplon scattering.

The temperature dependence of σ_{xx} is the most decisive factor for the electron-ripplon scattering regime. Therefore, it is important to compare the previously measured $\sigma_{xx}(T)$ data with the data reported here. It should be noted that the temperature dependence observed in Ref. 12 at $T > 0.5$ K is similar to the one shown in Fig. 7. Still, the minimum of σ_{xx} was situated at higher $T \approx 1$ K, and it was necessary to substantially reduce the level broadening $\Gamma \to \Gamma_{a,0}$ to fit the data. Regarding the $\rho_{xx}(T)$ data reported in Ref. 13 (ρ_{xx} and consequently σ_{xx} decrease with decreasing T), they are in contradiction with available theories of ripplon-induced magnetoconductivity including the DK theory.

Conclusions

We have investigated the inelastic quantum magnetotransport in a highly correlated 2*D* electron liquid of SE on superfliud helium. The theoretical concept presented and the magnetoconductivity data obtained from the damping of the EMP show that the usually used quasielastic approximation is valid only in a limited temperature range which narrows with increasing magnetic field. The inelastic effect which is important at LT drastically reduces the Landau level broadening and magnetoconductivity. The many-electron theory reported here reproduces the results of previous quasielastic theories [18,19] as the opposite limiting cases which cannot be separately applied to the real experimental situation.

We have shown that the EMP damping method of measuring the SE magnetoconductivity at LT can be an alternative to the conventional methods based on measuring the electron response to the ac voltage by means of the capacitive coupling techniques.

The perfect agreement achieved between the theory and experiment in the wide range of temperatures (including the vapor atom scattering regime) and magnetic fields supports the idea of the Coulomb correction to the broadening of the single-electron density of states and provides important clues about the behavior of highly correlated ($G \sim 100$) 2*D* electron liquids in quantizing magnetic fields.

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