Fizika Nizkikh Temperatur, 1997, v. 23, Nos. 5/6, p. 546–552

Orbital anisotropy of magnetically distorted superfluid ^{3}He-*B*

A. Gongadze, G. Kharadze, and G. Vachnadze

Institute of Physics, 6 Tamarashvili st., 380077 Tbilisi, Georgia
E-mail:khar@physics.iberiapac.ge

Submitted November 18, 1996

The orbital anisotropy of the magnetized superfluid *B*-phase of liquid ^{3}He is investigated theoretically for arbitrary fields and temperatures. The behavior of freely rotating *B*-phase under the action of magnetic field is considered.

PACS: 67.57.Fg; 67.57.De

1. Introduction

Ultralow-temperature superfluid phases of liquid ^{3}He are ordered states of a Fermi system which have lost the series of symmetries appropriate to the normal state of ^{3}He. The order parameters of superfluid *A*- and *B*-phases are complicated multidimensional objects with nontrivial topological structures that generate rich properties of the Bose-condensate of Cooper pairs with spins $S = 1$ and internal orbital moments $L = 1$ (Ref. 1). Along with the amplitudes that characterize the amount of the Bose-condensation energy, the order parameters of the superfluid phases of ^{3}He depend on the Goldstone variables (overall phase and various angles specifying orientation of spin and orbital degrees of freedom of Cooper pairs).

The external magnetic field has a strong influence on the properties of superfluid phases. Especially susceptible in this respect is the initially isotropic *B*-phase, which is unable (in contrast to the *A*-phase) to adjust to the applied magnetic field at the expense of the reorientation of the Goldstone degrees of freedom, thereby saving the condensation energy. Instead, the *B*-phase exhibits strong distortion even at moderately high magnetic fields that lose part of the condensation energy (at a given temperature and pressure): its longitudinal «gap» $\Delta_{||}$ along the direction of the field is suppressed [2,3] and the superfluid state acquires magnetic anisotropy. It is important that because of the relative spin-orbit coherence of the *B*-phase, the appearance of the magnetic anisotropy axis $\hat{\mathbf{h}}$ (along the direction of the field **H**) generates uniaxial anisotropy of the orbital properties of the magnetized ^{3}He-*B* along the axis $\hat{\mathbf{l}}_B = \hat{\mathbf{h}}\overleftrightarrow{R}$, where $\overleftrightarrow{R}$ is the matrix of 3*D* rotations of the spin space with respect to the orbital space. In particular, the energy spectrum of fermionic excitations becomes anisotropic so that the normal (as well as superfluid) component density exhibits tensor character.

At present, there is a considerable amount of experimental information concerning diverse properties of strongly magnetized *B*-phase. Acoustic measurements allowed us to observe directly the suppression of $\Delta_{||}$ by the external magnetic field [4,5] and to investigate orbital anisotropy of the magnetized stationary and rotating ^{3}He-*B* [6]. The anisotropic nature of magnetically distorted *B*-phase was observed also by means of the NMR techniques [7,8] and by measuring the ion mobility [9].

There is satisfactory understanding of the properties of ^{3}He-*B* in the case where the anisotropy parameter $\delta_B = (\Delta_\perp - \Delta_{||})/\Delta_\perp$ is small ($\delta_B << 1$). On the other hand, the magnetic distortion of the *B*-phase is very pronounced near the *B* to *A* transition (at low pressures), where $\Delta_{||} << \Delta_\perp$ (when the transformation is close to a continuous transformation). In this region there is still a gap in the detailed theoretical description of the behavior of ^{3}He-*B*. One of the goals of this study is the elaboration of the theoretical background for the interpretation of the properties of strongly distorted ($\delta_B \lesssim 1$) *B*-phase of superfluid ^{3}He. In Sec. 2 we consider the anisotropy of the superfluid density of the magnetized ^{3}He-*B* in the (T, H) plane. The results are used to construct T and H dependences of the dipolar velocity v_D in Sec. 3. The effect of the magnetic-field-dependent orbital anisotropy on the behavior of the freely rotating B-phase is analyzed in Sec. 4.

2. Anisotropy of the superfluid density of magnetized ^{3}He-B

Uniaxial orbital anisotropy of ^{3}He-B in the flow effects manifests itself, in the first place, in the tensorial character of superfluid density (below we drop the subscript at $\hat{\mathbf{l}}_B$):

$$\rho_{ij}^{(S)} = \rho_{||}^{(S)}\hat{l}_i\hat{l}_j + \rho_{\perp}^{(S)}(\delta_{ij} - \hat{l}_i\hat{l}_j)\ , \qquad (2.1)$$

so that in the presence of a superflow with the velocity $\mathbf{v}_S$ an $\hat{\mathbf{l}}$-dependent contribution in the kinetic energy density appears:

$$F_{\rm flow}^{\rm (an)} = -\frac{1}{2}\,\delta\rho_{\rm an}^{(S)}(\hat{\mathbf{l}}\mathbf{v}_s)^2\ , \qquad \delta\rho_{\rm (an)}^{(S)} = \rho_{\perp}^{(S)} - \rho_{||}^{(S)}\ . \qquad (2.2)$$

Anisotropic contribution (2.2) was extensively used to interpret peculiar properties of the rotating magnetized ^{3}He-B [7]. In the vortex-free state, which can be easily achieved before the formation of an equilibrium vortex lattice, large counterflows of normal and superfluid components have a pronounced influence on the $\hat{\mathbf{l}}$-field texture through the anisotropic interaction (2.2). After the equilibrium vortex state is established large counterflows are eliminated but the anisotropic interaction still survives due to the presence of superflows that circulate around individual, quantized, singular vortices (see, for example, Ref.10).

Anisotropic part of the superfluid density $\delta\rho_{\rm an}^{(S)}(T, H)$ of magnetically distorted B-phase is an even function of the applied magnetic field and in the low field limit is proportional to H^2. On the other hand, in the case where the magnetic field strongly deforms ^{3}He-B order parameter (especially near the $B \to A$ phase transition at low pressures) one should expect to observe a pronounced deviation of $\delta\rho_{\rm an}^{(S)}$ from linearity in H^2. At the same time, the temperature dependence of $\delta\rho_{\rm an}^{(S)}$ for the strong field case must be established.

Since $\delta\rho_{\rm an}^{(S)}$ is an equilibrium property of magnetized B-phase, its (T, H)-behavior is completely determined by the structure of the excitation spectrum $E_{k\sigma}$ of quasiparticles with momentum $\mathbf{k}$ and spin projection $(1/2)\sigma = \pm\,(1/2))$ (of course, the Fermi-liquid effects must be incorporated in a proper way).

We start with a standard expression for the normal component density tensor (disregarding for the moment the Fermi liquid corrections),

$$\rho_{ij}^{(n)} = \frac{1}{m}\sum_{k\sigma} k_i\, k_j(-\partial f/\partial E_{k\sigma}) = \rho Y_{ij}(T)\ , \qquad (2.3)$$

where $f(E)$ is the Fermi distribution and

$$Y_{ij}(T) = 3\,\langle\, \hat{k}_i\hat{k}_j Y(\,\hat{k};\,T)\,\rangle \qquad (2.4)$$

with the generalized Yosida function

$$Y(\hat{k};\,T) = \frac{1}{2}\sum_{\sigma}\int_{-\infty}^{\infty} d\xi_k(-\partial f/\partial E_{k\sigma}) = \frac{1}{8T}\sum_{\sigma}\int_{-\infty}^{\infty}\frac{d\xi_k}{\cosh^2(E_{k\sigma}/2T)}\ . \qquad (2.5)$$

In (2.4) the angle brackets denote averaging over the position on the Fermi surface and in (2.5) $\xi_k = (k^2 - k_F^2)/2m$.

Using (2.3), we conclude that

$$\delta\rho_{\rm (an)}^{(S)}/\rho = (\rho_{||}^{(n)} - \rho_{\perp}^{(n)})/\rho = \frac{3}{2}\,\frac{1}{8T}\sum_{\sigma}\int_{-\infty}^{\infty} d\xi_k\,\langle\,\frac{3(\,\hat{\mathbf{k}}\hat{\mathbf{l}}\,)^2 - 1}{\cosh^2(E_{k\sigma}/2T)}\,\rangle\ . \qquad (2.6)$$

We now must use the explicit form of $E_{k\sigma}$ for the magnetized ^{3}He-B. The order parameter of this superfluid state is described by the bivector $A_{\mu i} = \Delta_{\mu\nu}R_{\nu i}\exp(i\Phi)$, where the uniaxial «gap» tensor is

$$\Delta_{\mu\nu} = \Delta_{||}\hat{h}_\mu\hat{h}_\nu + \Delta_{\perp}(\delta_{\mu\nu} - \hat{h}_\mu\hat{h}_\nu)\ . \qquad (2.7)$$

The (T, H)-dependence of $\Delta_{||}$ and $\Delta_{\perp}$ was extensively studied theoretically [2,3] using the set of Gorkov equations for the spin-triplet p-wave superfluid. The fermions excitation spectrum of the magnetized B-phase is given by

$$E_{k\sigma}^2 = \left(\sqrt{\xi_k^2 + \Delta_{||}^2(\,\hat{\mathbf{k}}\hat{\mathbf{l}}\,)^2} + \frac{1}{2}\,\sigma\omega_0\right)^2 + \Delta_{\perp}^2(\,\hat{\mathbf{k}}\times\hat{\mathbf{l}}\,)^2 = \left(|\xi_k| + \frac{1}{2}\,\sigma\omega_0\right)^2 + |\Delta(\,\hat{\mathbf{k}}\,)|^2 + \sigma\omega_0\left(\sqrt{\xi_k^2 + \Delta_{||}^2(\,\hat{\mathbf{k}}\hat{\mathbf{l}}\,)^2} - |\xi_k|\right), \qquad (2.8)$$

where $\omega_0 = gH$ is the Larmor frequency of ^{3}He nuclear magnetic moments, and

$$\left|\Delta(\hat{\mathbf{k}})\right|^2 = \Delta_{\|}^2(\hat{\mathbf{k}}\,\hat{\mathbf{l}})^2 + \Delta_{\perp}^2(\hat{\mathbf{k}}\times\hat{\mathbf{l}})^2 = = \Delta_{\perp}^2\left[1 - \delta_B(2-\delta_B)(\hat{\mathbf{k}}\hat{\mathbf{l}})^2\right]. \quad (2.9)$$

Inspection of the second and third lines in (2.8) shows that the influence of the Zeeman slitting on the quasiparticle spectrum is twofold: the presence of ω_0, renormalizes the Fermi energy (via the first term) and changes the character of the dispersion relation (via the last term). Of course, ω_0, appears implicitly in $|\Delta|^2$ through the anisotropy parameter $\delta_B = \delta_B(\omega_0)$. In the low field limit δ_B is proportional to ω_0^2/Δ_0^2 where $\Delta_0(T)$ denotes the energy gap in the excitation spectrum of the zero-field isotropic B-phase.

Since in all practical cases $\omega_0 \ll \varepsilon_F$, the field renormalization of the Fermi energy is negligible. If in addition $\Delta_{\|} \ll T$, we can use a simple BCS-type dispersion relation,

$$E_{k\sigma}^2 \simeq \xi_k^2 + |\Delta(\hat{k})|^2 \quad (2.10)$$

with an anisotropic gap given by (2.9).

In Ref. 7, $\delta\rho_{\rm an}^{(S)}$ was estimated in the low field limit ($\omega_0 \ll \Delta_0$, $\delta_B \ll 1$) using the apporoximate expression (2.10) for $E_{k\sigma}$. As we have seen above, this consideration is justifiable for the case with $\Delta_{\|} \ll T$. More generally, an exact dispersion relation (2.8) should be considered. For the low field case the expansion of $Y(\hat{k}; T)$ to the lowest order in δ_B and ω_0^2 gives

$$Y(\hat{k}; T) \simeq Y(T) + a(T)(\hat{\mathbf{k}}\hat{\mathbf{l}})^2, \quad (2.11)$$

where the anisotropic contribution is described by

$$a(T) = \delta_B Z(T) + (\omega_0^2/\Delta_0^2)\tilde{Z}(T) \quad (2.12)$$

with

$$Z(T) = (\Delta_0/2T)^2 \int_{-\infty}^{\infty} \frac{\tanh(E_k^0/2T)\,d\xi_k}{\cosh^2(E_k^0/2T)E_k^0}, \quad (2.13)$$

$$\tilde{Z}(T) = \frac{1}{4}\left(\Delta_0/2T\right)^4 \int_{-\infty}^{\infty} \frac{3\tanh(E_k^0/2T) - 1}{\cosh^2(E_k^0/2T)}\left(\frac{T}{E_k^0}\right)\frac{d\xi_k}{E_k^0},$$

and $E_k^0 = \sqrt{\xi_k^2 + \Delta_0^2}$. Finally, from (2.3) and (2.4) it follows that in the low magnetic fields

$$\delta\rho_{\rm an}^{(S)}/\rho \simeq \frac{2}{5}\left[\delta_B Z(T) + (\omega_0^2/\Delta_0^2)\tilde{Z}(T)\right]. \quad (2.14)$$

When using the dispersion relation (2.10) the $\tilde{Z}$-contribution to $\delta\rho_{\rm an}^{(S)}$ is lost. Although near T_c ($\Delta_0 \ll T$) this term is negligible ($\tilde{Z} \ll Z$), on lowering the temperature it becomes increasingly important. This fact was noticed in Ref. 8, where $\delta\rho_{\rm an}^{(S)}$ was calculated for the low field limit for the magnetic energy density

$$F_{\rm mag}^{(S)} = \frac{1}{2}\chi_{\mu\nu}^{(S)} H_\mu H_\nu, \quad (2.15)$$

where $\chi_{\mu\nu}^{(S)}$ is the tensor of the magnetic susceptibility of ^{3}He-B in the presence of superflow with velocity $\mathbf{v}_s$. Because of above-mentioned spin-orbit coherence of ^{3}He-B, the presence of preferred direction in orbital space along $\hat{v}_s = \mathbf{v}_s/v_s$ induces a uniaxial magnetic anisotropy:

$$\chi_{\mu\nu}^{(S)} = \chi_{\|}^{(S)}\hat{s}_\mu\hat{s}_\nu + \chi_{\perp}^{(S)}(\delta_{\mu\nu} - \hat{s}_\mu\hat{s}_\nu), \quad (2.16)$$

where $\hat{s}_\mu = R_{\mu i}\cdot\hat{v}_{si}$. As a result, an anisotropy contribution appears in (2.15):

$$F_{\rm mag}^{(S)} = -\frac{1}{2}\,\delta\chi_{\rm an}^{(S)}(\hat{s}\,\mathbf{H})^2 = -\frac{1}{2}(\delta\chi_{\rm an}^{(S)}H^2)\,(\hat{h}_\mu R_{\mu i}\,\hat{v}_{si})^2, \quad (2.17)$$

with $\delta\chi_{\rm an}^{(S)} = \chi_{\perp}^{(S)} - \chi_{\|}^{(S)}$. One the other hand, conceptually the same contribution to the spin-orbit anisotropy energy density is contained in Eq. (2.2), from which

$$F_{\rm flow}^{(\rm an)} = -\frac{1}{2}\left(\delta\rho_{\rm an}^{(S)}v_s^2\right)(\hat{h}_\mu R_{\mu i}\,\hat{v}_{si})^2. \quad (2.18)$$

Equating (2.17) and (2.18), we find the relation

$$\delta\rho_{\rm an}^{(S)} = (H^2/v_s^2)\,\delta\chi_{\rm an}^{(S)}, \quad (2.19)$$

which is an alternative way to calculate $\delta\rho_{\rm an}^{(S)}$. Using (2.19), it can be shown (see Ref. 8) that in the lowest order in H^2

$$\delta\rho_{\rm an}^{(S)}/\rho = \frac{1}{2}\left(\frac{\omega_0^2}{\Delta_0^2}\right)\left[\left(1 - \frac{3}{5}\frac{Z_5}{Z_3}\right)\times \times\left(Z_3 - \frac{3}{2}Z_5\right) + \frac{6}{5}\left(Z_5 - \frac{5}{4}Z_7\right)\right], \quad (2.20)$$

where

$$Z_n(T) = \pi T\sum_{\omega}\frac{\Delta_0^{n-1}}{(\omega^2 + \Delta_0^2)^{n/2}}. \quad (2.21)$$

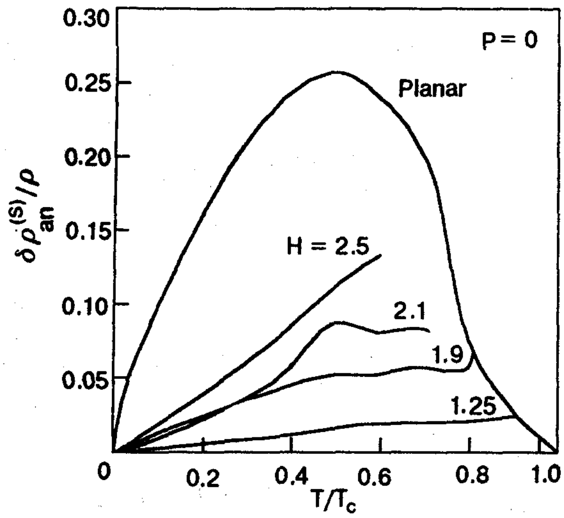

Fig. 1. Temperature dependence of $\delta\rho^{(S)}_{an}$ at $P = 0$ bar for various values of the magnetic field (in kGs).

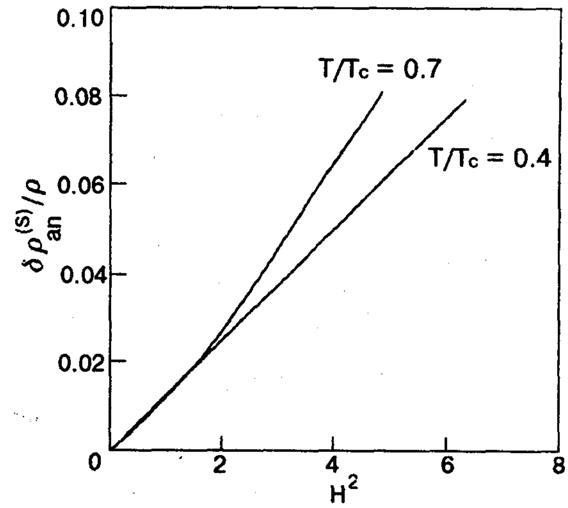

Fig. 2. Field dependence of $\delta\rho^{(S)}_{an}$ at $P = 0$ bar for various values of T/T_c. Deviation from linearity in H^2 is clearly seen at $T/T_c = 0{,}7$.

In (2.20) we have dropped the Fermi-liquid corrections for simplicity of presentation. The sum in (2.21) is taken over «odd» Matsubara frequencies. It can be shown (see the Appendix) that Eq. (2.20) is completely equivalent to our expression (2.14). The result (2.20) can be obtained by using the expressions for $\rho_{||}^{(S)}$ and $\rho_{\perp}^{(S)}$ given in Ref. 11.

Returning to the general expression (2.6) for $\delta\rho^{(S)}_{an}$ and using $E_{k\sigma}$ from (2.8), we calculate numerically the (T, H)-dependence of $\delta\rho^{(S)}_{an}$ for arbitrary magnetic fields and arbitrary temperatures.

In order to take into account the Fermi-liquid effects we must introduce the Landau molecular fields in a standard way. We will therefore use the following expression for $\delta\rho^{(S)}_{an}(T, H)$:

$$\delta\rho^{(S)}_{an}/\rho = \frac{(1 + \frac{1}{3} F_1^S)(Y_{||} - Y_{\perp})}{(1 + \frac{1}{3} F_1^S Y_{||})(1 + \frac{1}{3} F_1^S Y_{\perp})}, \quad (2.22)$$

where

$$Y_{||} = \frac{1}{2}\sum_{\sigma} \frac{1}{4T} \int_{-\infty}^{\infty} d\xi_k \left\langle \frac{3(\hat{\mathbf{k}}\hat{\mathbf{l}})^2}{\cosh^2(E_{k\sigma}/2T)} \right\rangle, \quad (2.23)$$

$$Y_{\perp} = \frac{1}{2}\sum_{\sigma} \frac{1}{4T} \int_{-\infty}^{\infty} d\xi_k \left\langle \frac{\frac{3}{2}(1 - (\hat{\mathbf{k}}\hat{\mathbf{l}})^2)}{\cosh^2(E_{k\sigma}/2T)} \right\rangle .$$

To take into account the Landau exchange parameter F_0^a it is enough to make a substitution $\omega_0 \to \tilde{\omega}_0$, where the renormalized Larmor frequency $\tilde{\omega}_0$ is defined by the equation

$$\tilde{\omega}_0 = \frac{\omega_0}{1 + F_0^a\left[\frac{2}{3} + \frac{1}{3} Y(T, \tilde{\omega}_0)\right]}, \quad (2.24)$$

where $Y(T, \omega_0)$ is the field-dependent Yosida function (introduction of higher-order exchange parameter requires a more complicated procedure [2]).

In Figs. 1 and 2 some results for $\delta\rho^{(S)}_{an}(T, H)$ are shown.

3. Dipolar velocity of the magnetized ^{3}He-*B*

In addition to the anisotropic part of the flow energy (2.2), the dipole-dipole potential also contains terms which depend on the orbital anisotropy axis $\hat{l}$. Starting from the expression of the dipolar energy density

$$F_D = \frac{1}{15} \chi_B(\Omega_B/g)^2 \Delta_B^{-2}\left\{|\mathrm{Sp}\hat{A}|^2 + \mathrm{Sp}(\hat{A}^+ \hat{A})\right\} \quad (3.1)$$

and using an explicit form of the order parameter of the magnetized *B*-phase, it can be shown that (up to a constant term)

$$F_D = \frac{2}{15} \chi_B(\Omega_B/g)^2(\Delta_{\perp}/\Delta_B)^2 \times$$

$$\times \left\{-\delta_B(2 - \delta_B)\hat{\mathbf{l}}\hat{\mathbf{h}} + \left[\left(2\cos\theta + \frac{1}{2}\right) + \delta_B\left(\frac{1}{2} - \hat{\mathbf{l}}\hat{\mathbf{h}}\right)\right]^2\right\}. \quad (3.2)$$

In (3.2) θ denotes the angle appearing in the matrix $\overleftrightarrow{R} = \overleftrightarrow{R}(\theta, \hat{\mathbf{n}})$ of the relative spin-orbit rotation

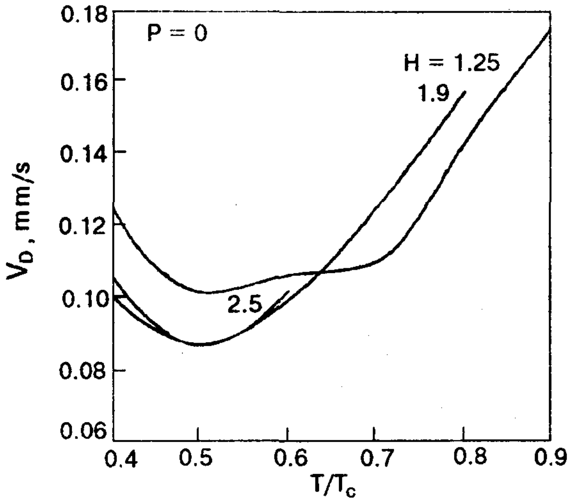

Fig. 3. Temperature dependence of the dipolar velocity at $P = 0$ bar for various values of H (in kGs).

about the axis $\hat{n}$. At the fixed value of $\hat{\mathbf{l}}\hat{\mathbf{h}} = \cos\beta$ the minimum of the dipolar energy is realized at

$$\cos\theta_0 = -\frac{1}{4}\left[1 + \delta_B(1 - 2\cos\beta)\right]. \quad (3.3)$$

Combining (3.2) with the flow contribution (2.2), we obtain an expression for the anisotropic part of the bulk energy density of the magnetically distorted ^{3}He-B:

$$F_B^{(\mathrm{an})} = F_D(\theta = \theta_0) + F_{\mathrm{flow}}^{(\mathrm{an})} =$$

$$= -\frac{4}{5}A(H)\left\{\hat{\mathbf{l}}\hat{\mathbf{h}} + \frac{1}{2}(v_s/v_D)^2(\hat{\mathbf{l}}\,\hat{\mathbf{v}}_s)^2\right\}, \quad (3.4)$$

where

$$A(H) = \frac{1}{6}\,\delta_B(2 - \delta_B)\chi_B(\Omega_B/g)^2(\Delta_\perp/\Delta_B)^2 \quad (3.5)$$

and the dipolar velocity v_D is defined by

$$v_D^{-2} = 5\,\delta\,\rho_{\mathrm{an}}^{(S)}/4A\,. \quad (3.6)$$

At the low fields ($\delta_B << 1$, $\delta_B \propto H^2$) we conclude $A(H) \simeq aH^2$ and recalling that $\hat{h}\hat{l} = \cos\theta_0 + (1 - \cos\theta_0)(\hat{\mathbf{n}}\hat{\mathbf{h}})^2$, we can write the anisotropy energy density in a conventional form:

$$F_B \simeq -aH^2 \times$$

$$\times\left\{(\hat{\mathbf{n}}\hat{\mathbf{h}})^2 + \frac{2}{5}(v_s/v_D)^2(\hat{h}_\mu R_{\mu i}(\theta_0, \hat{n})\hat{v}_{si})^2\right\} + \mathrm{const}\,. \quad (3.7)$$

In Ref. 6, an attempt to measure v_D at high fields was made using an ultrasonic probe. When the rotating vessel with strongly magnetized B-phase was slowly accelerated in the vortex-free Landau state, a critical angular velocity Ω_c signaling a textural transition was observed. Since v_D defines a characteristic velocity above which the superfluid counterflow takes over in the competition with the magnetic anisotropy (of the dipole-dipole origin), it was concluded that the experimentally observed critical velocity $v_c = \Omega_c R_0$ (where R_0 is the radius of the cylindrical container) is directly connected to v_D. Although an accurate interpretation of v_D in terms of v_c needs a detailed knowledge of the textural distribution, as a rough estimate we can set $v_D \simeq v_c$.

Using the definition (3.6), we have calculated numerically the dipolar velocity v_D for various fields and temperatures. Some of the results are presented in Fig. 3.

4. Angular momentum of the rotating, magnetized, vortex-free ^{3}He-B

The amount of the orbital anisotropy of ^{3}He-B depends on the strength of the applied magnetic field. This can be explored, in particular, in experiments with a freely rotating vessel. Since the angular momentum $\mathbf{L}$ of the rotating magnetized B-phase in the vortex-free state strongly depends on the magnetic field (see below), it would be possible to observe the variation in the angular velocity $\boldsymbol{\Omega}$ of the freely rotating sample when chanding H (due to the conservation of $\mathbf{L}$). In the case of the vortex-free rotation the angular momentum of a superfluid liquid is

$$\mathbf{L} = \mathbf{L}_n = \int(\mathbf{R}\times\mathbf{J}_n)d^3R\,, \quad (4.1)$$

where the mass current of the normal component is

$$\mathbf{J}_n = \overleftrightarrow{\rho}_n\boldsymbol{v}_n\,, \quad (4.2)$$

where $\boldsymbol{v}_n = \boldsymbol{\Omega}\times\mathbf{R} = (\Omega r)\hat{\varphi}$. Here r is the radial coordinate in the plane perpendicular to the angular velocity $\boldsymbol{\Omega}$, and $\hat{\varphi}$ is the unit vector in the circular direction. It is clear that

$$\mathbf{J}_n = (\Omega r)\left[\rho_\perp^{(n)}\hat{\varphi} + (\rho_\parallel^{(n)} - \rho_\perp^{(n)})(\hat{\mathbf{l}}\hat{\varphi})\hat{\mathbf{l}}\right]. \quad (4.3)$$

Setting

$$\hat{\mathbf{l}} = \cos\beta\hat{\mathbf{z}} + \sin\beta(\cos\alpha\hat{\mathbf{r}} + \sin\alpha\hat{\varphi}) =$$

$$= \cos\beta\hat{\mathbf{z}} + \sin\beta\left[\cos(\alpha + \varphi)\hat{\mathbf{x}} + \sin(\alpha + \varphi)\hat{\mathbf{y}}\right] \quad (4.4)$$

and considering a circular cylindrical vessel in the magnetic field oriented along the symmetry axis, we easily see that in the case of an axially symmetric $\hat{l}$-texture $[\alpha = \alpha(r),\ \beta = \beta(r)]$ $L_x = L_y = 0$ and

$$L_z/\Omega = = 2\pi \int \left\{\rho_\perp^{(n)} + (\rho_\parallel^{(n)} - \rho_\perp^{(n)}) \sin^2\alpha(r) \sin^2\beta(r)\right\} r^3 dr\, dz\,. \quad (4.5)$$

This result is quite transparent, L_z depends on the degree of the orbital anisotropy of the magnetized B-phase, but in the region where $\hat{l}$ has no circular component ($\alpha = 0$) only the transverse part $\rho_\perp^{(n)}$ of the density tensor is probed.

Introducing the moment of inertia $I_{zz}^{(0)} = \frac{1}{2}\rho R_0^2 V$ of the normal state ^{3}He liquid that fills a cylindrical container of radius R_0 (and of volume V), Eq.(4.5) can be rewritten as

$$L_z/\Omega = I_{zz}^{(0)} F\,, \quad (4.6)$$

where

$$F = 4\int_0^1 \left\{\rho_\perp^{(n)}/\rho + \left[(\rho_\parallel^{(n)} - \rho_\perp^{(n)})/\rho\right] \sin^2\alpha(x) \sin^2\beta(x)\right\} x^3 dx, \quad (4.7)$$

with $x = r/R_0$. For the isotropic B-phase (realized in zero magnetic field) $F = \rho_n(T)/\rho$, as it should be for the case of vortex-free rotation. For the magnetized ^{3}He-B $F = F(T, H, \Omega)$ and the Ω-dependence appears through the textural distribution which is sensitive to the superfluid counterflow orienting effects.

Using (4.6) and referring to the conservation of the angular momentum of an isolated system for the case of freely rotating magnetized ^{3}He-B, we conclude that due to the field-dependent orbital anisotropy one should observe the dependence of the angular velocity of rotation Ω on the strength of the applied field. In particular, if we start with a freely rotating state at $\Omega = \Omega_0$ and $H = 0$ and then apply the magnetic field H, the final state will be characterized by an angular velocity

$$\Omega(H) = \frac{1 + [\rho_n(T)/\rho]B}{1 + F(T, H, \Omega)B}\,\Omega_0\,, \quad (4.8)$$

where B is the ratio of moments of inertia of the normal ^{3}He liquid and of the container. It is to be remembered that the above-mentioned results refer to the case of a metastable, vortex-free rotation. When an equilibrium number of vortices fill the vessel, the anisotropy is washed out and the angular momentum in this case is $L_z = I_{zz}^{(0)}\Omega$ (on the average).

In order to calculate $F(T, H, \Omega)$ we must know the textural distribution across the rotating veseel. For a crude estimate of F it is instructive to use an approximate description with

$$\sin^2\alpha = 1\,, \qquad \cos\beta \simeq \begin{cases} 1\,, & \Omega r < v_D \\ (v_D/\Omega r)^2\,, & \Omega r > v_D\,. \end{cases} \quad (4.9)$$

Substituting (4.9) into (4.7), we easily find

$$F = \begin{cases} \rho_\perp^{(n)}\,, & \Omega < \Omega_c \\ \rho_\parallel^{(n)}/\rho - \left[\left(\rho_\parallel^{(n)} - \rho_\perp^{(n)}\right)/\rho\right](\Omega_c/\Omega)^4\left(1 + 4\ln(\Omega/\Omega_c)\right), & \Omega > \Omega_c\,, \end{cases} \quad (4.10)$$

where $\Omega_c = v_D/R_0$, as in Sec. 3. Since we are dealing with a vortex-free rotation of ^{3}He-B, his consideration is valid for $\Omega < v_{cr}/R_0$, where v_{cr} is the critical velocity of the vortex nucleation.

Acknowledgments

This work is a part of the ROTA Project. We are sincerely grateful to P. Hakonen, M. Krusius, E. Thuneberg, and G. Volovik for many useful discussions. The authors are greatly indebted to Prof. Olli Lounasmaa for continuous support and to the scientific staff of the Low-Temperature Laboratory of the Helsinki University of Technology for hospitality.

Appendix

Starting from the ω-sum representation of the Fermi distribution $f(E)$, which gives

$$\partial f/\partial E = T\sum_\omega \frac{E^2 - \omega^2}{(\omega^2 + E^2)^2}\,, \quad (A.1)$$

and using the identity

$$\frac{1}{2}\left[\frac{E^2-\omega^2}{(E^2+\omega^2)^2}+\frac{d}{d\xi}\left(\frac{\xi}{\omega^2+E^2}\right)\right]=\frac{E^2-\xi E(\partial E/\partial\xi)}{(\omega^2+E^2)^2}, \tag{A.2}$$

we obtain the useful formula

$$\int_{-\infty}^{\infty}(-\partial f/\partial E)d\xi = 1-2T\sum_{\omega}\int_{-\infty}^{\infty}d\xi\,\frac{E^2-\xi E(\partial E/\partial\xi)}{(\omega^2+E^2)^2}. \tag{A.3}$$

From (A.3) follows the ω-sum representation for the Yosida function

$$Y(\hat{k};T)=1-T\sum_{\sigma}\sum_{\omega}\int_{-\infty}^{\infty}d\xi_k\,\frac{E_{k\sigma}^2-\xi_k E_{k\sigma}(\partial E_{k\sigma}/\partial\xi_k)}{(\omega^2+E_{k\sigma}^2)^2}. \tag{A.4}$$

Taking into account that for the spectrum $E_{k\sigma}$ given by (2.8)

$$\partial E_{k\sigma}/\partial\xi_k=\frac{\xi_k}{E_{k\sigma}}\left(1+\frac{1}{2}\frac{\sigma\omega_0}{\left(\xi_k^2+\Delta_{||}^2(\hat{k}\hat{l})^2\right)^{1/2}}\right) \tag{A.5}$$

and explanding (A.4) with respect to δ_B and ω_0^2/Δ_0^2, we conclude that in the low field limit

$$Y(\hat{k};T)=Y(T)+a(T)(\hat{\mathbf{k}}\hat{\mathbf{l}})^2,$$

where

$$a(T)=2\delta_B\left(Z_3-\frac{3}{2}Z_5\right)+\frac{3}{2}\frac{\omega_0^2}{\Delta_0^2}\left(Z_5-\frac{5}{4}Z_7\right). \tag{A.6}$$

As a final step we must express the anisptropy parameter $\delta_B=(\Delta_\perp-\Delta_{||})/\Delta_\perp$ in terms of $Z_n(T)$. Using the equations for $\Delta_\perp$ and $\Delta_{||}$, it can be shown that in the low field limit

$$\delta_B(T,H)=\frac{5}{8}\frac{\omega_0^2}{\Delta_0^2}\left(1-\frac{3}{5}\frac{Z_5}{Z_3}\right). \tag{A.7}$$

Insertion of (A.7) into (A.6) restores the result (2.20) for $\delta\rho_{\mathrm{an}}^{(S)}$.

1. D. Vollhardt and P. Wölfle, *The Superfluid Phases of Helium-3*, Taylor and Francis (1990).
2. N. Schopohl, *J. Low Temp. Phys.* **49**, 347 (1982).
3. M. Ashida and K. Nagai, *Progr. Theor. Phys.* **74**, 949 (1985).
4. R. Movshovich, N. Kim, and D. M. Lee, *Phys. Rev. Lett.* **64**, 431 (1990); R. Movshovich and D. M. Lee, *J. Low Temp. Phys.* **89**, 515 (1992).
5. S. N. Fisher, A. M. Guenault, C. J. Kennedy and G. R. Pickett, *Phys. Rev. Lett.* **67**, 1270 (1991).
6. J. M. Kyynarainen, J. P. Pekola, K. Torizuka, A. J. Manninen, and A. V. Babkin, *J. Low Temp. Phys.* **82**, 325 (1991).
7. P. J. Hakonen, M. Krusius, M. M. Salomaa et al., *J. Low. Temp. Phys.* **76**, 225 (1989).
8. J. Korhonen, Yu. M. Bunkov, V. Dmitriev et al., *Phys. Rev.* **B46**, 13983 (1992).
9. K. K. Nummila, P. J. Hakonen, and J. Korhonen, *Europhys. Lett.* **11**, 651 (1990).
10. G. A. Kharadze, *in Helium-3*, W. P. Halperin and L. P. Pitaevskii (eds)., North-Holland, Amsterdam (1990).
11. G. Baramidze, G. Kharadze, and G. Vachnadze, *JETP Lett.* **63**, 107 (1996).