ABOUT CONDITIONS OF EFFECTIVE INTERACTION OF WAVES IN NON-UNIFORM, NON-STATIONARY AND NONLINEAR MEDIUM

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It is shown that in the distributed systems there are more general conditions for effective interaction of waves (synchronism) than the well known conditions. The well known conditions of synchronism are just the particular case of these more general conditions. It has been shown that these new conditions lead to new opportunities for effective interaction of wave, particularly at interaction of waves in plasma.

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1. INTRODUCTION

The processes of coherent interaction of the waves in periodic – non-uniform, periodic – non-stationary and in nonlinear medium are well investigated. At this the effective interaction occurs only at performance of conditions of synchronism. In a general form these conditions (see, for example, [1,2]) can be written down in such kind:

$$\sum_i \omega_i = 0,\ \sum_i \vec{k}_i = 0\,, \tag{1}$$

here ω_i - frequency of the interacting waves; $\vec{k}_i$ - wave vectors of these waves.

Among possible processes of interaction the greatest importance has the processes with participation of three waves (three-wave processes). Below we shall be limited by consideration only of three-wave processes. Most simple is the case, when the parameters of one of waves do not vary during interaction. It can be the waves of dielectric permeability, potential waves, initial stage of isintergration processes and others. Namely such processes we, first of all, will be considered.

2. INTERACTION OF WAVES IN MEDIUM WITH PERIODICALLY VARYING DIELECTRIC PERMEABILITY

Let plain transverse wave with frequency ω_0 and wave vector $\vec{k}_0$ is propagating in medium which dielectric permeability can be described by the formula:

$$\varepsilon = \varepsilon_0 + \tilde{\varepsilon},\ \tilde{\varepsilon} = q\cos(\vec{\kappa}\vec{r} - \Omega t),\ q << 1\,. \tag{2}$$

The presence of the second addend will result that the part of energy of a zero wave will be transformed in a wave with frequency ω_1 and wave vector $\vec{k}_1$. Let's name this wave by a wave with number 1. The conditions at which, the significant part of energy from a zero wave can be transformed in a first wave will be interesting for us. From Maxwell equations we can get such equation for a vector of an electrical field for waves that are propagating in medium with dielectric permeability (2):

$$\Delta\vec{E} - \frac{1}{c^2}\frac{\partial^2(\varepsilon\vec{E})}{\partial t^2} = -\vec{\nabla}\left(\frac{1}{\varepsilon}\vec{E}\cdot\vec{\nabla}\varepsilon\right). \tag{3}$$

The solution of the equation (3) we shall search as the sum of two components:

$$\vec{E} = \vec{A}_0(\vec{r},t)\exp(-i\omega_0 t + i\vec{k}_0\vec{r}) + \vec{A}_1(\vec{r},t)\exp(-i\omega_1 t + i\vec{k}_1\vec{r}). \tag{4}$$

First component in (4) describes a zero wave, second – the wave, which is born as a result of presence of periodic heterogeneity. At this we assume, that between wave vectors and frequencies of both waves exist usual connection: $k_0^2 = \omega_0^2\varepsilon_0/c^2$ $k_1^2 = \omega_1^2\varepsilon_0/c^2$. Substituting the solution (4) in the equation (3) after bulky, but simple calculations, we can receive the following system of the differential equations in partial derivatives of the first order for definition of dynamics in time and space of amplitudes $\vec{A}_0$ and $\vec{A}_1$:

$$\frac{\partial\vec{A}_0}{\partial l_0} = \frac{q\cdot i}{4N_0}\left\{\frac{\left(\omega_1^2+\Omega^2\right)}{c^2}\cdot\vec{A}_1 \pm \frac{1}{\varepsilon_0}\left(\vec{k}_1 \pm \vec{\kappa}\right)\left(\vec{\kappa}\vec{A}_1\right)\right\}\exp\left(i\delta\right)$$

$$\frac{\partial\vec{A}_1}{\partial l_1} = \frac{q\cdot i}{4N_1}\left\{\frac{\left(\omega_0^2+\Omega^2\right)}{c^2}\cdot\vec{A}_0 \mp \frac{1}{\varepsilon_0}\left(\vec{k}_0 \mp \vec{\kappa}\right)\left(\vec{\kappa}\vec{A}_0\right)\right\}\times \tag{5}$$

$$\times\exp\left(-i\delta\right).$$

Here $\delta(\vec{r},t)$ - function determining detuning between wave vectors and frequencies: $\delta(\vec{r},t) \equiv \Delta\vec{k}\cdot\vec{r} - \Delta\omega\cdot t$; $\Delta\vec{k} \equiv \left(\vec{k}_1 - \vec{k}_0 \pm \vec{\kappa}\right)$; $\Delta\omega \equiv \omega_1 - \omega_0 \pm \Omega$; $N_i = \sqrt{k_{ix}^2 + k_{iy}^2 + k_{iz}^2 + \varepsilon\cdot\omega_i^2/c^2}$ - norms for each of waves. Derivative in system (5) are taken along characteristic directions: $\vec{l}_{0,1} = \left\{k_{0,1,x}; k_{0,1,y}; k_{0,1,z}; \omega_0\cdot\sqrt{\varepsilon_0}/c\right\}/N_{0,1}$. $\vec{l}_0$ and $\vec{l}_1$ - the unit vectors along characteristic directions.

At getting (5) we assumed, that the amplitudes are slow functions of time and coordinates. Their changes are caused only by presence of the weak spatial - temporary heterogeneity ($q << 1$). For this reason components with second derivatives in system (5) had been omitted. This fact we should remember. The solutions which are not satisfied to these assumptions should be rejected.

For slow change of amplitudes it is necessary, that the right parts of the equations (5) were varied slowly. It will be occur when $\delta\left(\vec{r},t\right) = C = const$. This equation represents the equation of hyperplane in four dimensional space ($\vec{r},t$). This detuning will be not varying during an exchange of energy between waves, if the characteristic straight lines will be parallel to this hyperplane.

So, the characteristic lines

$$x = \alpha k_{0x} + x_0;\; y = \alpha k_{0y} + y_0;\quad z = \alpha k_{0z} + z_0;$$

$$x = \alpha_1 k_{1x} + x_1;\; y = \alpha k_{1y} + y_1;\; z = \alpha_1 k_{1z} + z_1; \qquad (6)$$

$$t = \alpha \cdot \left(\omega_0 \cdot \varepsilon_0 / c^2\right) + t_0; \qquad t = \alpha_1 \cdot \left(\omega_1 \cdot \varepsilon_0 / c^2\right) + t_1,$$

should be parallel to hyperplane $\delta(\vec{r},t) = C = const$. These conditions have such form:

$$\left(\Delta\vec{k}\cdot\vec{k}_0\right)c^2 - \Delta\omega\cdot\omega_0\cdot\varepsilon_0 = 0,$$

$$\left(\Delta\vec{k}\cdot\vec{k}_1\right)c^2 - \Delta\omega\cdot\omega_1\cdot\varepsilon_0 = 0. \qquad (7)$$

These conditions are represented on Fig.1. It is easy to check that derivations along $\vec{l}_0$ и $\vec{l}_1$ from detuning at conditions (7) will be equal to zero: $\partial\delta(\vec{r},t)/\partial l_0 = \partial\delta(\vec{r},t)/\partial l_1 = 0$.

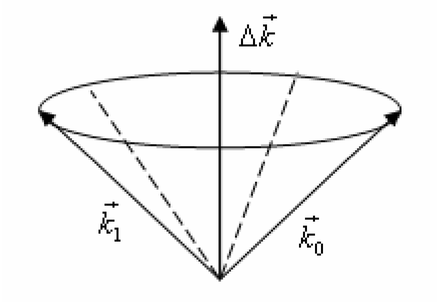

Fig. 1. The circuit of a mutual arrangement of vectors $\Delta\vec{k}$, $\vec{k}_0$, and $\vec{k}_1$ necessary for effective interaction of waves

It is interesting to analyze cases which are not managed by the known schema of interaction. The simplest of them, apparently, is the case of interaction of waves in stationary medium ($\Omega = 0$). Besides we shall assume that $\Delta\omega = 0$. From conditions (7) for this case, in particular, follows, that the effective interaction of waves will occur at performance of conditions:

$$\vec{k}_0 = -\vec{k}_1, \quad |\vec{\kappa}| = 2\left|\vec{k}_0\right| / \left|\cos\left(\vec{k}_1\cdot\vec{\kappa}\right)\right|. \qquad (7a)$$

It is easy to see, that the second condition differs from a well known Bragg condition ($|\vec{\kappa}| = 2\left|\vec{k}_0\right|$). It is easy to show, that at performance (7a) occur full reflection of waves from non-uniform medium (crystal). More details see in [3].

3. INTERACTION OF WAVES IN NONLINEAR MEDIUMS

The equations, which describe interaction of three waves ($\omega_0, \omega_1, \omega_2; \vec{k}_0, \vec{k}_1, \vec{k}_2$) in nonlinear mediums, are possible to present as:

$$\partial a_0/\partial l_0 \equiv \dot{a}_0 + \left(\vec{V}_0\vec{\nabla}\right)a_0 = -\sigma_0\cdot a_1\cdot a_2\cdot\exp\left(-i\cdot\delta\right);$$

$$\partial a_1/\partial l_1 \equiv \dot{a}_1 + \left(\vec{V}_1\vec{\nabla}\right)a_1 = \sigma_1\cdot a_0\cdot a_2^*\cdot\exp\left(i\cdot\delta\right);$$

$$\partial a_2/\partial l_2 \equiv \dot{a}_2 + \left(\vec{V}_2\vec{\nabla}\right)a_2 = \sigma_2\cdot a_0\cdot a_1^*\cdot\exp\left(i\cdot\delta\right). \qquad (8)$$

Here $\vec{V}_i$ - group velocity of waves; σ_i - matrix elements of nonlinear interaction.

The left part of each of the equations of system (8) is submitted as derivative along characteristic directions. For distinctness we shall examine interaction of waves with positive energy ($\sigma_i > 0$), and also we shall be guided by processes of disintegration. The conditions of synchronism of interacting waves will be a conditions of parallelism between characteristic lines and hyperplane $\delta(\vec{r},t) = const$ i.e. such conditions:

$$\Delta\omega - \Delta\vec{k}\cdot\vec{V}_i = 0. \qquad (9)$$

The linear stage of the disintegration process proceeds as instability. At this stage it is possible to consider a wave with the maximal frequency as don't changed wave ($a_0 = const$). In this case the systems (8) are convenient to rewrite as:

$$\partial a_1/\partial l_1 = \sigma_1\cdot a_0\cdot a_2^*\cdot\exp\left(i\cdot\delta\right);$$

$$\partial a_2^*/\partial l_2 = \sigma_2\cdot a_0^*\cdot a_1\cdot\exp\left(-i\cdot\delta\right). \qquad (10)$$

Substituting in (10) the solution in form $\sim\exp(i\Omega t - i\vec{\kappa}\vec{r})$, we will get such dispersion equation:

$$\Omega^2 - \Omega\cdot\vec{\kappa}\left(\vec{V}_1 + \vec{V}_2\right) + \left[\kappa^2\cdot\vec{V}_1\cdot\vec{V}_2 + \sigma_1\sigma_2\left|a_0\right|^2\right] = 0. \quad (11)$$

At the solving (11) we should from all set of the possible solutions to consider only that which frequency Ω and vector $\vec{\kappa}$ will be small sizes ($\Omega \sim \kappa \sim |a_0|$). Solving the equation (11) for Ω in this case we shall get:

$$\Omega = \vec{\kappa}\vec{V} \pm i\cdot|a_0|\sqrt{\sigma_1\sigma_2}. \qquad (12)$$

The imaginary part of frequency $\left(\operatorname{Im}\Omega = |a_0|\sqrt{\sigma_1\sigma_2}\right)$ determines the increment of the decay instability.

It is interesting to find those new opportunities for disintegrations, which are not management by known conditions of disintegrations. Let's consider simplest. Let there is a disintegration of a transverse wave on a transverse wave and on one of the own waves of the magnetized plasma waveguide ($t_0 \to t_1 + l$). Let's consider that all three waves are inside a linear part of dispersion curve (Fig. 2). In this case the group velocities of all three waves coincide ($\vec{V}_0 = \vec{V}_1 = \vec{V}_2 \equiv \vec{V}$). Moreover, the group velocities in this case coincide with phase velocity. The conditions of synchronism (9) will be carried out in this case for any three of the waves. Really, the conditions (9) in this case get the kind of identity:

$$\omega_0 - \omega_1 - \omega_2 = \left(\frac{\omega_0}{V} - \frac{\omega_1}{V} - \frac{\omega_2}{V}\right)V. \qquad (13)$$

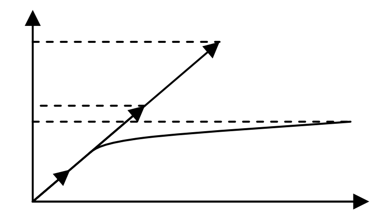

Fig. 2. The circuit of disintegration of a cross wave on a cross wave and on a wave of magnetized plasma waveguide

Thus, in a considered case in process disintegration can to take part the large number ensembles from three waves. And, the characteristics of these waves can a little differ from each other. In this case, as shown in work [4], the process of disintegration becomes chaotic. It is necessary to notice, that the considered case of coinciding of all velocity can be solved analytically not only at a linear stage, but also on nonlinear. For this purpose it is enough to use a method of effective potential (see, for example, [5]). However, because this process became chaotic character, this solution loses interest.

4. CONCLUSIONS

In the conclusion we shall formulate the basic reason of an opportunity of occurrence of new conditions of synchronism. If in the equations (5) and (8) we carry out the averaging, for example, on a small interval of time $\Delta t \sim 1/\Delta\omega$, the slowness of change of amplitudes of interacting waves will be only at performance of a usual condition $\Delta\omega = 0$. Similarly, integrating on a small spatial interval, we shall get habitual conditions: $\Delta\vec{k} = 0$.

However process of wave interaction occurs not only in time, but also in space. He occurs along the characteristics. Therefore it is enough (for effective interaction of waves) that the right parts of the equations (5) and (8) changed slowly along these directions. This fact is expressed by equality to zero derivatives from detuning along characteristic directions (see (7)). It is significant also that the right parts in equations (5) and (10) have detuning phases ($\delta(\vec{r},t)$) with opposite signs. That is why this detuning was disappeared from dispersion equation (11).

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ОБ УСЛОВИЯХ ЭФФЕКТИВНОГО ВЗАИМОДЕЙСТВИЯ ВОЛН В НЕОДНОРОДНЫХ, НЕСТАЦИОНАРНЫХ И НЕЛИНЕЙНЫХ СРЕДАХ

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Показано, что в распределенных системах существуют более общие условия эффективного взаимодействия волн (синхронизма), чем известные условия. Из этих более общих условий, как частный случай, следуют известные условия. Показано, что новые условия могут приводить к неизвестным ранее процессам взаимодействия волн, в частности, при взаимодействии волн в плазме.

ПРО УМОВИ ЕФЕКТИВНОЇ ВЗАЄМОДІЇ ХВИЛЬ У НЕОДНОРІДНИХ, НЕСТАЦІОНАРНИХ ТА НЕЛІНІЙНИХ СЕРЕДОВИЩАХ

В.О. Буц

Показано, що в розподілених системах існують більш загальні умови ефективної взаємодії хвиль (синхронізму), чим відомі умови. Із цих більш загальних умов, як окремий випадок, випливають відомі умови. Показано, що нові умови можуть приводити до невідомих раніше процесам взаємодії хвиль, зокрема, при взаємодії хвиль у плазмі.