LONG SEQUENCE OF RELATIVISTIC ELECTRON BUNCHES AS A DRIVER IN WAKEFIELD METHOD OF CHARGED PARTICLES ACCELERATION IN PLASMA

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Using *LCODE* 2.5D-simulation of wakefield excitation in plasma by a long sequence of relativistic electron bunches was performed. For the resonant sequence wakefields add coherently until the wave nonlinearity comes into play. The mechanism is found out which enables resonant excitation of the wakefield even if the bunch repetition frequency appreciably differs from the plasma frequency. Conditions for enhancement of excitation efficiency, acceleration gradient, and transformation ratio were investigated.

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1. INTRODUCTION

High gradient electric field excited in a plasma by intense relativistic electron bunch or train of bunches proposed in [1,2] and firstly experimentally tested in [3-5] has already been demonstrated to offer new promising techniques for acceleration [6], focusing [7, 8], and deflection [9] of charged particle beams.

The bunch of the density comparable with or greater than the plasma density n_0 is required for excitation of a high amplitude wave with the electric field of the order of the wavebreaking limit $E_0=c\omega_p/e$, where m is the electron mass, e is the elementary charge, c is the speed of light, and $\omega_p = \sqrt{4\pi n_0 e^2/m}$ is the plasma frequency. If a high density beam is not available, the wave can be resonantly driven by a train of short low density electron bunches providing the same total charge [3]. The multibunch scheme has also been tested experimentally [3-5,10-14] though accelerating gradients achieved so far are not so impressive compared to single bunch experiment [6].

We present results of numerical simulation of plasma wakefield excitation by a sequence of relativistic electron bunches, made with 2.5D quasi-static code LCODE [15] that treats the bunches as ensembles of macro-particles and plasma as a cold electron fluid, since particle models cannot treat very long bunch trains due to error accumulation. The code is quasi-static, that is, the plasma response is calculated as a function of the co-moving coordinate $\xi = z-ct$. The quasi-static approximation is fully justified for short highly relativistic beams which evolve slowly on the time scale of beam passage through a plasma cross-section; in this case ξ-dependencies define both spatial portrait of the plasma response and its temporal evolution at a certain cross-section. In our case, the beam is long and the dualism in interpretation of ξ-dependencies disappears. Each bunch still interacts with the plasma quasistatically, that is, as a rigid object moving with the speed of light. As to the whole beam, corresponding times and distances are to be compared with times and distances of non-quasistatic processes. At plasma temperatures of interest (electron-volts), both collisions and energy drift with the group velocity occur at times greater than the time of beam passage through a given cross-section. Thus, quasi-static ξ-dependencies characterize the temporal behavior of the plasma response at a fixed point. As to the instant spatial portrait of the system, it cannot be obtained from ξ-dependencies by putting t constant, since the beam itself is longer than the distance of beam evolution.

Parameters are taken close to those of plasma wakefield experiments [3], in which electron beam represented by a regular sequence of 6000 electron bunches, each of energy 2 MeV, charge 0.32 nC, rms length $2\sigma_z$=1.7cm, rms radius σ_r=0.5cm, and rms angular spread σ_θ=0.05 mrad excites wakefield in the plasma of density $n_p=10^{11}$ cm^{-3} and length of about 1m, so that the repetition frequency of the bunches coincides with the plasma frequency ω_p (so called resonant sequence).

The multibunch scheme imposes heavy demands on accuracy of the plasma density. We have found, however, a robust mechanism that, at the expense of bunch population, enables resonant excitation of the wakefield even if the bunch repetition rate appreciably differs from the plasma frequency. There are experimental evidences of this effect: in KIPT experiments [3] up to 5000 bunches coherently build up plasma oscillations thus causing a linear growth of the wakefield amplitude. To provide this field growth without the effect of frequency synchronization, the plasma density must be controlled to the precision of roughly 1/5000, which is absolutely impossible with the decaying plasma used in the experiments.

2d3v-investigation of transformation ratio has been carried out. The cases of bunches placing on phases and ramping of bunches intensity, which leads to values of transformation ratio, exceeding limiting value 2, which follows from Wilson theorem, have been considered.

2. RESULTS OF SIMULATION

2.1. RESONANT TRAIN OF BUNCHES

We consider dynamics of first 31 bunches in the plasma. Bunches and plasma densities in the cylindrical coordinate system (r,z) at some z as functions of the dimensionless time $\tau=\omega_p t$ are shown in Figs. 1, 2. From Fig. 1 we see that, at the middle of the plasma, the bunches are already focused by the wakefield, and the focusing is non-uniform. This looks like compression of bunches both in radial and longitudinal directions, though, of course, at these times and beam energies, radial relative shifts of beam particles prevail. Because of the complicated shape of bunches, the excited wave (Fig. 2) looks like a nonlinear one, with the wave period being longer near the axis. However, it is not nonlinear yet, that is, the period of remaining wakefields will be exactly $2\pi/\omega_p$ if we break the sequence after the 31-th bunch. As the bunch sequence evolves, the wakefield also evolves, and location of defocusing regions shifts with respect to the bunches. For a bunch slice to be defocused, it is sufficient to fall into the defocusing field only once for a relatively short period of time. As a consequence, at the end of the plasma the bunches are mostly defocused (Fig. 3), and the wakefield is lower.

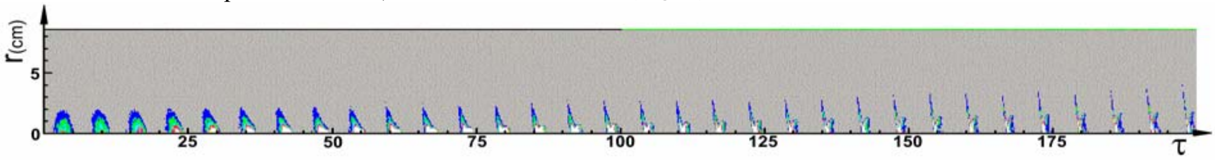

Fig. 1. Temporal evolution of the bunches density in the middle of the plasma (at z=50 cm from the injection point)

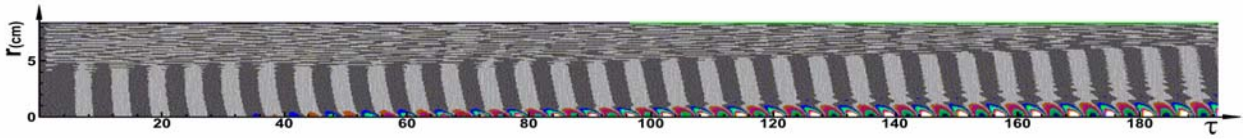

Fig. 2. Temporal evolution of the plasma electron density (z=50 cm)

For the sequence of 500 bunches (see Fig. 3), we observe that 100 bunches lose their energy linearly, i.e. coherently deposit energy in plasma wakefield excitation (Fig. 4).

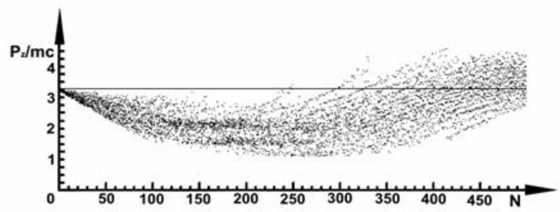

Fig. 3. Longitudinal momenta of 500 bunches as they pass the middle of the plasma (z=50cm)

The next portion of bunches (up to approximately 300-th bunch) continues to lose their energy and contribute to wakefield build-up, but at a smaller rate. Subsequent bunches fall in deceleration and acceleration phases of the excited wakefield, so that the wakefield amplitude saturates at the magnitude of 3 MeV/m.

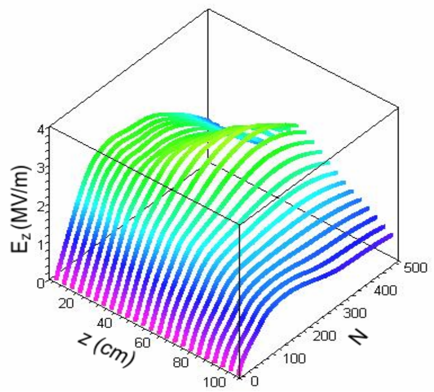

Fig. 4. The amplitude of the on-axis electric field as a function of the coordinate along the plasma and the number of bunches

The overall picture of wakefield excitation is seen from Fig. 4 that shows the temporal growth of longitudinal electric field E_z in different plasma cross-sections. Near the entrance, the bunches have a perfect Gaussian-like shape, and the field grows linearly until the wave gets nonlinear and goes out of resonance with the sequence. At z~50 cm, the effect of bunch pinching comes into play, and we observe faster field growth and a higher saturation level. The maximum electric field here is as high as 10% of the wavebreaking limit. Near the end of the plasma, the bunches are mostly defocused, and the excited wakefield is low.

2.2. NONRESONANT TRAIN OF BUNCHES

Fig. 5 shows how the frequency synchronization manifests itself if the plasma density is 5% lower than the resonant one.

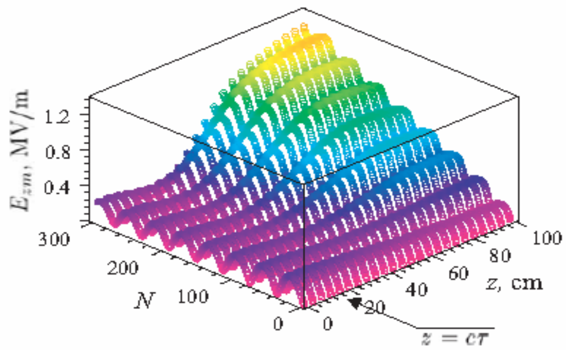

Fig. 5. Wakefield amplitude as a function of coordinate and number of past bunches for n_0 = 0.95 n_{res}

Here we plot the on-axis amplitude of the longitudinal electric field *Ezm* versus the distance *z* along the plasma and the number of bunches *N* past through these cross-sections. At small *z*, we observe beating of the field, as it should be for a harmonic oscillator driven by a periodic force of a slightly different frequency. Some distance downstream, the shape of bunches changes, and we see the linear wakefield growth composed by small equal steps that follow with the beat frequency.

To visualize the underlying physics, we reduce the plasma density to 75% of the resonance value and look at the phasing of bunches with respect to the wakefield (Fig. 6, a). Just after entering the plasma, the bunches are fresh, and the beating is nearly periodic. The bunches at the beginning of beating pulses are mainly in the decelerating phase of the wave; the ones near the end are in the accelerating phase; the bunches near the field maximum, on the average, do not exchange energy with the wave. In the linear wakefield considered, intervals of focusing are $\pi/2$ shifted forward in time with respect to the acceleration intervals.

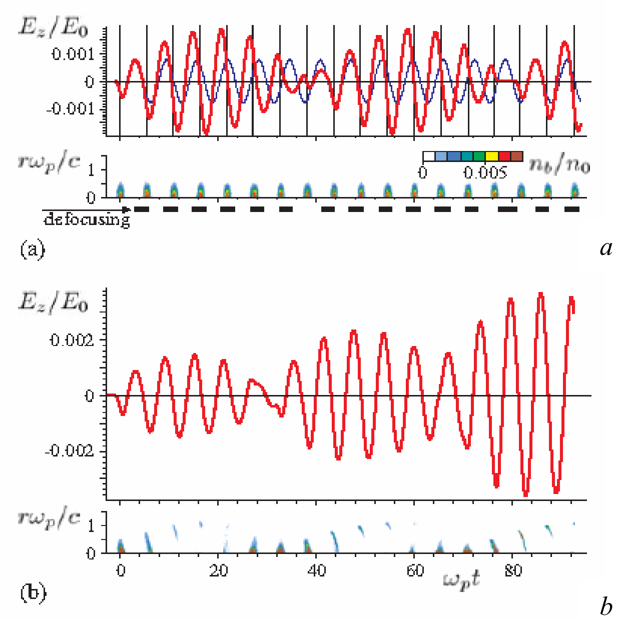

Fig. 6. The on-axis field Ez(t) (top) and the beam density $n_b(r, t)$ (bottom) near the entrance to the plasma at $z = 0$ (a) and at the distance of frequency synchronization $z = 70c/\omega_p$ (b) for $n_0 = 0.75\ n_{res}$. Black dashes under the beam density map in (a) indicate cross-sections of the defocusing transverse force. The thin sinusoid in (a) is the wakefield of the first bunch. Vertical thin lines in (a) show the relative location of bunch centroids with respect to the wave

The bunches near the field maximum thus fall into the defocusing phase of the wave and quickly leave the wakefield area (Fig. 6, b). The bunches which build up or damp the wave (thereby defining its structure) are in the phase of a small transverse force and preserve their shape. Due to this fact, the wave remains unchanged until the defocused bunches get completely destroyed. With respect to the plasma-frequency sinusoid, all survived bunches are in the decelerating phase, while the destroyed bunches were in the accelerating phase. Consequently, as the latter disappear, the plasma-frequency sinusoid becomes the dominant mode that monotonically increases its amplitude with each group of survived bunches. In other words, the survived bunches form a sequence which is strictly resonant with the plasma wave. It is particularly remarkable that the beam rearrangement occurs identically in all beating periods, just with a time delay.

We can estimate the time of frequency synchronization from the linear wakefield theory [16]. For the discussed beam parameters, this time corresponds to the distance of about 15 cm that is marked in Fig. 1 by the arrow.

2.3. TRASFORMATION RATIO

For plasma wakefield accelerator (PWFA) concept three parameters are of great importance: accelerating rate, efficiency and transformer ratio. For the simplest case of PWFA – one-dimensional collinear along *z* two "point" bunches (driver and witness) with number of particles N_1, N_2 and particle energy E_1, E_2, respectively, these parameters are defined by following relations: accelerating rate $G=dE_2/dz$; transformer ratio $T = \Delta E_2/E_1$; and efficiency $\eta = \Delta(N_2E_2)/ N_1E_1$

High accelerating rate allows to reduce length of accelerator but energy gain of accelerated bunch is limited because the higher gradient of excited wakefield the higher retarding field and smaller length on which driver bunch loses its whole energy. As it has obtained in [17] (Wilson's theorem) for this case

$$T = \Delta E_2/E_1 \le (2\text{-}N_2/N_1), \tag{1}$$

$$\eta \le N_1/N_2\ (2\text{-}N_2/N_1). \tag{2}$$

There some possibility to overcome this limit, including the use multibunch driver. So for resonant train of M bunches due to different stoppage distances of various bunches, namely 1st, 2nd, 3rd,……M bunch loses its whole energy on distance L, L/3, L/5,….L/(2M-1), respectively, transformer ratio [17]

$$T\le\Sigma 2/(2k\text{-}1)\text{-}N_2/N_1,\ \text{i.e. } T\sim \ln M, \tag{3}$$

though wakefield increases linearly with number of bunches M.

If bunches are placed in zeros of summarized wakefield then T grows faster, namely linearly with M [18].

$$T\le 2\sqrt{M}\text{-}N_2/N_1, \tag{4}$$

because all bunches are in the equal its own decelerating field and lose whole energy on the equal distance.

If the sequence is ramped then [18]:

$$T\le 2M\text{-}N_2/N_1. \tag{5}$$

We carried out 2d3v simulation of these cases and investigated influence of focusing/defocusing of bunches on value of transformer ratio.

For resonant ($\omega_{mod}=\omega_p$) sequence of 7 bunches of equal intensity ($I_M= I_0$) and 8th accelerated bunch the results are presented in Figs. 7, 8.

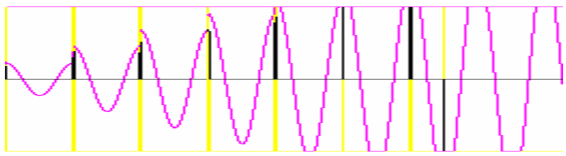

Fig. 7. Longitudinal electric field E_z (red) and coupling coefficient (black) at γ_b=1000, $I_b=0{,}3\times10^{-3}mc^3/4e$, $r_b=0{,}3c/\omega_p$

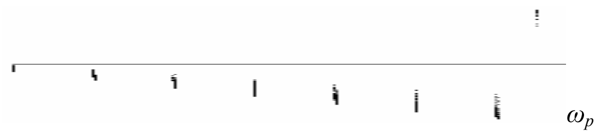

Fig. 8. Longitudinal momenta of bunches

From Figs. 7, 8 it follows that for the resonant train transformer ratios, defined by field T_E and by energy T_E are different $T_E \neq T_E$, namely $T_E\approx 1.17$, $T_E \sim \ln M$ [17] and are not high enough. To enhance transformer ratio all bunches should be in the equal decelerating field and consequently lose its energy on the same distance and consequently $T_E=T_E$. For this aim each bunch should be placed in phase where total wakefield of previous bunches is zero. It gives transformer ratio $T_E \sim \sqrt{M}$ [17].

We proposed to place bunches only in phases of wakefield zeros, where bunches experience focusing [19]. To manifest influence of focusing/defocusing of bunches we considered mildy relativistic case γ_b=10. In Figs. 9,10 there are shown E_z wakefield (red), coupling coefficient (black) and bunch density (yellow) for bunches placing in focusing (see Fig. 9) and defocusing (Fig. 10) phases where wakefield is zero (I_b=0,3×10^{-3}mc^3/4e, r_b=0,3c/ω_p).

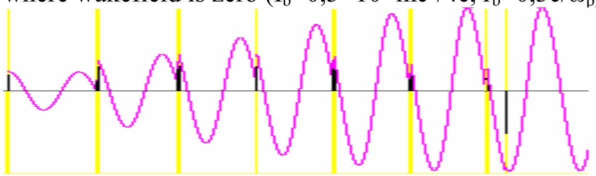

Fig. 9. Focusing phases T_E = 4.2, $T_E > 2\sqrt{M}$

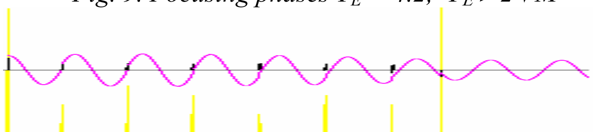

Fig. 10. Defocusing phases T_E = 1, $T_E < \sqrt{M}$

From Fig. 9 it is seen that placing bunches in focusing zeros of wakefield results in linear increase of total wakefield simultaneously with high transformer ratio. Meanwhile in defocusing case (see Fig. 10) there is no wakefield growth nor transformer ratio enhancement with number of bunches.

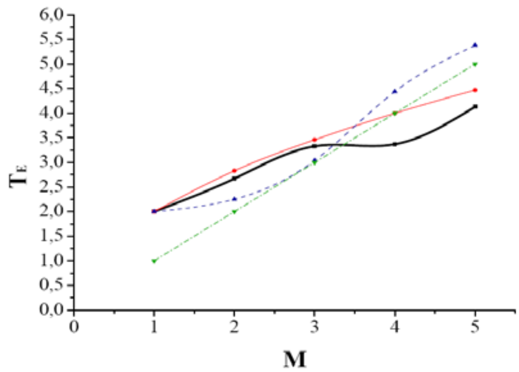

Fig. 11. Dependence of T_E on number of point bunches, placed in wakefield zeros phases, where bunches are focused: γ=1000 (black), γ=10 (blue dashed-line); 2√M (red), M (green chain line)

Dependence of transformer ratio on number of bunches with taking into account radial dynamic of bunches at 2d3v simulation is shown in Fig. 11 for bunches of equal charge and in Fig. 12 for ramped sequence of bunches. For bunches experiencing focusing (γ=10) transformer ratio is higher comparing to T_E=2√M for identical bunches and T_E = 2M for ramped sequence.

In [14] the similar scheme of placing bunches in zeros of Ez wakefield was proposed but additionally the train of bunches should be profiled (e.g. ramped). For such scheme the transformer ratio increases linearly with the number of bunches ($T=MT_1$), while the peak accelerating field is only E_{final}=ME1 (at the expense of the total charge which scales as M^2Q_1 for R_1=2). E_1 is the wake excited by the first bunch with charge Q_1. From an energy standpoint, each bunch transfers as much energy to the accelerating wake as the first bunch does, and the rest of its energy is transferred through the plasma to the subsequent bunches to prevent them from decelerating at higher rate.

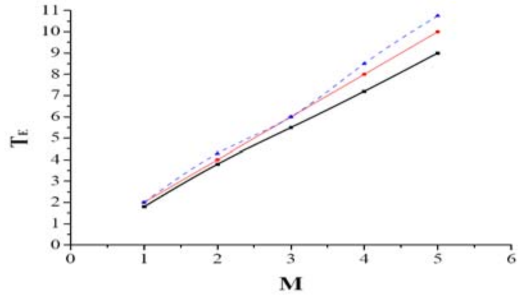

Fig. 12. Dependence of T_E on number of bunches for ramped sequence: γ=1000 (black), γ=10 (blue dashed-line); 2M (red)

Fig. 13 shows an example of a case where 4 drive bunches with l=λ_p/2 whose charges scale as 15:45:75:105 pC drive a 110 MV/m wake when they are placed 1.5λ_p apart. The wake amplitude is small given the total charge provided, but the transformer ratio is almost quadrupled.

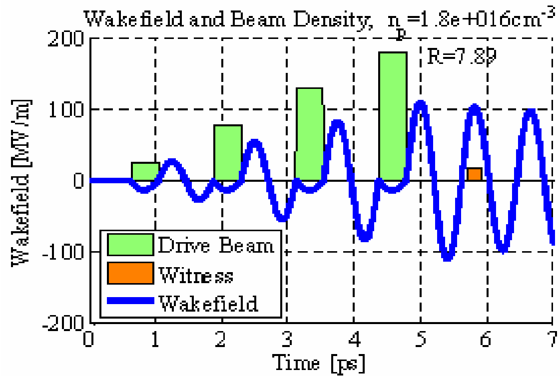

Fig. 13. Ramped bunch train distribution for maximum transformer ratio. The drive bunches are separated by 1.5 plasma wavelengths

2.4. PLASMA-BASED AFTERBURNER DESIGN FOR ILC

An afterburner from 100 to 500 GeV for International Linear Collider can be designed by extending the ramped bunches scheme within the limits of linear regime; an exact design will have to account for the nonlinear effects at ILC. A small witness bunch can gain 400 GeV (per particle) by sampling a ~15 GV/m accelerating wakefield over ~27 m of plasma. This wakefield can be excited by a train of 4 ramped drive bunches if the plasma density is increased to n_p=2×10^{17} cm^{-3} and the beam charge is increased 5 times, while the focused beam size is σ_r~10 μm. The total charge is 1.2 nC and needs to be distributed as 75:225:375:525 pC while the bunches are separated by 1.5λ_p=112 μm. Finally, in this model, if the witness bunch only loads the wake by 30% in order to minimize the energy spread, its charge would have to be 0.3Q_{total}/T=0.3×1.2nC/8=45 pC [20]. If the same amount of charge was distributed into 4 identical equidistant bunches (or one single drive bunch), then the maximum accelerating wakefield could be 4 times higher, ~60 GV/m, but the fourth drive bunch would lose all its energy in less than 2 m in the plasma, thus limiting the possible energy gain of the witness particles to 120 GeV.

However, using the 4 ramped bunches, the transformer ratio in principle can be close to R=8 and the decelerating wakefield inside any drive bunch is only (15 GV/m)/R = 1.875 GV/m.

This example demonstrates the advantages of using transformer ratio enhancement techniques in plasma accelerators [21].

CONCLUSIONS

1. It is shown that sequence of only about 300 relativistic electron bunches contributes to wakefield growth until the wave nonlinearity comes into play. For experiment [3] the maximal wakefield of the order of 3 MV/m, i.e., 10% of the wavebreaking limit, is achieved in the middle of the plasma length. The electron density perturbation up to 60% is observed.
2. The effect of frequency synchronization makes possible resonant wakefield excitation by very long bunch train: requirement of sharp frequency matching removed.
3. It was proposed and simulated the way to increase transformer ratio by means of placing bunches in those zeros of WF where they are focusing by radial component of WF.
4. PWFA based on multi-bunch driver has a perspective as a concept of creating more compact collider comparatively to conventional ones.

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ДЛИННАЯ ПОСЛЕДОВАТЕЛЬНОСТЬ РЕЛЯТИВИСТСКИХ ЭЛЕКТРОННЫХ СГУСТКОВ КАК ДРАЙВЕР В КИЛЬВАТЕРНОМ МЕТОДЕ УСКОРЕНИЯ ЗАРЯЖЕННЫХ ЧАСТИЦ В ПЛАЗМЕ

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С использованием *LCODE* проведено 2.5-мерное численное моделирование возбуждения кильватерных полей в плазме длинной последовательностью релятивистских электронных сгустков. Для резонансной цепочки кильватерные поля складываются когерентно, пока существенной не становится нелинейность волны. Обнаружен механизм, который делает возможным резонансное возбуждение кильватерного поля, даже если частота следования сгустков отличается от плазменной частоты. Исследованы условия повышения эффективности возбуждения, темпа ускорения и коэффициента трансформации.

ДОВГА ПОСЛІДОВНІСТЬ РЕЛЯТИВІСТСЬКИХ ЕЛЕКТРОННИХ ЗГУСТКІВ ЯК ДРАЙВЕР В КІЛЬВАТЕРНОМУ МЕТОДІ ПРИСКОРЕННЯ ЗАРЯДЖЕНИХ ЧАСТИНОК У ПЛАЗМІ

К.В. Лотов, В.І. Маслов, І.М. Онищенко

З використанням *LCODE* проведено 2.5-вимірне чисельне моделювання збудження кільватерних полів у плазмі довгою послідовністю релятивістських електронних згустків. Для резонансної послідовності кільватерні поля додаються когерентно, поки не стає суттєвою нелінійність хвилі. Виявлено механізм, який забезпечує резонансне збудження кільватерного поля, навіть якщо частота слідування згустків суттєво відрізняється від плазмової частоти. Досліджено умови збільшення ефективності збудження, темпу прискорення та коефіцієнта трансформації енергії.