OSCILLATION SPECTRUM OF ELECTRON PLASMA, CONTAINING ADDITIVE OF BACKGROUND GAS IONS (AZIMUTH NUMBER m=2)

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The spectrum of nonneutral plasma oscillations with azimuth number $m = 2$ is computed. Plasma, completely filling a wave guide, is composed of "cold" magnetized electrons and a small density fraction of ions, produced by ionization of background gas atoms by electron impact. The distribution function of ions is anisotropic one. The spectrum obtained consists of the families of "modified" ion cyclotron (*MIC*) modes and also of the families of electron upper hybrid (*UH*) and low hybrid (*LH*) modes (Trivelpiece–Gould modes) Doppler shifted by the electron rotation. Due to the Doppler shift the frequencies of electron modes fall into a region of ion frequencies and instabilities arise. The growth rates of UH modes are much faster than the growth rates of LH and MIC modes.

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The nonneutral plasma with magnetized electrons and unmagnetized ions, produced by ionisation of atoms (molecules) of background gas by electron impact, is created and used in many experiments and technological devices (plasma lenses, ion sources made on the basis of Penning sell, beam experiments etc). Oscillations excited in such plasma, affect the operation regimes of such devices and even hinder them. An interpretation of oscillation spectra of such a plasma, observed over a wide range of electric and magnetic fields, electron and ion densities, is still lacking.

For determining the spectrum of plasma oscillations in such devices it is necessary to solve the non local stability problem in cylindrical geometry, using the kinetic description of equilibrium and stability of unmagnetized ions of background gas concerning helical perturbations $\tilde{\Phi} = \tilde{\Phi}_m(r)\exp\left[i(m\varphi + k_z z - \omega t)\right]$. Attempts to solve this problem, go back to the known article [1] (see also [2]). However, only in [3] the adequate kinetic description of an equilibrium state of unmagnetized ions of the background gas, taking into account the peculiarity of ion production, has been yielded, in [4] their perturbed state has been worked out, in [5, 6] the non local dispersive equation for oscillation frequencies of the electron plasma, containing a small density fraction of such ions, has been obtained. It is valid over the entire range of allowable electric and magnetic field strengths, for magnetized and unmagnetized ions. In [5, 6] the electrons are supposed to be "hot" and "cold" consequently.

As it has been shown in [3, 6], the frequencies of volume electron modes, which are high-frequency in neutral plasma, in nonneutral plasma can fall into the region of low (ion) frequencies due to the Doppler shift caused by the electron rotation in crossed fields. This peculiarity of behaviour of electron modes is important for the stability of nonneutral plasma. If there is even a small additive of ions in plasma it leads to interaction of electron and ion modes and plasma instability This peculiarity has not been revealed in [1, 2].

The frequencies of volume oscillations of the waveguide completely filled with the homogeneous nonneutral "cold" electron plasma are equal (neglecting the ions influence) the upper hybride (UH) and low hybride (LH) frequencies with Doppler shift [2]:

$$\omega = m\omega_e \pm$$
$$\pm\frac{1}{2^{1/2}}\left\{\Omega_e^2 + \omega_{pe}^2 \pm \left[(\Omega_e^2 + \omega_{pe}^2)^2 - 4\omega_{pe}^2\Omega_e^2\cos^2\theta\right]^{1/2}\right\}^{1/2}. \quad (1)$$

In (1) ω_{pe} is Lengmuir frequency of electrons, $\Omega_e = \left(\omega_{ce}^2 + 4eE_r / m_e r\right)^{1/2}$ - their "modified" cyclotron frequency in the crossed fields, $e > 0$, radial electric field $E_r < 0$, ω_{ce} is the electron cyclotron frequency, $\cos^2\theta = k_z^2 / (k_z^2 + k_\perp^2)$, $k_\perp^2 = \kappa_{m,l}^2 / a^2$, k_z is a longitudinal wave vector, $\kappa_{m,l}$ – the lth root of Bessel function J_m, a is the plasma radius, $\omega_e > 0$ is the angular velocity of electron rotation in the crossed fields. For unambiguity it is considered $m > 0$, and a frequency sign ω is arbitrary. The signs «$\pm$» in braces define the frequencies of UH and LH modes, and signs «$\pm$» before braces – the frequencies of the fast (*F*) and slow (*S*) modes.

Thus, in the nonneutral plasma four families of electron modes – FUH, FLH, SUH, SLH - are available (Fig. 1). The frequencies of fast modes (FUH and FLH) are always positive. The frequencies of slow modes (SUH and SLH) due to Doppler shift can change a sign, crossing zero of frequency and region of ion frequencies. The SLH modes cross the region of ion frequencies at any azimuth wave numbers m (region I in Fig. 1 and Fig. 2), the SUH mode – at azimuth numbers $m \geq 2$ (region II in Fig. 1).

For azimuth wave number $m = 1$ the spectrum of modes of plasma, containing an additive of background gas ions, has been computed in [6]. In the present work the computation results of oscillation spectrum for azimuthal wave number $m = 2$ are presented.

The normalized frequencies ω'/Ω_i ($\omega' = \omega - m\omega_+$, $\omega_+ = \left(-\omega_{ci} + \Omega_i\right)/2$, $\Omega_i = \left(\omega_{ci}^2 - 4eE_r / m_i r\right)^{1/2}$ – "modified" ion cyclotron (MIC) frequency) versus parameter $q = 2\omega_{pe}^2 / \omega_{ce}^2$ were computed at the same numerical values of parametres, as in [6]: the ion mass is chosen equal to the mass of atomic nitrogen ion ($m_i = 14$ au), $k_z a = 0.1$, coefficient of charge neutralisation [2] $f = N / n_e = 0.01$ (N, n_e – the ion and electron densities).

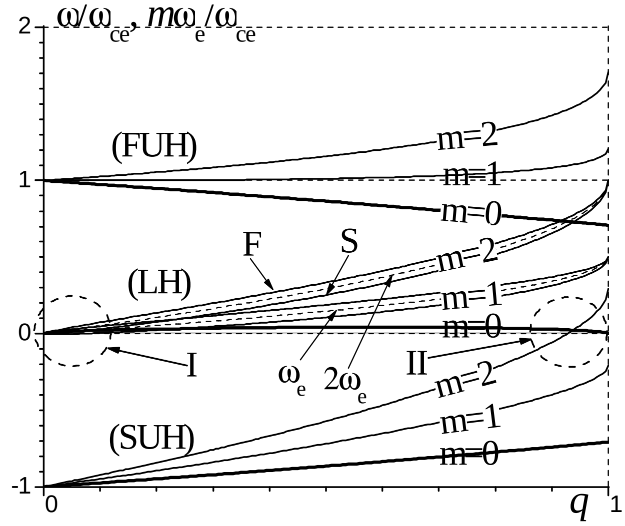

Fig. 1. Behaviour of modes (1) with azimuth numbers $m = 0, 1, 2$ in pure electron plasma ($f = 0$). The modes intersect the frequency zero in regions I and II. The shaped lines denote Doppler shifts $m\omega_e$, $\cos^2\theta = 10^{-2}$

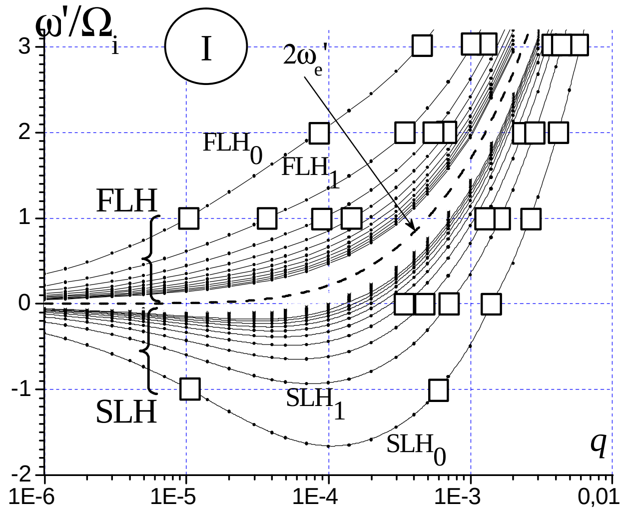

Fig. 2. Spectrum of electron modes with $m = 2$ in region I. MIC modes on the scale of figure coincide with the harmonics of MIC frequency. Square markers denote the regions of intersection of electron modes with MIC modes, $\omega_e' = \omega_e - \omega_+$

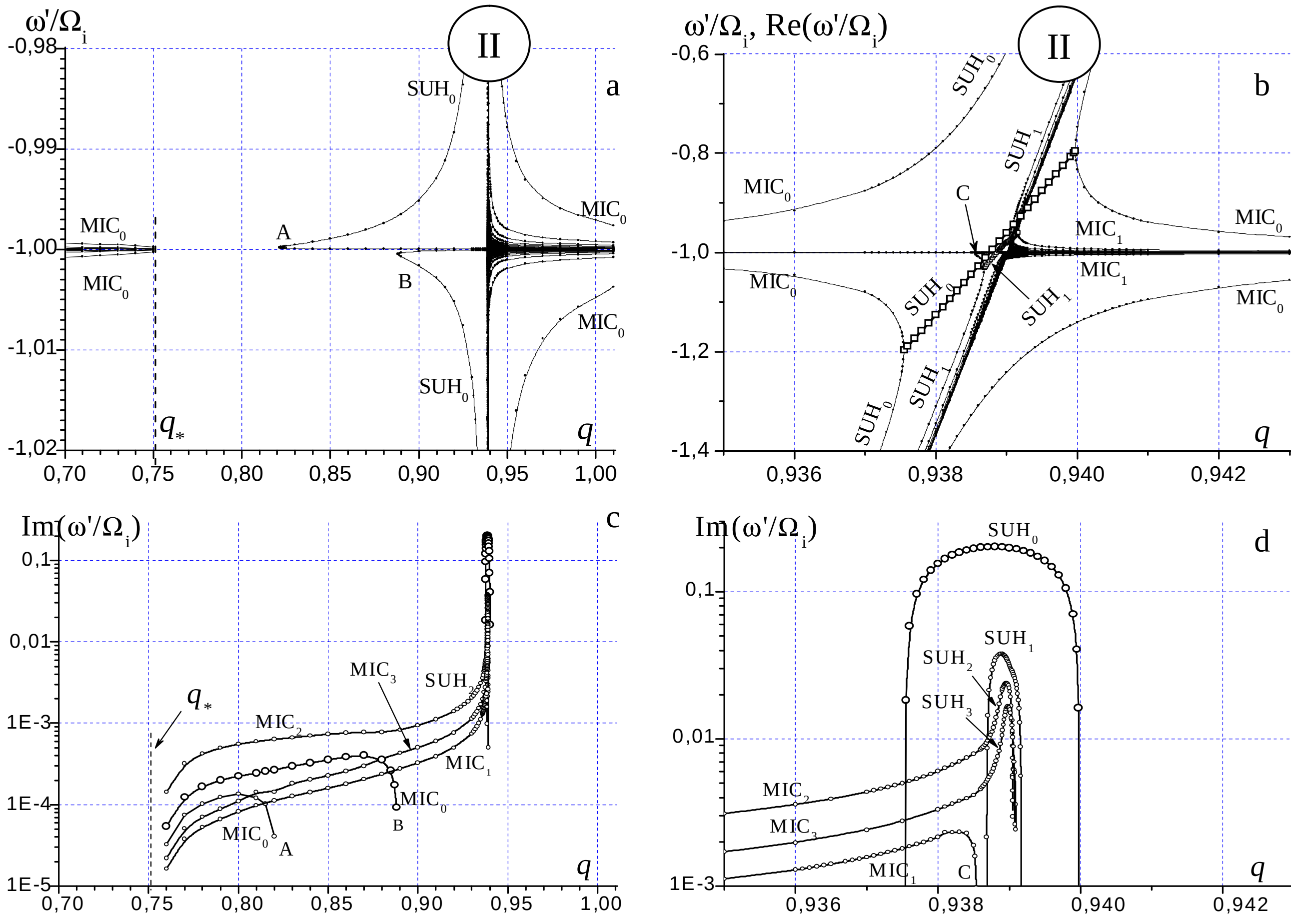

Fig. 3. The spectrum of oscillations with azimuthal number $m = 2$ in a vicinity of MIC frequency harmonic $\omega' / \Omega_i = n = -1$ in region $q > q_*$: a) the modes with the real frequencies are shown, b) the real frequencies and real parts of complex frequencies of SUH modes in the region II are presented on a large scale in q-axis, c) the imaginary parts of complex frequencies (growth rates) of SUH modes are shown, d) the same growth rates, as in (c), are presented on a large scale, corresponding to the scale of (b). Real and imaginary parts of complex frequencies are denoted by open markers. Indexes 0, 1, 2, 3 denote the numbers of radial modes. The shaped lines $q = q_*$ in (a) and (c) specify the position of the peculiarity of MIC modes behaviour, where the relation $m^2\omega_e^2 \approx \Omega_e^2$ is satisfied

Results of computations on dispersion equation are presented in Fig. 2, 3. The LH mode behavior in the region of intersection with MIC frequencies (region I in Fig. 1, Fig. 2) is similar to the case $m=1$ [6]. The spectrum consists of the families of FLH and SLH modes and families of MIC modes. Owing to the smallness of ion density the MIC modes are located in a narrow vicinity of harmonics of MIC frequency and coincide with them on a scale of Fig. 2. Because of the strictly circumscribed paper size we do not present here the figures, demonstrating behavior of modes within the region I on a large scale. We enumerate here only main results. FLH modes cross only positive harmonics of MIC frequency (Fig. 2) and remain stable. SLH modes cross the harmonics of MIC frequency with numbers $n \geq -1$ and become unstable in a vicinity of intersection with the non-negative harmonics of MIC frequency. The fastest growth rate, equal $\mathrm{Im}(\omega'/\Omega_i) \approx 0.04$, has the lowest (zero) radial mode SLH_0 near to intersection with zero harmonic of MIC frequency. It is almost twice less than the corresponding growth rate in the case $m=1$.

The MIC modes are unstable over a wide range of field changing in region I. Their fastest growth rates reach the value $\mathrm{Im}(\omega'/\Omega_i) \approx 0.002$. The behavior of MIC modes is similar to their behavior in the case $m=1$. The oscillations of small amplitude are observed on frequency profiles. They are similar to oscillations on dispersive curves of plasma in metals. This peculiarity is caused by the resemblance of the distribution function of background gas ions with the degenerate Fermi-Dirac distribution. There is also another peculiarity of MIC mode behavior, located in a case $m=2$ near the value $q=q_* \approx 0.75$ (Fig. 3,a, c). At this value of q the relation $m^2\omega_e^2 \approx \Omega_e^2$ is satisfied and a transverse component of a tensor of dielectric permeability of electrons ε_1 has a pole. The interaction of SUH and MIC modes in the region of stronger electric fields ($q > q_*$), including region II, is presented in Fig. 3. Because of the circumscribed paper size we present behavior of modes in a vicinity of the harmonic of MIC frequency $n=-1$ only. Near the other harmonics they behave in the same manner. Different radial SUH modes are arranged very closely to each other (Fig. 3,a, b).They cross the region of ion frequencies nearly erect. The SUH modes are unstable near the intersection with all harmonics of MIC frequency – both positive, and negative, and zero. The growth rates of SUH modes (Fig. 3,c, d) are much faster than the growth rates of SLH and MIC modes. The fastest value ($\mathrm{Im}(\omega'/\Omega_i) \approx 0.27$) has the lowest radial mode SUH_0 in a vicinity of intersection with zero harmonic of MIC frequency ($n=0$).

The causes of instabilities of both electron and MIC modes are a relative azimuth motion of electron and ion components of nonneutral plasma (a cross current) and anisotropy of the distribution function of ions, produced by ionization of background gas in crossed fields.

The modes with $m>2$ behave in the same manner as the modes with $m=2$. The modes with azimuth numbers m = 1, 2 exhaust the all variety of behavior types of nonneutral plasma modes. Thus, the spectra, obtained in [6] and in present article, yield the complete solution of nonlocal stability problem of nonneutral plasma with an additive of background gas ions, discussed in [1], in that specific case of plasma, completely filling a waveguide.

REFERENCES

1. R.H. Levy, J.D. Daugherty, O. Buneman // *Phys. Fl.* 1969, v. 12, p. 2616.
2. R.C. Davidson. *Theory of Nonneutral Plasmas,* New York: “Benjamin”, 1974.
3. V.G. Dem’yanov, Yu.N. Eliseev, Yu.A. Kirochkin et al. // *Fiz. Plazmy*. 1988, v. 14, p. 840.
4. D.B. Chibisov, V.S. Mikhailenko, K.N. Stepanov // *Plasma Phys. Controlled Fusion*. 1992, v. 34, p. 95.
5. Yu.N. Yeliseyev // *Plasma Phys. Rep.* 2006, v. 32, p. 927.
6. Yu.N. Yeliseyev // *Plasma Phys. Rep.* 2010, v. 36, p. 607.

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СПЕКТР КОЛЕБАНИЙ ЭЛЕКТРОННОЙ ПЛАЗМЫ С ДОБАВКОЙ ИОНОВ ФОНОВОГО ГАЗА (АЗИМУТАЛЬНОЕ ЧИСЛО *m = 2*)

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Для азимутальной моды m = 2 численно определен спектр колебаний заряженной плазмы, полностью заполняющей волновод, состоящей из «холодных» замагниченных электронов и добавки ионов, образовавшихся ионизацией атомов фонового газа электронным ударом, и имеющих анизотропную функцию распределения. Спектр состоит из семейств «модифицированных» ионных циклотронных (*MIC*) мод и семейств электронных верхнегибридных (*UH*) и нижнегибридных *(LH*) мод (мод Трайвелписа-Гоулда) с допплеровским сдвигом, обусловленным вращением электронов, из-за которого электронные моды попадают в область ионных частот, и возникает неустойчивость. Инкременты нарастания UH-мод значительно больше инкрементов LH-и MIC-мод.

СПЕКТР КОЛИВАНЬ ЕЛЕКТРОННОЇ ПЛАЗМИ З ДОБАВКОЮ ІОНІВ ФОНОВОГО ГАЗА (АЗИМУТАЛЬНЕ ЧИСЛО *m = 2*)

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Для азимутальної моди m = 2 чисельно визначено спектр коливань зарядженої плазми, яка повністю заповнює хвилевід та складається з «холодних» замагнічених електронів і добавки іонів, що утворилися іонізацією атомів фонового газу електронним ударом, і мають анізотропну функцію розподілу. Спектр складається із сімейств «модифікованих» іонних циклотронних (*MIC*) мод і сімейств електронних верхньогібридних (*UH*) і нижньогібридних (*LH*) мод (мод Трайвелпіса-Гоулда) з допплеровським зрушенням, обумовленим обертанням електронів, через що електронні моди потрапляють в область іонних частот, і виникає нестійкість. Інкременти наростання UH-мод значно більше інкрементів LH- і MIC-мод.