UDC 629.7

A. P. ALPATOV, Yu. M. HOLDSHTEIN

ON THE CHOICE OF THE BALLISTIC PARAMETERS OF AN ON-ORBIT SERVICE SPACECRAFT

Institute of Technical Mechanics
of the National Academy of Sciences of Ukraine and the State Space Agency of Ukraine,
15 Leshko-Popel St., Dnipro 49005, Ukraine; e-mail: aalpatov@ukr.net; jura_gold@meta.ua

В даний час спостерігається суттєве зростання вартості космічних апаратів (КА). У зв’язку з цим зростають вимоги до збільшення тривалості строків їх активного існування, надійності функціонування й зниженню експлуатаційних витрат. Багатообіцяючим шляхом задоволення цих вимог є впровадження технології орбітального сервісного обслуговування (ОСО). ОСО дозволяє вирішувати технічні та економічні проблеми за рахунок виконання сервісних операцій у космосі. Впровадження ОСО сприяє збільшенню строків активного існування КА, підвищенню надійності їх функціонування, зниженню витрат на підтримку в експлуатації орбітальних супутникових систем різного призначення. Програми ОСО з усунення КА, що завершили активне існування, або ушкоджених КА з робочих орбіт допомагають зм’якшити проблему нагромадження космічного сміття. Склад системи ОСО залежить від плануємих до виконання сервісних завдань і балістичних можливостей одноразових або багаторазових сервісних космічних апаратів (СКА). Реалізуємість і характер балістичних маневрів СКА багато в чому визначаються типом і характеристиками їх маршових двигунів. Метою статті є оцінка балістичного потенціалу сучасних і перспективних орбітальних сервісних апаратів і розробка методики планування раціональних маршрутів ОСО. Розглянуто й проаналізовано різні системи ОСО наземного та космічного базування. Проведено оцінку доцільності їх використання в залежності від плануємих для виконання сервісних завдань. Виявлено найбільш перспективні схеми ОСО. Запропоновано методику планування раціонального маршруту послідовного обходу орбіт КА, що обслуговуються, й наведено тестовий приклад розрахунку. Методика базується на розв’язанні багатокритеріальної задачі комівояжера. Багатокритеріальна задача комівояжера сформульована в термінах целочисельного лінійного програмування й приведена до однокритеріальної задачі методом адитивної згортки. Новизна запропонованої методики полягає у приведенні вихідної задачі до виду багатокритеріальної задачі комівояжера. Отримані результати можуть знайти застосування при обґрунтуванні, плануванні й здійсненні сервісних космічних операцій.

В настоящее время наблюдается существенный рост стоимости космических аппаратов (КА). В связи с этим возрастают требования к увеличению длительности сроков их активного существования, надежности функционирования и снижению эксплуатационных расходов. Многообещающим путём удовлетворения этих требований является внедрение технологии орбитального сервисного обслуживания (ОСО). ОСО позволяет решать технические и экономические проблемы за счет выполнения сервисных операций в космосе. Внедрение ОСО способствует увеличению сроков активного существования КА, повышению надежности их функционирования, снижению затрат на поддержание в эксплуатации орбитальных спутниковых систем различного назначения. Программы ОСО по устранению завершивших активное существование или поврежденных КА с рабочих орбит помогают смягчить проблему накопления космического мусора. Состав системы ОСО зависит от планируемых для выполнения сервисных задач и баллистических возможностей одноразовых или многоразовых сервисных космических аппаратов (СКА). Реализуемость и характер баллистических маневров СКА во многом определяются типом и характеристиками их маршевых двигателей. Целью статьи является оценка баллистического потенциала современных и перспективных орбитальных сервисных аппаратов и разработка методики планирования рациональных маршрутов ОСО. Рассмотрены и проанализированы различные системы ОСО наземного и космического базирования. Проведена оценка целесообразности их использования в зависимости от планируемых для выполнения сервисных задач. Выявлены наиболее перспективные схемы ОСО. Предложена методика планирования рационального маршрута последовательного обхода орбит обслуживаемых КА и приведен тестовый пример расчета. Методика базируется на решении многокритериальной задачи коммивояжера. Многокритериальная задача коммивояжера сформулирована в терминах целочисленного линейного программирования и приведена к однокритериальной задаче методом аддитивной свертки. Новизна предложенной методики состоит в приведении исходной задачи к виду многокритериальной задачи коммивояжера. Полученные результаты могут найти применение при обосновании, планировании и осуществлении сервисных космических операций.

At present, a significant increase in the cost of spacecraft is observed. Due to this fact the requirements for their active life duration, operational reliability, and operational cost reduction become more and more stringent. A promising way to meet these requirements is the introduction of on-orbit service (OOS). OOS allows one to solve technical and economic problems by performing service operations in space. The introduction of OOS contributes to extending the active life of spacecraft, increasing their operational reliability, and reducing the service maintenance cost of orbital satellite systems of various purposes. OOS programs on the deorbit of used or damaged spacecraft help in the mitigation of space debris problem. The composition of an OOS system depends on the service tasks to be performed and the ballistic capabilities of disposable or reusable service spacecraft. The realizability and character of the ballistic maneuvers of service spacecraft are largely determined by the type and characteristics of their sustainer engines. The aim of this paper is to assess the ballistic potential of modern and prospective OOS spacecraft and to develop a methodology for planning rational OOS routes. Various ground- and space-based OOS systems are considered and analyzed. The expediency of their use is estimated depending on the service tasks to be performed. The most promising OOS schemes are identified. A technique for planning a rational sequence of orbit transfers between the orbits of the spacecraft to be serviced is proposed and illustrated by the example of a test calculation. The technique is based on the solution of a multi-criteria traveling salesman problem, which is formulated in terms of integer linear programming and reduced to a single-criterion problem by the additive convolution method. The novelty of the proposed technique lies in reducing the original problem to a multi-criteria traveling salesman problem. The results obtained may be used in the justification, planning, and implementation of service space operations.

Keywords: *traveling salesman problem, spacecraft, multi-criteria optimization, on-orbit service, route planning***.**

Introduction. Due to the constant increase in the cost of spacecraft operating in orbit, the need for their servicing increases. Orbital servicing support (OSS) is an important direction for the increment of the efficiency of space activities. It provides the solution of technical and economic problems through the implementation of orbital service operations. The introduction of OSS increases the duration of the spacecraft lifetime, improves the reliability of their operations and reduces the cost for maintenance work of the orbital satellite systems for various purposes. The OSS also contributes to the solution of the space debris problem. Programs to eliminate damaged satellites from work orbits help mitigate this problem.

Orbital Service Schemes. The composition and design of the OSS system are determined by the planned servicing tasks [1 3]. The OSS system may consist of the following elements:

- disposable or reusable launch vehicle and overclocking unit;
- disposable or reusable service spacecrafts (SSC);
- serviced spacecrafts (SC);
- orbital technical centers - base stations;
- technical means of flight control and service operations.

The same SSC can be used in a disposable or reusable mode during the performance of various tasks. The location of the SSC is essential for the usage of its reusable mode.

When ground-based, after servicing the SSC on Earth, it is launched from the spaceport to the orbit of the serviced spacecraft

After the completion of service operations on the serviced SC, the SSC can relocate to the basic trajectory to refill the operating supply and wait for the next service request or terminate its existence.

Ground-based SSC can be used in the maintenance mode of orbital objects by the pattern of a shuttle or a sequential traversal.

For a space-based stage, the SSC is delivered in its unfilled state, entirely or in a disassembled form, to a low near-earth orbit, where it is finally assembled and filled with fuel. All subsequent refueling and repair works are also carried out in orbit.

There are great prospects for the creation of infrastructures in space, which include reusable SSC and orbital base stations designed for servicing the spacecraft when complex repairs are necessary.

Reusable SSC can serve a spacecraft by a shuttle or sequential traversal pattern. Shuttle pattern option assumes that, after each service operation, the SSC is returned to the base orbit or orbital station. In the case of a sequential bypass scheme of several serviced spacecraft, a return to the base orbit or the orbital base station is carried out after the completion of the bypass. Spacecraft service operations can be performed by direct SSC operation with the serviced spacecraft, if not, by using it for transportation of the spacecraft from its operational orbit to the orbital servicing base, after that the spacecraft is serviced it returns back to the orbit by the delivery platform. The implementation of these methods can be carried out by one, two or multiple SSC.

The following options are possible for two SSC:

- one of the SSC is used as an observer, and the second - for the direct performance of the task;

- one of the SSC is used as a base, and the second - for the direct performance of the task.

Group application of SSC is also possible in two versions: accomplishment of the task with the use of “swarm” tactics with prompt decision-making on the choice of a servicing SC; in the form of a functionally complete object with a rigid distribution of functional responsibilities between the SSC

Group application of SSC is also possible in two options:

execution of a task by using the "hive" tactics with immediate decision of servicing SSC;

in the form of a functionally complete object with a rigid distribution of functional responsibilities between the SSC.

The group application of the SSC seems to be more effective for the solution of target objectives of the OSS and increment of the stability of the functioning of orbital complexes and systems, from the efficiency point of view.

Thrust engine of a servicing spacecraft. The feasibility and the pattern of the SSC ballistic maneuvers are largely determined by the characteristics of their thrust engines, which are the essential system of any SSC. SSC marching engines are divided into: thermochemical, nuclear and electric, due to the type of energy used. [4].

Currently, thermo-chemical liquid rocket engines (LRE) and solid rocket engines (SRE) are mainly used in SSC, the electro rocket engines (ERE) are less commonly used. The nuclear engine for rocket vehicle applications (NERVA) are marked for further intend to use. Fig. 1 shows the classification of the SSC propulsion engines.

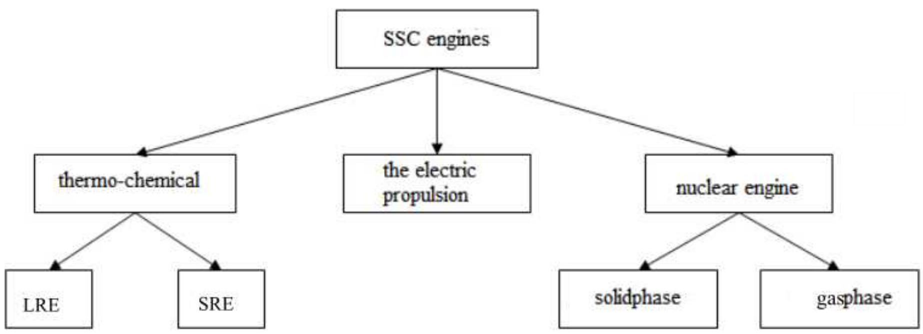

Figure 1 – Classification propulsion engines SSC

One of the most important characteristics of SSC engines is the specific thrust impulse. Table 1 shows the ranges of changes of the specific thrust impulse for various types of SSC engines.

Table 1 – Ranges of changes of specific thrust impulses

Engine type	SRE	LRE	NERVA	ERE
Specific thrust impulse, km/s	2 – 3	3 – 5	7 – 25	5 – 100

Upper stages of missiles, overclocking units or space tugs are currently mainly used to perform the service operation of transporting a spacecraft from a “base near-earth orbit” to a “high-energy orbit of destination”. Engines with high-boiling components are used for long flights and engines with cryogenic components are used for short-time flights. Table 2 shows the characteristics of LRE of overclocking units [5].

Table 2 – Characteristics of the LRE of overclocking units

Name	D, DM	Briz M, KM	Briz K, KS, Fregat	Agena D	Centaur D
Engine	11D58M	14D30	S5.92	Bell 8096	RL10A31
Thrust, kN	85	19,62	19,91	71,6	131
Specific impulse, km/s	3,538	3,255	3,270	2,855	4,356
Number of switchings	6	10	20		
Combustion time, s	720	3000			
Operational endurance, h	<7	Unlimited	Unlimited		<7
Mass, kg	310	95	76,5		

Existing overclocking units are characterized by a high unit cost of transporting payloads (PL) to high orbits, as well as a low mass of PL.

In this regard, reusable SSC with ERE are actively developed. The potential benefits of SSC with ERE overused resources are shown in [6 8]. Reusable SSC with low-power, but economically efficient ERE is ten times more economical than traditional overclocking units.

Distinctive features of SSC with ERE are:

– significantly greater impulse and extremely low level of flight and, as a result, a very long flight time, as compared with the LRE, SRE and NERVA;

– high level of energy consumption.

Two main types of power plants are currently being considered: solar and nuclear power plants and, accordingly, the solar electric propulsion engine (SEPE) and the nuclear electric propulsion engine (NEPE).

In electro-thermal ERE, electric power is used only for heating of the working mass the combustion chamber; its acceleration is performed due to the effect of gas-dynamic forces in the same way as in an LRE and NERVA. It is considered to distinguish electric heating, electric arc and induction motors, due to the principles used for heating of the working mass.

The wide use of electro-thermal ERE is constrained by a low specific thrust impulse (compared to other types of electric propulsion), which is not much higher than that of chemical engines (up to 9000 m/s in promising electric propulsion projects with hydrogen as a working mass).

In an electrostatic engine, thrust is created by the acceleration of the like-charged particles of the working mass in an electric field. Ionic and colloidal engines are distinguished by type of accelerated particles.

In an ion engine - the working mass is first ionized, by the ion engine [9], after those positive ions are accelerated in an electrostatic field (using an electrode system) and, create a thrust, by flowing out of the nozzle (the electrons are injected into it to neutralize the charge of the jet stream).

In a colloidal engine, the working mass executed in the form of positively charged microscopic particles, which are accelerated by an electrostatic field.

In an electromagnetic ERE the acceleration of the working mass is performed by the electromagnetic force. The thrust is created by the interaction of charged particles with a magnetic field. Depending on the method of the power supply, electromagnetic engines are divided into stationery (non-stop action) and pulsed.

The aggregated indicators of ERE of various types are shown in Table 3.

Table 3 – The aggregated indicators of ERE

Characteristics	Engines		
	Electro-thermal	Electro-magnetic	Electro-static
Thrust, N	0,01…1	0,001…2,5	0,001…0,1
Specific impulse, km/s	1…25	10…70	30…100
Power, W	Tens - thousands	Ones-thousands	Tens-hundreds

Currently, much attention is paid to stationary plasma (SPT) and ion engines [8, 9]. Table 4 and Table 5 present the technical characteristics of SPT and ion engines.

Table 4 – SPT characteristics

Characteristics	SPT-50	SPT-100	SPT-140
Thrust, mN	20	83	80-280
Specific impulse, km/s	12,5	25	15 - 26
Power, W	0,350	1,221	1,2 - 6,0
Operating life, h	2250	7500	10000
Mass, kg	0,8	3,5	7

Table 5 – Characteristics of ion engines

Characteristics	XIPS-13	XIPS-25	RIT-10
Thrust, mN	18	165	15
Specific impulse, km/s	23,5	35	34
Power, kW	0,45	4,2	0,59
Country	USA	USA	EU

It should be noted that the ERE is the most promising for reusable SSC, used for multiple flights between the base and high-energy orbits. The efficiency of the use of ERE will increase with increasing frequency of use of the SSC. The analysis of the field of application of SSC with ERE shows their advantage versus SSC on thermochemical engines in problems where flight time is not a critical parameter. SSC based on ERE is preferably used for flights that require relatively high energy costs and are unlimited in time. For example, for transportation from a low near-earth orbit to geostationary heavy oversized structures with a limited level of permissible overloads.

Mathematical models of interorbital flights. Due to fuel consumption, during the planning of SSC interorbital flights with engines of high and low thrust, in the first approximation it will be possible to use [10] a system of differential equations for the controlled undisturbed motion of the SSC in the central field of one attracting center with the equation of mass change:

When planning SSC interorbital flights with high and low thrust engines, in the first approximation, you can use [10] a system of differential equations for the controlled undisturbed motion of the SSC in the central field of one attracting center along with the mass change equation due to fuel consumption:

$$\frac{d^2\bar{r}}{dt^2} = \frac{\mu}{r^3} r + \frac{P\bar{e}}{m}\delta, \qquad \frac{dm}{dt} = -q\delta = -\frac{P}{w}\delta, \tag{1}$$

where $\bar{r}$ – - is the current radius vector; μ – the gravitational constant of the Earth; P – rocket engine thrust; m – the current mass of SSC; q and w – fuel consumption and outflow rate of the working mass; $\delta(t)$ – relay on-off function of the engine; $\bar{e}(t)$ – single thrust vectoring function.

The duration of the operation, the direction of the thrust vector and the number of motor starts depend on the parameters of the initial and final orbits. During the maneuver calculation, such conditions are determined for it that minimizes fuel consumption or other criteria of optimality, such as, the time of flight from one orbit to another. The system of equations (1) is supplemented by the initial conditions of the maneuver and the change of the thrust vector time law. Models of perfectly regulated and unregulated engines are considered. Ideally adjustable engines are subject only to the most important limitations for the type of engines that are under consideration, such as the limits on their thrust or rate of flow. Unregulated engines can either be turned on, and then the thrust and flow rates are constant, or off, and then the thrust and flow rates are zero. There are no restrictions on changing the direction of thrust.

The equations of the system (1) contain variables that oscillate rapidly due to the periodicity of motion along the trajectory. This complicates the numerical analysis of the SSC multi-turn trajectories with low-thrust engines. To eliminate this deficiency, the equations in equinoctial elements [11] are used. They have no peculiarities with zero eccentricity and inclination.

Along with direct numerical integration of systems of differential equations, the approaches based on simplifying assumptions are used to simulate SSC interorbital flights. Depending on the magnitude and duration of the action of the thrust force, pulsed maneuvers and low-pressure maneuvers are considered.

These types of maneuvers, as a rule, determined by the energy capabilities and the operating principle of the used SSC main engines, which are divided into large or intermediate and low thrust engines. SSC with high or intermediate propulsion engines have the initial thrust-to-weight ratio, $n_0 \in [0{,}03, 1]$, and those ones with low-thrust engines have the initial thrust-to-weight ratio $n_0 < 0{,}03$.

During inter-orbital maneuvering usage of high thrust engines, the duration of active flight segments is negligible, as compared with the duration of passive flight segments. Therefore, when calculating interorbital flights using high-thrust engines, you can use a pulsed approximation of the thrust action, which reduces to an abrupt change in flight speed without changing the coordinates of the SSC during engine operation. The calculation of such a maneuver involves the determination of the number of orientation, and points of application of velocity pulses.

The usage of SSC with a low-thrust engine for a flight between near-circular orbits, near the Earth, leads to multiple spiral paths. The time of active flight of such SSC is very long and in many cases may coincide with the time of flight

A characteristic feature of low thrust engines consists of the fact that their control acceleration is quite small in comparison with local gravitational acceleration. This eliminates the possibility of using the pulse approach.

Equations in osculating elements are often used as equations of motion [10], for occasions of interorbital maneuvering using low thrust engines.

The model in osculating elements is the most convenient one for simplifying the methods of asymptotic separation of variables[12], [13]. Firstly, this caused, by the presence of a small parameter in an explicit form – the reactive acceleration from thrust, which is less than the gravitational one, by several orders of magnitude. Secondly, the presence of a cyclic variable – the angular coordinate characterizing the position of the spacecraft in orbit relative to the line of nodes or the pericenter. The used approach allows obtaining a substantially simpler averaged system of equations for simulation of interorbital flights of SSC with low thrust engines.

The route optimization of the sequential bypass of the orbits of the servicing spacecraft. The selection of a rational route of the SSC involves the solution of the corresponding route-trace optimization problem. The problem of optimizing the bypass of a sequence of serviced spacecraft belongs to the type of multicriteria route-trace discrete-continuous problems.

The general direction for the solution of the route-trace discrete-continuous problems is their splitting by decomposing discrete and continuous parts and independent solution of those parts. Initially, optimization of all possible individual flights between orbits performed, and then the optimal sequence of flights selected. Decomposition reduces the task to the sequence of the solution of two simpler problems and allows choosing the most efficient route for the sequential service of spacecraft group. For cases when the decomposition of the problem of independent continuous and discrete subtasks is impossible, we should pick the method of dynamic programming.

In the particular case when the sequence of serviced spacecraft is predetermined, the solution of the discrete part is eliminated. After that, it remains to solve the sequence of continuous optimization problems for interorbital flights by methods of optimal control on each of the trajectory segments.

The trace part consists of the construction of the trajectory that satisfies a set of constraints and minimizes the functionality for the chosen sequence of visits to the serviced spacecraft. It is an optimal control problem. The complexity of such problems lies in the evaluation of the functional for a large number of considered combinations in solving the discrete part of the problem.

Optimization of the flight trajectories of spacecraft for a long time has been the subject of intensive research [14 - 18]. There are two main types of path optimization methods: indirect and direct, or combinations thereof. Indirect methods are reduced to the solution of a two-point boundary value problem based on the maximum principle of L. S. Pontryagin. In such a boundary value problem, the unknown variables are conjugate variables that have high sensitivity, which complicates the search for the initial approximation. Direct methods transform the original problem to a parametric optimization problem, which is usually solved with help of non-linear programming methods.

The possibility of application of pulse approximation for simulating the controlled motion of SSC with high thrust engines allows using numerical methods of nonlinear programming for the optimization of the trajectories of interorbital flights, along with the methods of the theory of optimal control.

The route part of the problem is discrete in nature and enables mathematical formalization in the form of a generalized traveling salesman problem, in which the SSC must go around the n orbits, starting with the reference orbit, on which it is located, and finish its route, returning to the original orbit without having been anywhere twice. The n destination orbits and the flight cost matrix C between any pair of them are considered as given. Expenses for the flight between any pair of orbits can be used in form of energy (characteristic speed or motor flight time for an SSC with a low thrust engine), time or cost. In this problem, the total cost of the full path starting and ending in the reference orbit presented as the evaluation function, the presence or absence of a flight between the individual orbits of the list in question, as well as the need to visit all of these orbits, are the limitations.

The matrix C is generally accepted as asymmetric. The size of this matrix $n \times n$, and its diagonal elements can be arbitrary, since they are not used anywhere in the solution. The remaining elements of the matrix C in the sense of the problem are non-negative.

We can formulate the considered traveling salesman problem formulated in terms of integer linear programming [19–20]. We denote e_{ik} as integer variables associated with arcs. There are as many variables as there are arcs. – $m = n(n-1)$. Each of them can take only one of two possible values: 1 or 0, depending on whether or not the arc is in the desired cycle

$$e_{ik} = 0 \vee 1,$$
$$i,k = \overline{1,n, i \neq k}. \tag{2}$$

The objective function in this problem is the total weight of the arcs included in the cycle:

$$z = \sum_{\substack{i,k=1 \\ i \neq k}}^{n} c_{i,k} e_{i,k} \to \min. \tag{3}$$

In addition, to the variables e_{ik} (2) the following restrictions are imposed. From each vertex there should be exactly one arc:

$$\sum_{\substack{k=1 \\ k \neq i}}^{n} e_{i,k} = 1, \quad i = \overline{1,n}. \tag{4}$$

Each vertex must also include one arc:

$$\sum_{\substack{i=1 \\ i \neq k}}^{n} e_{i,k} = 1, \quad k = \overline{1,n}. \tag{5}$$

Finally, it is necessary that the cycle was complete, that is, consisting of n vertices and n arcs. After all, constraints (4), (5) are satisfied, for example, by a system of several smaller cycles, which together cover all of the n vertices. To eliminate such subcycles, unbounded real variables v_i, associated with vertices are introduced. The system of constraints that eliminates subcycles has the following form:

$$v_i - v_k + (n-1)e_{ik} \leq n-2, \quad 2 \leq i \neq k \leq n. \tag{6}$$

Equations (2) - (6) define the mixed problem of integer linear programming corresponding to the original traveling salesman problem. The problem is mixed because the parts of variables are an integer, and parts are unlimited real. In total, the problem uses $n(n-1)$ integer variables with $e_{i\,k}$ constraints (4), (5) and n unbounded real variables v_i. They are subject to restrictions (6). Target function (3) is minimized.

The considered traveling salesman problem lies in the NP class of complete tasks that cannot be solved with polynomial algorithms. Its computational complexity grows exponentially with the size of the problem.

To solve it, exact and heuristic algorithms are used, and exact algorithms are applicable only for small-sized tasks. The complexity of the problem under consideration leads to a wide use of heuristic algorithms that give approximate solutions in less time than exact methods.

Optimization of the route part of the problem simultaneously by several criteria (for example, the Delta-v and time of the orbital flight) leads to the necessary solution of the two-criteria salesman problem. Assignment n orbits and two matrices $C^{\Delta V}$ and C^T with the Delta-v and flight times between any pair of them are considered as given. Conditions (2) and restrictions (4) - (6) are satisfied. The following objective task functions are minimized:

$$z^{\Delta V} = \sum_{\substack{i,k=1 \\ i \neq k}}^{n} c_{i,k}^{\Delta V} e_{i,k} \rightarrow \min, \quad z^{T} = \sum_{\substack{i,k=1 \\ i \neq k}}^{n} c_{i,k}^{T} e_{i,k} \rightarrow \min \tag{7}$$

The solution of the two-criterion minimization problem can be performed by the methods of additive convolution, successive concessions or the minimum deviation from the ideal point. Let's consider in more detail the method of additive convolution.

The method of additive convolution allows to bring up the two-criteria optimization problem (7) to a single-criterion one. If we set some values of weight coefficients α_1 and α_2 objective functions that satisfy the conditions $\alpha_1, \alpha_2 \in [0, 1]$ and $\alpha_1 + \alpha_2 = 1$, then the additive convolution of two criteria into one is performed according to the formula:

$$z = \alpha_1 z^{\Delta V} + \alpha_2 z^{T}. \tag{8}$$

Different in a physical sense, the elements of matrices $C^{\Delta V}$ and C^{T} can vary greatly in absolute values. It is necessary to standardize them, to eliminate this distinction and perform the convolution of equivalent criteria by formula (8). Standardization allows to bring elements of matrices $C^{\Delta V}$ and C^{T} to the same area of their change. Elements $\tilde{c}_{i,k}^{\Delta V}$ and $\tilde{c}_{i,k}^{T}$ of linearly standardized matrices $\tilde{C}^{\Delta V}$ and $\tilde{C}^{T}$ are calculated by the formulas (9)

$$\tilde{c}_{i,k}^{\Delta V} = \frac{c_{i,k}^{\Delta V} - \min\left(c_{i,k}^{\Delta V}\right)}{\max\left(c_{i,k}^{\Delta V}\right) - \min\left(c_{i,k}^{\Delta V}\right)}, \quad \tilde{c}_{i,k}^{T} = \frac{c_{i,k}^{T} - \min\left(c_{i,k}^{T}\right)}{\max\left(c_{i,k}^{T}\right) - \min\left(c_{i,k}^{T}\right)}. \tag{9}$$

Thus, the initial minimization problem (7) is reduced to the one-dimensional minimization problem of the objective function (10)

$$z = \sum_{\substack{i,k=1 \\ i \neq k}}^{n} \left(\alpha_1 \tilde{c}_{i,k}^{\Delta V} + \alpha_2 \tilde{c}_{i,k}^{T}\right) e_{i,k} \rightarrow \min. \tag{10}$$

An example of optimizing the route of a sequential bypass of orbits serviced by spacecraft. Let's present an example of optimizing the route of a sequential bypass of five orbits of spacecraft serviced, to demonstrate the proposed methodology for choosing a rational orbital service route. The orbit parameters are shown in table 6.

Table 6 – Parameters of the orbits of the servicing spacecraft

Number of the orbit	Major semi-axis, km	Ellipticity	Inclination of orbit, deg
1	7303,80	0,010	67,84
2	7725,86	0,052	68,75
3	8566,31	0,106	67,66
4	7885,64	0,084	68,97
5	7061,74	0,056	67,80

Let us consider the orbital flight between a pair of non-coplanar elliptical orbits in the form of a sequence of five single-pulse flights. The first impulse is necessary for an orbital flight between the initial elliptical orbit and the circular with a radius equal to the radius of the apogee of the initial orbit in cases when the radius of the apogee of the initial orbit is greater than the radius of the apogee of the final orbit. The magnitude of the accelerating pulse applied at the apogee ΔV_1 is calculated by the formula

$$\Delta V_1 = \sqrt{\frac{\mu}{r_{ан}}}\left(1 - \sqrt{\frac{2 r_{пн}}{r_{пн} + r_{ан}}}\right), \tag{11}$$

where μ - is the gravitational constant of the Earth, $r_{ан}$ and $r_{пн}$ - the radii of the apogee and perigee of the initial orbit.

The second velocity impulse implements the rotation of the plane of the initial orbit by the angle of non-coplanarity χ until it coincides with the plane of the final orbit. The formula for the impulse calculation of the characteristic velocity, that is necessary to rotate the plane of the orbit by the noncoplanar angle χ, has the form of

$$\Delta V_2 = 2\sqrt{\frac{\mu}{r_{ан}}}\cos\theta\,\sin\frac{\chi}{2}, \tag{12}$$

where θ - is the angle between the velocity vectors and the plane of the local horizon at the time of the pulse

A two-pulse Hohmann control program is used for the implementation of a coplanar orbital flight between a circular orbit with a radius that is equal to the radius of the apogee of the initial orbit, and a circular orbit with a radius equal to the radius of the apogee of the final orbit. The transition trajectory represents a semi-ellipse of Hohmann, the perigee of which is in the initial orbit, and the apogee on the final. The cost of the Delta-v for the Hohmann maneuver is calculated by the formulas:

$$\Delta V_3 = \sqrt{\frac{\mu}{r_{ак}}}\left(1 - \sqrt{\frac{2 r_{ан}}{r_{ан} + r_{ак}}}\right), \tag{13}$$

$$\Delta V_4 = \sqrt{\frac{\mu}{r_{ан}}}\left(\sqrt{\frac{2 r_{ак}}{r_{ан} + r_{ак}}} - 1\right), \tag{14}$$

where $r_{ан}$, $r_{ак}$ — are the apogee radii, respectively, of the initial and final orbits. The flight time for the Hohmann semi-ellipse is determined by the formula

$$T = \pi\sqrt{\frac{a^3}{\mu}}, \tag{15}$$

where $a = (r_{ан} + r_{ак})/2$ - is the length of the major semiaxis of the Hohmann ellipse.

The fifth impulse of the characteristic speed is calculated by the formula (16) and is intended for the flight between a circular orbit with a radius equal to the radius of the apogee of the final orbit and the final elliptical orbit with the perigee $r_{пк}$ и and apogee $r_{ак}$:

$$\Delta V_5 = \sqrt{\frac{\mu}{r_{ак}}} \left(\sqrt{\frac{2 r_{пк}}{r_{пк} + r_{ак}}} - 1 \right). \quad (16)$$

The total characteristic flight speed is determined by the expression:

$$\Delta V = \sum_{i=1}^{5} \left| \Delta V_i \right|. \quad (17)$$

In the case where the radius of the apogee of the initial orbit is less than the radius of the apogee of the final orbit, the interorbital flight between them can be performed in a similar way.

Tables 7 and 8 show the matrices formed by the initial data of table 6 using formulas (11) – (16) of the matrix $C^{\Delta V}$ and C^{T} and the characteristic speeds and transit times between any pair of orbits of servicing spacecraft. The units of the matrix elements are the characteristic speeds km/s, and the matrix elements of the minute flight.

Table 7 – Matrix of Delta-v between pairs of orbits of serviced by spacecraft

Orbit number	1	2	3	4	5
1	0,00	0,68	1,24	0,99	0,29
2	0,68	0,00	1,18	0,68	0,82
3	1,27	1,18	0,00	1,14	1,40
4	0,99	0,68	1,14	0,00	1,12
5	0,29	0,82	1,40	1,12	0,00

Table 8 – The matrix of flight times between pairs of orbits of serviced by spacecraft

Orbit number	1	2	3	4	5
1	0,00	56,63	64,15	58,94	52,97
2	56,63	0,00	68,48	63,15	57,04
3	64,15	68,48	0,00	70,95	64,59
4	58,94	63,15	70,95	0,00	59,36
5	52,98	57,04	64,59	59,36	0,00

As a result of $C^{\Delta V}$ and C^{T} matrices standardization accordingly to formulas (9), the additive convolution of two criteria into one in accordance with (10) and solving the single-criterion traveling salesman problem obtained by using the branch and bound method, the results are presented in table 9 for different values of weights. α_1 and α_2 objective function.

Table 9 – Calculation results

α_1	α_2	Route	The cost route of the characteristic speed, km/s	Duration of flight, min
1,0	0,0	1 2 4 3 5 1	4,18	308,29
0,0	1,0	1 3 5 2 4 1	5,15	307,87
0,5	0,5	1 2 4 3 5 1	4,18	308,29

The calculation option with α_1 =1 и α_2 =0 corresponds to the minimization of the Delta-v of the flight, the calculation option with α_1 =0 and α_2 =1 corresponds to the minimization of the flight time, and the calculation option with α_1 =0,5 and α_2 =0,5 corresponds to the two-criteria minimization with the same weight coefficients for the Delta-v and flight time. The cost of the Delta-v for the flight depends significantly on the chosen route, and the duration of the flight was insensitive to the choice of route, for the considered group of close orbits of the servicing spacecraft and the type of maneuver of interorbital flights.

Conclusion. From the point of view of the choice of ballistic parameters, variants of OSS systems and advanced main engines SSC are considered and analyzed. An assessment of the feasibility of their use depends on the planned to perform service tasks. The schemes of most promising serviced objects are identified. Method for planning a rational route of a sequential bypass of the orbits of spacecraft has been proposed. The calculation by the proposed method is presented as an example. The obtained results can be used for the justification, planning and implementation of space service operations.

This work was funded by the Ukrainian Budget Program “Support of the Development of Priority Lines of Research” (KPKVK 6541230).

1 *Ivanov V. M.* Conceptual foundations of orbital servicing for advanced automatic space vehicles (*in Russian*). Spacecraft and Rockets. Vestnik Moskovskogo Aviatsionnogo Instituta. 2008. V. 15. No. 3. Pp. 5 – 7.

2 *Barbee B.* Design of Spacecraft Missions to Remove Multiple Orbital Debris Objects. Specialist Conference, Paper AAS 15-598, August 2015.

3 *Stephen J.* Design for on-orbit spacecraft servicing. Specialist Conference, Paper AAS 14-374, October 2014.

4 *Caveny L.* Space Engines: State of the Art and Prospects (*in Russian*). Moscow : Mir, 1988. 454 pp.

5 *Serdyuk V. K., Tolyarenko N. V., Khlebnikov N. N.* Space program support vehicles (*in Russian*). Rocket Production and Space Engineering: VINITI. Moscow. 1990. V. 11.

6 *Gusev Yu. G., Pilnikov A. V.* Pole and place of electrojet engines in the Russian Space Program (*in Russian*) / Trudy MAI (electronic journal). Issue 60. https:// www.mai.ru/science/trudy/.

7 *Vasin A. I., Koroteev A. S., Lovtsov A. S.* Overview of the work on electrojet engines at the Keldysh Research Center (*in Russian*). Trudy MAI (electronic journal). Issue 60. https:// www.mai.ru/science/trudy/

8 *Kim V.* Stationary plasma engines in Russia: problems and prospects (*in Russian*). Trudy MAI (electronic journal). Issue 60. https:// www.mai.ru/science/trudy/

9 *Gorshkov O. A., Muravlev V. A., Shagaida A. A.* Hall and Ion Plasma Engines for Spacecraft (*in Russian*). Moscow: Mashinostroyeniye, 2008. 280 pp.

10 *Narimanov G. S.* Basics of Spacecraft Flight and Design Theory (*in Russian*). Moscow.: Mashinostroyeniye, 1972. 607 pp.

11 *Petukhov V. G.* Robust suboptimal feedback control for low-trust transfer between noncoplanar elliptical and circular orbits (*in Russian*). Spacecraft and Rockets. Vestnik Moskovskogo Aviatsionnogo Instituta. 2010. V. 17, No. 3. Pp. 50 – 58.

12 *Lebedev V. N.* Low-Thrust Spacecraft Flight Calculation (*in Russian*). Moscow: Coputing Center of the USSR Academy of Sciences, 1968. 105 pp.

13 *Grishin S. D., Zakharov Yu. A., Odolevsky V. K.* Low-Thrust Spacecraft Design (*in Russian*). Moscow: Mashinostroyeniye, 1990. 223 pp.

14 *Ilyin V. A., Kuzmak G. E.*. Optimal Spacecraft Orbit Transfers (*in Russian*). Moscow: Nauka, 1976. 740 pp.

15 *Grozdovsky G. L., Ivanov Yu. N., Tokarev V. V.* Space Flight Mechanics (Optimization Problems) (*in Russian*). Moscow: Nauka, 1975. 704 pp.

16 *Salmin V. V.* Methods for the Solution of Variational Problems in Low-Thrust Space Flight Mechanics (*in Russian*). Samara : Samara Research Center of the Russian Academy of Sciences, 2006. 127pp.

17 *Petukhov V. G.* Optimization of interplanetary trajectories for spacecraft with ideally regulated engines using the continuation method. Cosmic Research. 2008. V. 4. No. 3. Pp. 219–232.

18 *Grigoriev I. S., Zapletin M. P.* Choosing promising sequences of asteroids. Automation and Remote Control. 2013. V. 74. Issue 8. Pp. 1284 – 1296.

19 *Papadimitriou C., Steiglitz K.* Combinatory Optimization. Algorithms and Complexity (*in Russian*) / Moscow: Mir, 1982. – 510 pp.

20 *Sigal I. Kh*. Algorithms for solving the two-criterion large-scale travelling salesman problems. Computational Mathematics and Mathematical Physics. 1994. V. 34. No. 1. Pp. 33–43.

Received on 10.10.2018,
in final form on 26.11.2018