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This paper presents a marching algorithm for the numerical solution of "viscous layer" equations (simplified Navier–Stokes equations in which the second derivatives with respect to the longitudinal coordinate are

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dropped and dissipative terms in the equation for the radial velocity component are discarded) to calculate the turbulent flow in a multicomponent supersonic jet of a rocket engine outflowing into a submerged space or a subsonic cocurrent air flow. In this algorithm, the equations are divided into two groups. The first group includes the equations for the longitudinal momentum component, energy, gas mixture component transfer, and turbulence characteristics. These equations are of the parabolic type, and they can be solved with a marching method by an implicit absolutely stable iterative scheme using scalar marching. The second group includes the equation for the radial momentum component and the continuity equation. In combination with the state equation, these equations are considered as equations in the pressure and the transverse velocity component. An algorithm for solving this group of equations is formulated. In this algorithm, a tridiagonal matrix system of equations for the grid values of the pressure is formed in solving the equations of the second group. A boundary-value problem is formulated for this system: the radial coordinate derivative of the pressure on the jet axis is zero, and the pressure on the jet boundary is set equal to that in the submerged space or in the concurrent subsonic flow. The solution of this problem is used in the determination of the pressure in the supersonic flow region. In the outer subsonic part of the jet, the pressure is put to be equal to that in the outflow space. The transverse velocity component in the super- and the subsonic part of the jet is determined using a relation that results from the difference approximation of the continuity equation.

The use of the proposed marching algorithm in the calculation of a rocket engine jet outflowing into a submerged space allows a continuous transition from a nonisobaric fully supersonic flow at the nozzle outlet to a subsonic flow in the jet to the point of its compete mixing with the air in the outflow space.

To verify the proposed algorithm, the results calculated thereby are compared with experimental data.

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— — [4], :

$$\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \nu \frac{\rho v}{y} = 0, \qquad (1)$$

x

$$...u \frac{\partial u}{\partial x} + ...v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{1}{y^{\epsilon}} \frac{\partial}{\partial y}\left(y^{\epsilon} \sim_{t} \frac{\partial u}{\partial y} \right), \qquad (2)$$

y

$$...u \frac{\partial v}{\partial x} + ...v \frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y}, \qquad (3)$$

18

$$\rho u \frac{\partial H}{\partial x} + \rho v \frac{\partial H}{\partial y} = \frac{1}{y^{\nu}} \frac{\partial}{\partial y}\left(y^{\nu}\mu_t \frac{\partial H}{\partial y}\right) + \frac{1}{y^{\nu}} \frac{\partial}{\partial y}\left[\left(\frac{1}{\Pr_t} - 1\right)y^{\nu}\mu_t \frac{\partial h}{\partial y}\right], \quad (4)$$

$$\rho u \frac{\partial X_k}{\partial x} + \rho v \frac{\partial X_k}{\partial y} = \frac{1}{y^{\nu}} \frac{\partial}{\partial y}\left(y^{\nu}\left(\frac{\mu}{Sm}\right)_{ef} \frac{\partial X_k}{\partial y}\right) + \dot{\omega}_k, \quad k = \overline{1, N_k - 1}, \quad (5)$$

u, v – x, y (x

            , y –   ), ρ –   -

, p –   , X_k – k -   -

        , $H = h + 0{,}5\left(u^2 + v^2\right)$ –   , h –

      , μ_t –   ,

$\Pr_t, Sm$ –   ,   , $\dot{\omega}_k$ –   -

                                                    k -

, $\nu = 0$; $\nu = 1$.

                                    (1) – (5)   -

$$p = R_0 \sum \frac{X_k}{m_k} \rho T, \qquad R_0, \ T, \ m_k \ -$$

, k -   .

                                                                              -

                    . .   -

                $\mathfrak{E}_t$ [5]   [6]

$$\rho u \frac{\partial \nu_t}{\partial x} + \rho v \frac{\partial \nu_t}{\partial y} = \beta_o f_M(M)\mu_t\left|\frac{\partial u}{\partial y}\right| + \frac{1}{y^{\nu}} \frac{\partial}{\partial y}\left(\frac{y^{\nu}\mu_t}{Kr_{\nu_t}} \frac{\partial \nu_t}{\partial y}\right), \qquad (6)$$

$\mu_t = \rho\nu_t$, $\beta_o, \xi_o, Kr_{\nu_t}$ –   , $\beta_o = 0{,}2$; $Kr_{\nu_t} = 0{,}5$;

$$f_M(M) = \begin{cases} 1 & M < 1, \\ 1/M & M \geq 1 \end{cases}$$

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2.   .

                                    (1) – (5)   -

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    (2),   (4)   (6).

        (5).   (1)

                                    (3).

                        , P

v .   ,   -

                                    u , H -

                    ν_t ,

            ,

–   [7]

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p   v,   ,   u , H , X_k   v_t   ,

...   .

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,   p

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$$\eta = \frac{y}{S(x)} \quad (y = S(x) - \qquad\qquad\qquad\qquad),$$

$$\frac{\partial(y^{\nu} S\, p/f \cdot u)}{\partial x} + \frac{\partial(y^{\nu}\rho v)}{\partial \eta} = \frac{\partial(y^{\nu}\eta S_x'\rho u)}{\partial \eta}\ ,$$

$$S\rho u \frac{\partial v}{\partial x} + \rho v_n \frac{\partial v}{\partial \eta} + \frac{\partial p}{\partial \eta} = 0\ ,$$

(7)

$$v_n = v - S_x'\eta u\ ,\ S_x' = \frac{dS}{dx}\ ,\ f = R_0 \sum_{k=1}^{K} \frac{X_k}{m_k} T\ .$$

-   ,

[4]   -

$$a_{1j} v_{j+1} + b_{1j} v_j + c_{1j} p_{j+1} + d_{1j} p_j = e_{1j}\ ,$$
$$a_{2j} v_{j+1} + b_{2j} v_j + c_{2j} p_{j+1} + d_{2j} p_j = e_{2j}\ ,$$

$$j = \overline{1, NJ - 1}$$

(8)

:

$$a_{1j} = 2\frac{\Delta x}{\Delta y}\left(y^{\nu}\rho\right)_{i,j+1}\ ,\ b_{1j} = -2\frac{\Delta x}{\Delta y}\left(y^{\nu}\rho\right)_{i,j}\ ,\ c_{1j} = \left(y^{\nu} u/F\right)_{i,j+1}\ ,\ d_{1j} = \left(y^{\nu} u/F\right)_{i,j}\ ,$$

$$e_{1j} = \left[y^{\nu}\rho u\right]_{-1,j+1} + \left[y^{\nu}\rho u\right]_{-1,j}\ ,\ a_{2j} = \left(\rho u\right)_{i-1,j+1} + 2\frac{\Delta x}{\Delta y}\left(\rho v\right)_{i-1,j+1/2}\ ,$$

$$b_{2j} = \left(\rho u\right)_{i-1,j} - 2\frac{\Delta x}{\Delta y}\left(\rho v\right)_{i-1,j+1/2}\ ,\ c_{2j} = 2\frac{\Delta x}{\Delta y}\ ,\qquad d_{1j} = -2\frac{\Delta x}{\Delta y}\ ,$$

$$e_{2j} = 2\left(\rho u v\right)_{i-1,j+1/2}\ ,$$
$$NJ\ -$$

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                                                     -

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(8),

$$[j, j+1] \quad [j-1, j]$$

$$a_{1j}v_{j+1} + b_{1j}v_j + c_{1j}p_{j+1} + d_{1j}p_j = e_{1j},$$
$$a_{2j}v_{j+1} + b_{2j}v_j + c_{2j}p_{j+1} + d_{2j}p_j = e_{2j}, \qquad j = \overline{1, NJ-1} \qquad (9)$$

$$a_{1j-1}v_j + b_{1j-1}v_{j-1} + c_{1j-1}p_j + d_{1j-1}p_{j-1} = e_{1j-1},$$
$$a_{2j-1}v_j + b_{2j-1}v_{j-1} + c_{2j-1}p_j + d_{2j-1}p_{j-1} = e_{2j-1}. \qquad j = \overline{2, NJ} \qquad (10)$$

$$v_{j+1},$$

$$- v_{j-1},$$

$$k_{1j}v_j + k_{2j}p_{j+1} + k_{3j}p_j = k_{4j},$$
$$k_{5j}v_j + k_{6j}p_j + k_{7j}p_{j-1} = k_{8j}, \qquad j = \overline{2, NJ-1} \qquad (11)$$

$$k_{1j} = b_{1j}a_{2j} - b_{2j}a_{1j}, \ \ k_{2j} = c_{1j}a_{2j} - c_{2j}a_{1j};$$
$$k_{3j} = d_{1j}a_{2j} - d_{2j}a_{1j}, \ \ k_{4j} = e_{1j}a_{2j} - e_{2j}a_{1j};$$
$$k_{5j} = a_{1j-1}b_{2j-1} - a_{2j-1}b_{1j-1}, \ \ k_{6j} = c_{1j-1}b_{2j-1} - c_{2j-1}b_{1j-1};$$
$$k_{7j} = d_{1j-1}b_{2j-1} - d_{2j-1}b_{1j-1}, \ \ k_{8j} = e_{1j-1}b_{2j-1} - e_{2j-1}b_{1j-1}.$$

(11) v_j,

$$A_j p_{j+1} + B_j p_j + C_j p_{j-1} = D_j, \qquad (j = \overline{2, NJ-1}) \qquad (12)$$

$$A_j = k_{5j}k_{2j}, \ B_j = k_{5j}k_{3j} - k_{6j}k_{1j}, \ C_j = -k_{7j}k_{1j}, D_j = k_{5j}k_{4j} - k_{8j}k_{1j}.$$

(12)

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[7].

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$$p_j = L_j p_{j+1} + K_j, \ \ j = \overline{(NJ-1),1}. \qquad (13)$$

$$L_j, K_j \qquad\qquad (-$$

)

$$L_j = -\frac{A_j}{B_j + C_j L_{j-1}}, \quad K_j = \frac{D_j - C_j K_{j-1}}{B_j + C_j L_{j-1}}, \ j = \overline{2, NJ-1}. \qquad (14)$$

$$j = 1 \qquad\qquad \frac{\partial p}{\partial y} = 0, \qquad\qquad p_1 = p_2$$

$$L_1 = 1, \ K_1 = 0. \qquad\qquad j = N_j \qquad\qquad p_j = p_e,$$

$$p_e - \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad .$$

.   (14)

$$j = J_e,$$

$M > 1,$ $j = J_e + 1$ $M < 1.$ (13) (

)

$$j = \overline{1, J_e}.$$ $p_{J_e} = p_e.$ $j = \overline{J_e, NJ}$

$$\boldsymbol{p}_j = \boldsymbol{p}_e = \text{const}.$$,

$$p_e.$$

,   (8),

(1)

$$v_{j+1} = \frac{1}{a_{1j}}(e_{1j} - b_{1j}v_j + c_{1j}p_{j+1} + d_{1j}p_j), (\, j = \overline{1, NJ - 1}, v_1 = 0).$$ (15)

,   , $M > 1,$

(2), (4) − (6)

(1)

(3) −   «   ».   ($M < 1$)

(2), (4) − (6)

(1) −   «   ».   ,   -

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$$n = p_c / p_e = 1{,}45$$

$M = 2.$: 1 −   [8]
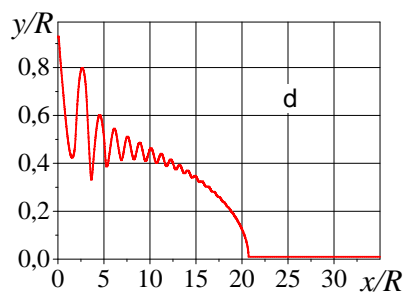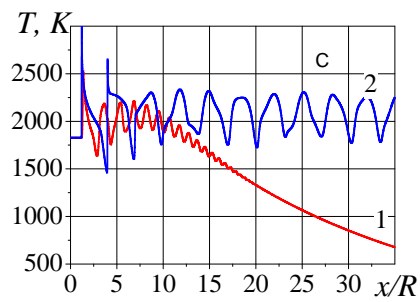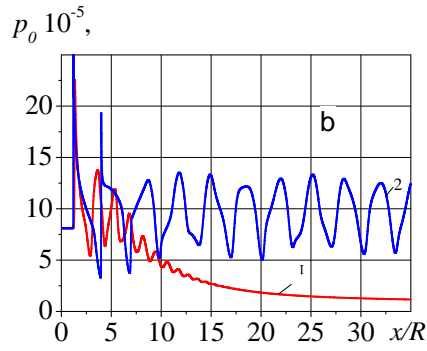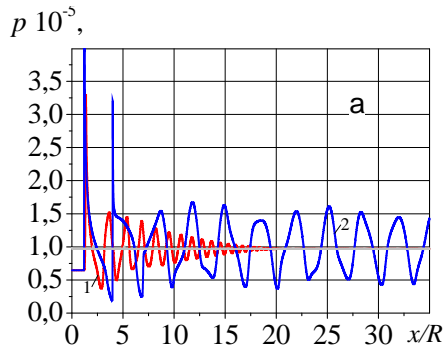
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: $p_c = 0,65 \cdot 10^5$; $M = 3,24$;   -

$T_0 = 21000$ K.

$p_e = 1 \cdot 10^5$ -

$T = 2930$ K.

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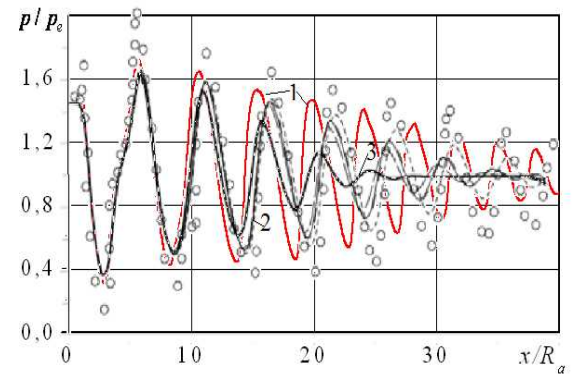
    (a),   (b)   ,   (c)

(d)

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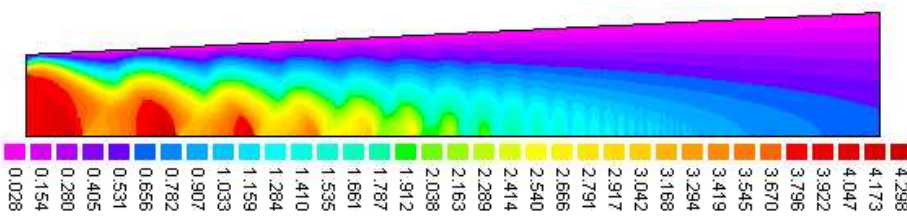
1.   . .   .   . 2002. Т. 42,   9.   . 1413–1424.
2.   . .   .   .   . 2006.   4. URL: htpp//chemphys.edu.ru/2006-10-23-001.pdf
3.   . .   .   . 2006.   .42,   2. С. 3–9.
4.   . .,   . .   . 2008.   . 1,   . 1.   . 15–23.
5.   . .   .   . 1971.   5. С. 114–127.
6.   . .,   . .,   . .   . 1983.   . 2193. 36 .
7.   . .,   *В. С.*   :   . .:   , 1973. 400 .
8.   . .,   . .   2017.   23,   6.   3–11
9.   . .   . .,   . .   . URL: www.chemphys.edu.ru/pdf/2010-01-12-023.pdf.
10. *Seiner J. M., Norum T. D.* Experiments of shock associated noise on supersonic jets. AIAA. 1979. Pap. 79–1526