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TWO-PROBE IMPLEMENTATION OF MICROWAVE INTERFEROMETRY FOR MOTION SENSING AND COMPLEX REFLECTION COEFFICIENT MEASUREMENT

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У статті наведено результати досліджень з мікрохвильових зондових вимірювань, що проводилися у відділі функціональних елементів систем керування Інституту технічної механіки Національної академії наук України і Державного космічного агентства України на протязі останніх п’яти років. У результаті цих досліджень розроблено двозондовий варіант надвисокочастотної інтерферометрії, який дозволяє вимірювати як переміщення механічного об’єкта, так і комплексний коефіцієнт зразка матеріалу. Зменшення кількості зондів з трьох (загальновживаний випадок) до двох спрощує конструкцію та виготовлення хвилеводної секції та послаблює паразитний ефект перевідбиттів між зондами. Можливість використання тільки двох зондів показано шляхом аналізу коренів рівняння, яке пов’язує модуль невідомого комплексного коефіцієнта відбиття зі струмами з’єднаних із зондами напівпровідникових детекторів. Аналіз показує, що теоретично переміщення визначається точно для модуля коефіцієнта відбиття, що не перевищує $2^{-1/2}$, та з максимальною похибкою близько 4,4 % від довжини хвилі зондуючого електромагнітного випромінювання у вільному просторі в загальному випадку, і дає умови, за яких комплексний коефіцієнт відбиття однозначно визначається зі струмів детекторів. Як показали експерименти, при довжині хвилі електромагнітного випромінювання 3 см, подвійній амплітуді коливань об’єкта 10 і 15 см та частоті коливань об’єкта близько 2 Гц запропонований метод вимірювання переміщення дозволяє визначати миттєве значення переміщення об’єкта з максимальною похибкою близько 3 мм та середньою похибкою близько 1 мм без будь-якої попередньої обробки даних вимірювань, такої як фільтрація, згладжування, тощо. Результати, представлені в цій роботі, можуть бути використані при розробці надвисокочастотних датчиків переміщення та векторних рефлектометрів.

В данной статье приведены результаты исследований по микроволновым зондовым измерениям, которые проводились в отделе функциональных элементов систем управления Института технической механики Национальной академии наук Украины и Государственного космического агентства Украины в течение последних пяти лет. В результате этих исследований разработан двухзондовый вариант сверхвысокочастотной интерферометрии, который позволяет измерять как перемещение механического объекта, так и комплексный коэффициент отражения образца материала. Уменьшение количества зондов с трех (общепринятый случай) до двух упрощает конструкцию и изготовление волноводной секции и ослабляет паразитный эффект переотражений между зондами. Возможность использования только двух зондов продемонстрирована с помощью анализа корней уравнения, связывающего модуль неизвестного комплексного коэффициента отражения с токами соединенных с зондами полупроводниковых детекторов. Этот анализ показывает, что теоретически перемещение определяется точно для модуля коэффициента отражения, не превышающего $2^{-1/2}$, и с максимальной погрешностью около 4,4 % от длины волны зондирующего электромагнитного излучения в свободном пространстве в общем случае, и дает условия, при которых комплексный коэффициент отражения однозначно определяется из токов детекторов. Как показали эксперименты, при длине волны электромагнитного излучения 3 см, двойной амплитуде колебаний объекта 10 и 15 см и частоте колебаний объекта около 2 Гц предложенный метод измерения перемещения позволяет определить мгновенное значение перемещения объекта с максимальной погрешностью около 3 мм и средней погрешностью около 1 мм без какой-либо предварительной обработки данных измерений, такой как фильтрация, сглаживание и т. п Результаты, приведенные в данной работе, могут быть использованы при разработке сверхвысокочастотных датчиков перемещения и векторных рефлектометров.

This paper presents the results of the investigations into microwave probe measurements conducted at the Department for Functional Elements of Control Systems of the Institute of Technical Mechanics of the National Academy of Sciences of Ukraine and the State Space Agency of Ukraine over the past five years. These investigations resulted in a two-probe implementation of microwave interferometry that allows one to measure both the displacement of a mechanical object and the complex reflection coefficient of a material specimen. Reducing the number of probes from three (the conventional case) to two simplifies the design and manufacture of the waveguide section and alleviates the problem of interprobe interference. The possibility of using as few as two probes is demonstrated by analyzing the roots of the equation that relates the magnitude of the unknown complex reflection coefficient to the currents of the semiconductor detectors connected to the probes. The analysis shows that, theoretically, the displacement is determined exactly for reflection coefficient magnitudes no greater than the inverse of the square root of two and to a worst-case accuracy of about 4.4 % of the free-space operating wavelength in the general case and gives conditions under which the complex reflection coefficient is unambiguously determined from the detector currents. As shown by experiments, at an operating wavelength of 3 cm, a target double amplitude of 10 cm and 15 cm, and a target vibration frequency of about 2 Hz the proposed displacement measurement method allows one to determine the instantaneous target displacement with a maximum error of about 3 mm and an average error of about 1 mm without any preprocessing of the measured data, such as filtering, smoothing, etc. The results presented in this paper may be used in the development of microwave displacement sensors and vector reflectometers.

Keywords: *complex reflection coefficient, displacement, electrical probe, microwave interferometry, semiconductor detector, waveguide section.*

Microwave measurements are widely used in the determination of various parameters such as distance, displacement, speed, dielectric permittivity, etc. Microwave interferometry is an ideal means in terms of the development of motion sensors [1]. This is due to its ability to provide fast noncontact measurements and its applicability to dusty or smoky environments (as distinct from laser Doppler sensors [2–4] or vision-based systems using digital image processing techniques [5]). An important advantage over radar methods (both traditional pulse ones and recently developed continuous-wave step-frequency ones [6, 7]) is its simple hardware implementation. In microwave interferometry, the displacement of the object under measurement (target) is extracted from the phase shift between the signal reflected from the target and the reference signal, i. e. from the phase of the complex reflection coefficient. A characteristic feature of such measurements is phase ambiguity, which can be resolved by using two quadrature signals in combination with a phase unwrapping method. At present, the usual way to form the quadrature signals is to use special hardware incorporating a power divider and a phase-detecting processor, which is an analog [8] or a digital [9] quadrature mixer. On the other hand, information on the phase of the complex reflection coefficient is also contained in the electric field amplitude of the standing wave between the emitter and the target, which can be measured using an electrical probe and a semiconductor detector connected thereto. The hardware implementation of probe measurements is much simpler. The investigations into microwave probe measurements conducted at the Department for Functional Elements of Control Systems of the Institute of Technical Mechanics of the National Academy of Sciences of Ukraine and the State Space Agency of Ukraine over the past five years resulted in a two-probe displacement measurement method. In that method, the quadrature signals needed for the determination of the phase shift are extracted from the outputs of two probes placed in a waveguide section one eighth of the guided operating wavelength λ_g apart. Its distinctive feature is the possibility of displacement measurement at an unknown reflection coefficient with as few as two probes, while since the classic text by Tischer [10] it has been universally believed that at least three probes are needed to determine or eliminate the unknown reflection coefficient [11]. The theoretical basics of the method are as follows [12 – 15].

Consider two probes 1 and 2 connected to square-law semiconductor detectors. The probes are placed $\lambda_g/8$ apart in a waveguide section between a microwave oscillator and a target, probe 2 being closer to the target. A measurement schematic is shown in Fig. 1.

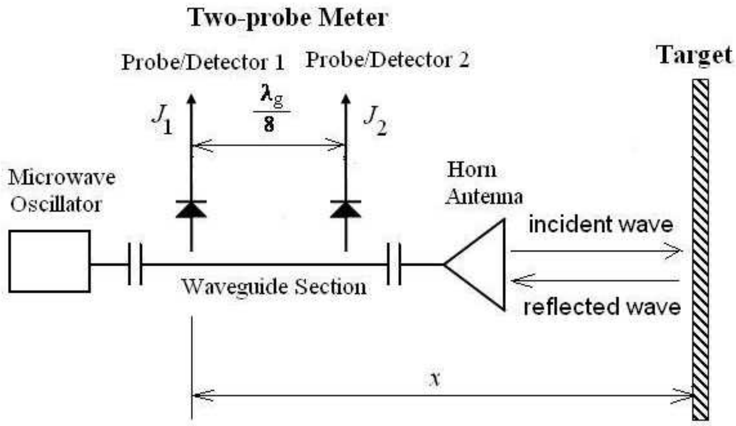

Fig. 1

The detector currents J_1, J_2 normalized to their values in the absence of a reflected wave are expressed in terms of the magnitude R and phase ψ of the complex reflection coefficient at the location of probe 1

$$J_1 = 1 + R^2 + 2R\cos\psi \ , \tag{1}$$

$$J_2 = 1 + R^2 + 2R\sin\psi \ . \tag{2}$$

Information on the distance x between the target and probe 1 is contained in the phase of the complex reflection coefficient

$$\psi = \frac{4\pi x}{\lambda_0} + \phi \, , \tag{3}$$

where λ_0 is the free-space operating wavelength and ϕ is the phase component that is governed by the waveguide section and horn antenna geometry and the phase shift caused by the reflection and does not depend on the distance *x*.

Let it be desired to find the displacement Δx of the target at time t relative to its initial position $x(t_0)$. As indicated above, for phase ambiguity resolution in relative displacement determination it is sufficient to have the quadrature signals $\cos\psi$ and $\sin\psi$. According to Eqs. (1) and (2), these signals are expressed in terms of the unknown magnitude of the reflection coefficient as follows

$$\cos\psi = \frac{a_1 - R^2}{2R} \, , \tag{4}$$

$$\sin\psi = \frac{a_2 - R^2}{2R} \, , \tag{5}$$

where $a_1 = J_1 - 1$ and $a_2 = J_2 - 1$.

The following biquadratic equation in R results from Eqs. (4) and (5)

$$R^4 - (a_1 + a_2 + 2)R^2 + \frac{a_1^2 + a_2^2}{2} = 0 \, . \tag{6}$$

This equation has two positive roots (the plus sign before the radical corresponds to the root R_1, and the minus sign corresponds to the root R_2)

$$R_{1,2} = \left[\frac{a_1 + a_2 + 2}{2} \pm \sqrt{\frac{(a_1 + a_2 + 2)^2}{4} - \frac{a_1^2 + a_2^2}{2}} \right]^{1/2} .$$

Clearly one of these roots is extraneous. So the phase ambiguity resolution problem reduces to the choice between the root R_1 and the root R_2.

An explicit expression for the extraneous root may be obtained by rearranging the absolute term of Eq. (6). From Eqs. (4) and (5) we have

$$a_1^2 = R^4 + 2R^3 \cos\psi + 4R^2 \cos^2 \psi , \tag{7}$$

$$a_1^2 = R^4 + 2R^3 \sin\psi + 4R^2 \sin^2 \psi . \tag{8}$$

Substituting Eqs. (7) and (8) into the expression for the absolute term of Eq. (6) gives

$$\frac{a_1^2 + a_2^2}{2} = R^2 \left[R^2 + 2\sqrt{2} R \sin(\psi + \pi/4) + 2 \right]. \tag{9}$$

Since the absolute term of a quartic equation is equal to the product of its roots, it follows from Eq. (9) that the positive extraneous root R_{ext} of Eq. (6) is

$$R_{ext} = \left[R^2 + 2\sqrt{2} R \sin(\psi + \pi/4) + 2 \right]^{1/2} . \tag{10}$$

Let us find the condition under which the inequality $R_{ext} \geq R$ is satisfied. It follows from Eq. (10) that this condition is

$$\sin\left(\psi + \frac{\pi}{4} \right) \geq -\frac{1}{\sqrt{2} R} . \tag{11}$$

This inequality is satisfied at any value of the phase ψ if $R \leq 1/\sqrt{2}$. Since $R_1 \geq R_2$, in this case the reflection coefficient magnitude R will always be given by the root R_2. In the case $R > 1/\sqrt{2}$, the condition of (11) will not be necessarily satisfied. Because of this, the reflection coefficient magnitude R will be given by the root R_2 if the condition of (11) is satisfied; otherwise it will be given by the root R_1.

First consider the case $R \leq 1/\sqrt{2}$. In this situation, the reflection coefficient magnitude R is unambiguously determined from Eq. (6) as its root R_2, and thus $\cos\psi$ and $\sin\psi$ are unambiguously determined from Eqs. (4) and (5). If $\cos\psi$ and $\sin\psi$ are known, the displacement Δx of the target at time t_n, $n = 0, 1, 2, ...,$ from its initial position $x(t_0)$ can be found by the following phase unwrapping algorithm [16]

$$\varphi(t_n)=\begin{cases}\arctan\dfrac{\sin\psi(t_n)}{\cos\psi(t_n)}, & \sin\psi(t_n)\ge 0, \cos\psi(t_n)\ge 0,\\ \arctan\dfrac{\sin\psi(t_n)}{\cos\psi(t_n)}+\pi, & \cos\psi(t_n)<0\,,\\ \arctan\dfrac{\sin\psi(t_n)}{\cos\psi(t_n)}+2\pi, & \sin\psi(t_n)<0, \cos\psi(t_n)\ge 0,\end{cases} \tag{12}$$

$$\Delta\varphi(t_n)=\varphi(t_n)-\varphi(t_{n-1})\,, \tag{13}$$

$$\theta(t_n)=\begin{cases}0, \; n=0,\\ \theta(t_{n-1})+\Delta\varphi(t_n), \quad |\Delta\varphi(t_n)|\le\pi, \quad n=1, 2, \ldots,\\ \theta(t_{n-1})+\Delta\varphi(t_n)-2\pi\,\mathrm{sgn}[\Delta\varphi(t_n)], \; |\Delta\varphi(t_n)|>\pi, \; n=1, 2, \ldots,\end{cases} \tag{14}$$

$$\Delta x(t_n)=\frac{\lambda_0}{4\pi}\theta(t_n)\,, \quad n=0,1,2,\ldots, \tag{15}$$

where φ and θ are the wrapped and the unwrapped phase, respectively.

Now consider the case $R>1/\sqrt{2}$. In this case, R_2 will be equal to R only for the values of ψ that satisfy the condition of (11). However, as will be shown below, the displacement can also be determined to sufficient accuracy using the root R_2 as the reflection coefficient magnitude. It follows from the condition of (11) that the root R_2 will be extraneous if $\sin(\psi+\pi/4)<-1/\sqrt{2}R$. In terms of the wrapped phase φ, this condition becomes

$$\frac{3\pi}{4}+\arcsin\frac{1}{\sqrt{2}R}<\varphi<\frac{7\pi}{4}-\arcsin\frac{1}{\sqrt{2}R}\,,$$

whence it follows that the wrapped phase that corresponds to the condition $\sin(\psi+\pi/4)<-1/\sqrt{2}R$ lies in the third quadrant.

Let us find the phase error that is introduced when the extraneous root R_{ext} is used as the reflection coefficient magnitude. In this case, Eqs. (4) and (5) will give the apparent values $\cos\psi_{ap}=\left(a_1-R_{ext}^2\right)/2R_{ext}$ and $\sin\psi_{ap}=\left(a_2-R_{ext}^2\right)/2R_{ext}$, which on substitution into Eq. (12) will give the apparent wrapped phase φ_{ap}. The final expression for the apparent wrapped phase is

$$\varphi_{ap}=\arctan\frac{1+R\cos\varphi}{1+R\sin\varphi}+\pi\;.$$

The use of the apparent wrapped phase φ_{ap} instead of the actual wrapped phase φ introduces the phase error $\Delta\varphi_{er}(\varphi,R)=\varphi_{ap}-\varphi$. The function $\Delta\varphi_{er}(\varphi,R)$ possesses the following properties:

– is antisymmetric in φ about $\varphi=\dfrac{5\pi}{4}$;

– becomes zero at $\varphi = \frac{3\pi}{4} + \arcsin\frac{1}{\sqrt{2}R}$, $\varphi = \frac{5\pi}{4}$, and $\varphi = \frac{7\pi}{4} - \arcsin\frac{1}{\sqrt{2}R}$;

— at a fixed φ, increases in magnitude with R;

– at a fixed R, has a negative minimum at $\varphi_1 = \frac{3\pi}{4} + \arcsin\frac{\sqrt{2}(1+R^2)}{3R}$ and a positive maximum at $\varphi_2 = \frac{7\pi}{4} - \arcsin\frac{\sqrt{2}(1+R^2)}{3R}$, which are equal in magnitude by virtue of the antisymmetry of the function.

It follows from these properties that the greatest possible phase error $\Delta\varphi_{er\,\max}$ is reached at $R = 1$ and is equal to

$$\Delta\varphi_{er\,\max} = \arctan\frac{\sqrt{2}+1}{\sqrt{2}-1} + \arcsin\frac{2\sqrt{2}}{3} - \frac{3\pi}{4}.$$

As can be seen from the algorithm of (12) – (15), the displacement determination error is governed by the phase error only at the initial and the current measurement point because the errors at the intermediate points cancel one another. Because of this, the greatest possible displacement determination error $\Delta x_{er\,\max}$ will be reached at $R = 1$ in the case where the initial measurement point corresponds to one extremum of the function $\Delta\varphi_{er}(\varphi)$ and the current measurement point corresponds to the other. As follows from the aforesaid, this error will be

$$\Delta x_{er\,\max} = \frac{\lambda_0}{4\pi} 2\Delta\varphi_{er\,\max} = 0.044\lambda_0 . \qquad (16)$$

As can be seen from Eq. (16), the greatest possible error $\Delta x_{er\,\max}$ is about 4.4 % of the free-space operating wavelength λ_0 (notice that this is the worst-case error, which occurs when the reflection coefficient magnitude is equal to unity, the initial measurement point corresponds to one extremum of the function $\Delta\varphi_{er}(\varphi)$, and the current measurement point corresponds to the other). So the proposed displacement measurement method, in which the reflection coefficient magnitude is taken to be equal to the smaller positive root of Eq. (6), allows the displacement to be determined to sufficient accuracy at any value of the reflection coefficient magnitude.

A technique that allows one to verify this method for a moving target without recourse to complex photorecording equipment is described in [17]. This technique is based on using a target put in motion by a crank mechanism as shown in Fig. 2.

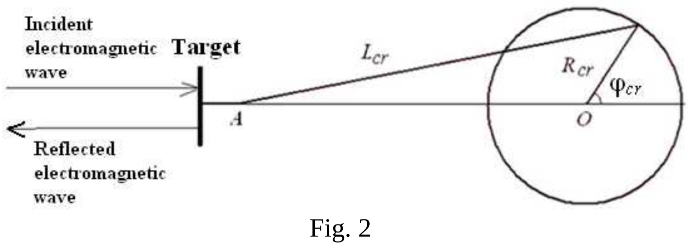

Fig. 2

As can be seen from Fig. 2, the displacement Δx of the target at time t relative to its initial position at $t = 0$ is

$$\Delta x(t) = OA(\varphi_{cr0}) - OA[\varphi_{cr}(t)], \quad (17)$$

$$OA(\varphi_{cr}) = \sqrt{L_{cr}^2 - R_{cr}^2 \sin^2 \varphi_{cr}} - R_{cr} \cos\varphi_{cr}\ , \quad (18)$$

$$\varphi_{cr}(t) = \varphi_{cr0} + \frac{2\pi t}{T} \quad (19)$$

where φ_{cr} is the crank angle, φ_{cr0} is the crank angle at $t = 0$, OA is the distance from the rotation center to the end of the crank arm, L_{cr} is the crank arm length, R_{cr} is the crank radius, and T is the rotation period.

As can be seen from Eqs. (17) to (19), the displacement is a periodical time function of period T that reaches one minimum and one maximum over a period. So the crank rotation period may be determined from the measured dependence $\Delta x(t)$ as the distance along the abscissa axis between two adjacent minima or two adjacent maxima, and $\ddot{}_{cr0}$ may be determined from the measured dependence $\Delta x(t)$ as follows

$$\varphi_{cr0} = -\frac{2\pi t_1}{T} \quad (20)$$

where t_1 is the time at which the measured dependence $\Delta x(t)$ shows its first maximum.

Given T and t_1, the actual target displacement can be calculated from Eqs. (17) to (20) and compared with the measured one. However, T and t_1 can be determined from the measured time dependence of the displacement only approximately. Because of this, the displacement measurement error, i. e. the difference of the measured displacement and the actual one, may be found by the following algorithm.

1. From the measured time dependence of the target displacement $\Delta x(t)$, estimate the crank rotation period T and the time t_1 at which the measured dependence $\Delta x(t)$ shows its first maximum (in the following, the estimated values of T and the time t_1 will be denoted as T_{ap} and t_{1ap}, respectively).

2. Vary T and t_1 with a specified step on the intervals $0.9T_{ap} \le T \le 1.1T_{ap}$ and $0.9t_{1ap} \le t_1 \le 1.1t_{1ap}$.

3. For each pair (T, t_1), calculate the target displacement at each time point from Eqs. (17) to (20).

4. For each time point, calculate the displacement measurement error Δx_{er} as the difference of the measured displacement $\Delta x(t)$ and the calculated displacement $\Delta x_c(t)$.

5. Find the maximum value $|\Delta x_{er}|_{max}$ of the displacement error magnitude for the given pair (T, t_1).

6. Find the pair (T, t_1) such that $|\Delta x_{er}|_{max}$ is a minimum and take these values of T and t_1 as the actual values T_{act} and t_{1act} of the crank rotation period T and the time t_1.

7. For $T = T_{act}$ and $t_1 = t_{1act}$`, calculate the target displacement at each time point from Eqs. (17) to (20).

8. Run Step 4 to find the actual displacement measurement error Δx_{er}.

In [17], the two-probe displacement measurement method was verified by the above algorithm using a target (a brass disc or a brass square) put into a reciprocal motion by an electrically driven crank mechanism.

The measuring setup comprised a microwave oscillator, a circulator with a dummy load, a waveguide section with two probes installed therein and two semiconductor detectors connected to the probes, a horn antenna mounted at the end of the waveguide section, two amplifiers, an analog-to-digital converter, and a personal computer. A schematic of the setup is shown in Fig. 3.

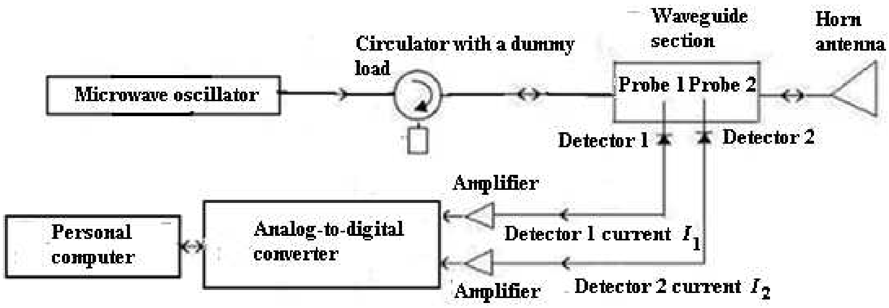

Fig. 3

The experiments were conducted at different values of the target double amplitude equal to twice the crank radius and the minimum distance between the antenna and the target. In all the cases, the free-space operating wavelength was 3 cm, with corresponds to an operating frequency of 10 GHz.

In Experiment 1, the target was a brass disc of diameter 128 mm, the target double amplitude was 15 cm, and the minimum distance between the antenna and the target was 100 cm. In Experiment 2, the target was the same as in Experiment 1, the target double amplitude was 10 cm, and the minimum distance between the antenna and the target was 15 cm. In Experiment 3, the target was a 70x70 mm brass square, the target double amplitude was 10 cm, and the minimum distance between the antenna and the target was 5 cm.

Figs. 4 to 6 show the target displacement Δx measured by the method proposed in [12, 13] and the actual target displacement Δx_{act} found by the algorithm described above for Experiments 1, 2, and 3, respectively. As can be seen from the figures, the target vibration period is about 0.5 sec, i. e. the vibration frequency is about 2 Hz. It can also be seen that the curves of the measured and the actual displacement coincide to within the line thickness. The peak-to-peak amplitude was determined to an accuracy of 0.7 mm in Experiment 1, 1.1 mm in Experiment 2, and 0.2 mm in Experiment 3.

Figs. 7 to 9 show the displacement measurement error Δx_{er} equal to the difference of the measured displacement Δx and the actual displacement Δx_{act} versus the time and the root R_2 of Eq. (6) (the measured reflection coefficient) versus the target displacement Δx_0 from the position closest to the antenna for Experiments 1, 2, and 3, respectively.

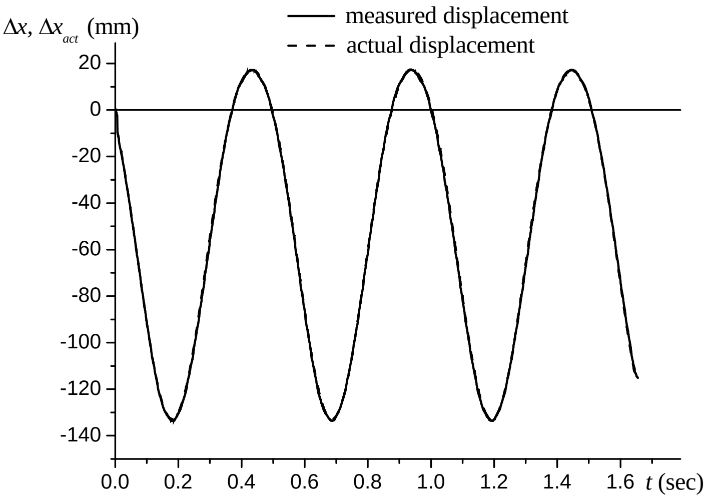

Fig. 4

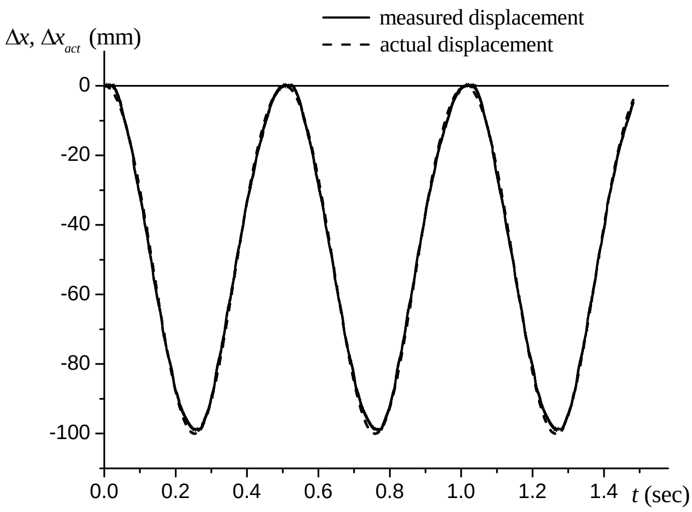

Fig. 5

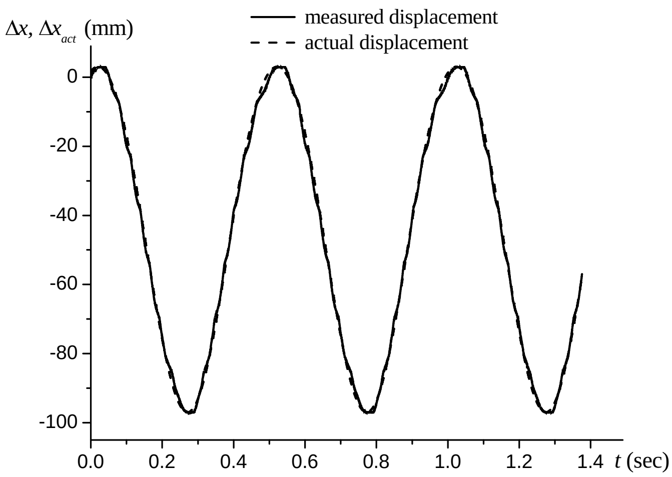

Fig. 6

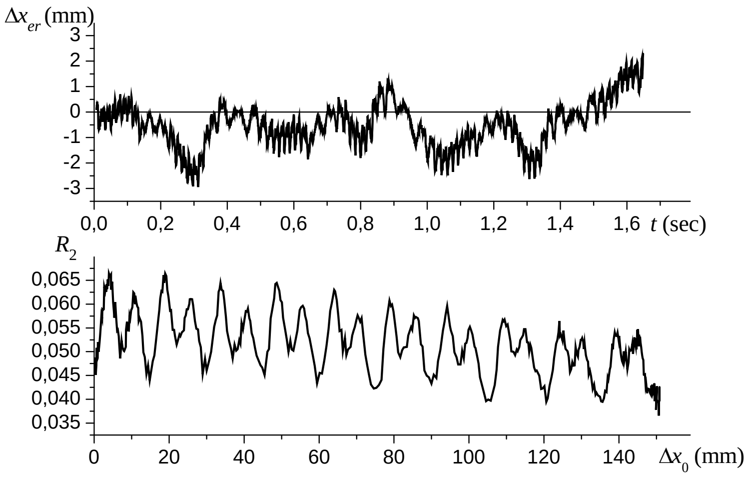

Fig. 7

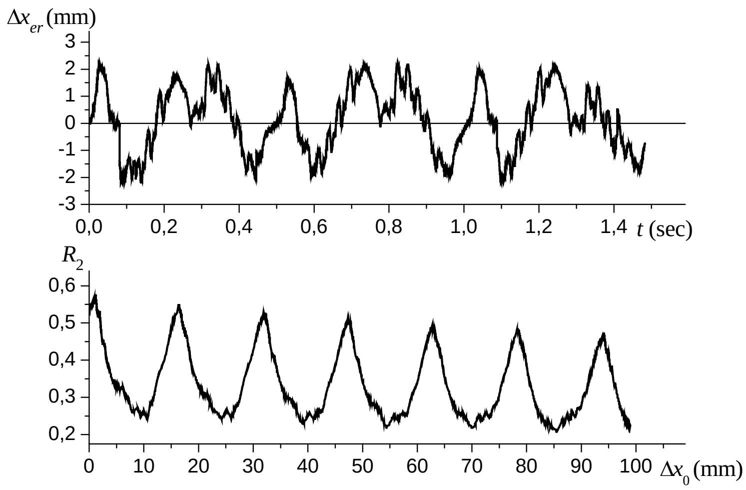

Fig. 8

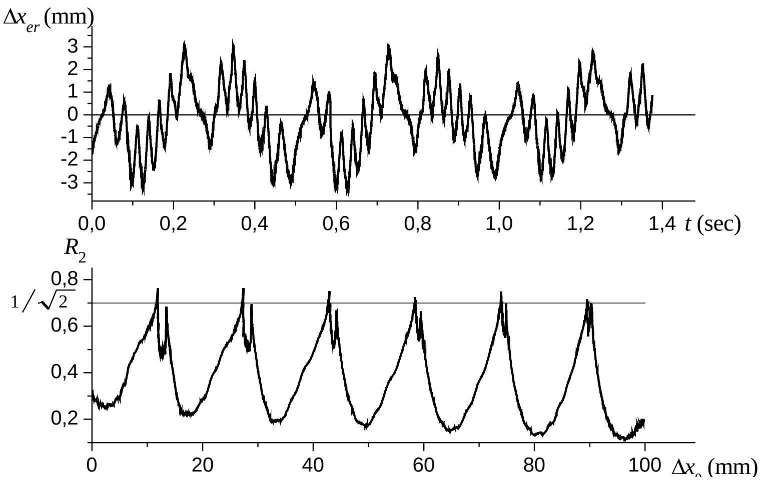

Fig. 9

The maximum and the average error in the determination of the instantaneous relative displacement was 2.9 mm and 0.8 mm in Experiment 1, 2.2 mm and

1.0 mm in Experiment 2, and 3.3 mm and 1.1 mm in Experiment 3. In Experiments 1 and 2, the measured reflection coefficient varied between 0.04 and 0.066 and between 0.12 and 0.58, respectively, i. e. it was less than $1/\sqrt{2} \approx 0.707$. Because of this, in those experiments the root R_2 gave the actual reflection coefficient, and thus the error was due to other factors, such as deviation of the reflected wave from the plane waveform, reflections from the antenna, noise, etc. In Experiment 3, the measured reflection coefficient varied between 0.2 and 0.76, i. e. at some of the measurement points the root R_2 might be extraneous. However, as can be seen from the data given above, this did not contribute much to the error in comparison with Experiments 1 and 2. As can be seen from Figs. 6 and 9 (Experiment 3), the two-probe method described above performs well for a minimum antenna–target distance of 5 cm too, while the standing-wave radar proposed in [18] fails to operate at distances less than 14 cm due to positional interference between the target and the antenna.

The above-described two-probe implementation of microwave interferometry may also be applied to determining the complex reflection coefficient, whose measurements are widely used in material characterization. To measure the complex reflection coefficient over a frequency range, this implementation should be generalized to the case of an arbitrary interporbe distance. Such a generalization is described in [19, 20]. A measurement schematic is shown in Fig. 10.

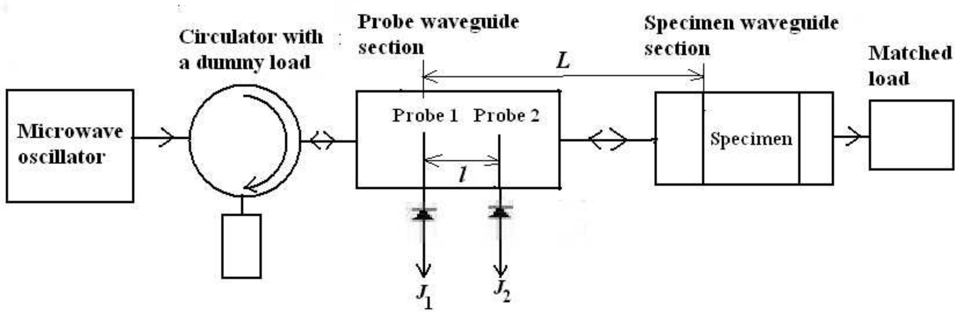

Fig. 10

In this case, Eq. (1) remains as before, while Eqs. (2) and (3) become

$$J_2 = 1 + R^2 + 2R\sin(\psi - \beta), \quad \beta = \frac{\pi}{2}\left(\frac{8l}{\lambda_g} - 1\right),$$

$$\psi = \frac{4\pi L}{\lambda_{g\gamma}} + \phi_{ref} \tag{21}$$

where l is the interprobe distance, L is the distance between the specimen and probe 1, and R and ϕ_{ref} are the magnitude and the phase of the complex reflection coefficient of the specimen).

The expression of (4) for $\cos\psi$ remains as before, while the expression of (5) for $\sin\psi$ becomes

$$\sin\psi = \frac{a_2 + a_1 \sin\beta - R^2(1 + \sin\beta)}{2R\cos\beta}. \tag{22}$$

Given R, the phase ϕ_{ref} of the complex reflection coefficient may be found from Eqs. (4), (12), (21), and (22) as follows

$$\phi_{ref} = \varphi - \frac{4\pi L}{\lambda_g} + 2\pi n ,$$

where φ is the wrapped phase given by Eq. (12), and the integer n is chosen such that the phase ϕ_{ref} lies between zero and 2π.

From Eqs. (4) and (22) we get the following biquadratic equation in R

$$R^4 - R^2[a_1 + a_2 + 2(1-\sin\beta)] + \frac{a_1^2 + a_2^2 + 2a_1a_2\sin\beta}{2(1+\sin\beta)} = 0 .$$

An analysis similar to that described above shows that the reflection coefficient magnitude is given by the smaller positive root of this equation if the following conditions are satisfied:

$$l \le \lambda_g/8 , \quad (23)$$

$$0 \le \varphi \le \pi \quad \text{or} \quad 3\pi/2 \le \varphi < 2\pi . \quad (24)$$

Because of this, the specimen must be positioned so that the wrapped phase given by Eq. (12) satisfies the condition of (24), and for measurements over a frequency range the interprobe distance must be such that the condition of (23) is satisfied at the shortest wavelength of that frequency range.

So the proposed probe implementation of microwave interferometry allows one to measure both the displacement of a mechanical object and the complex reflection coefficient of a specimen using as few as two probes. Theoretically, the displacement is determined exactly for reflection coefficient magnitudes no greater than the inverse of the square root of two and to a worst-case accuracy of about 4.4 % of the operating wavelength in the general case. This implementation also allows one to determine the complex reflection coefficient of the horn antenna at the end of the probe waveguide section, which may be then accounted for to reduce the displacement measurement error in cases where the reflection coefficients of the target and the horn antenna are comparable. The results presented in this paper may be used in the development of microwave displacement sensors and vector reflectometers.

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