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Measured Hardness by an Indenter Having Ellipsoidal Shape

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Измерение твердости с помощью индентора эллипсоидальной формы

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Если при исследовании твердости в качестве индентора использовалось твердое тело эллипсоидальной формы, выражение для статической твердости рассматривалось как функция от глубины и радиусов отпечатка индентора. При этом применялась общая формула, связывающая статическую твердость с отношением нормальной силы, прилагаемой к индентору, к реальной площади его отпечатка. При получении конечной формулы для расчета твердости использовались геометрический и математический подходы.

Ключевые слова: измерение твердости, механические испытания, объемная деформация, геометрическая модель.

Notation

A, B, C	–	semi-axes of ellipsoid
d	–	diameter of the area of the project imprint of a revolution ellipsoid indenter
H_e	–	static hardness of the ellipsoidal indenter
H_{ec}	–	measured hardness by a body of revolution, in the case of a circular imprint
H_{er}	–	measured hardness by a body of revolution, in the case of a real imprint
F	–	applied load
S	–	imprint surface
a^-, b^-	–	semi-axes of the projected elliptic surface
h	–	depth of the imprint
r	–	radius of the projected surface of the imprint

Introduction. The different dynamic and static hardness tests play a major role to define the hardness of massive and covered materials. They consist to penetrate a harder indenter in a less hard body [1].

The hardness of material is a very important property in the fields of materials industry. It is considered as a significant mechanical characteristic in a world of technologies. It is defined as the mechanical resistance of material under test, which opposes the penetration of another harder material [2–10].

The hardness of covered and massive materials is an old problem, which had already been a subject of many theoretical and experimental studies. There are a lot of geometrical and mathematical models, to determine the hardness of different materials. The static hardness H_e is expressed by the ratio of the applied load F to the imprint surface S [2–10]. Its mathematical expression is given by the following expression:

$$H_e = \frac{F}{S}.$$

In the hardness tests, the geometry of the indenter (pyramid, cone, sphere, etc.) is a very significant factor because of the geometrical form of the resulting imprint and the phenomena occurring during and after the tests (cracking, deformation, etc.).

In the present theoretical study, an indenter of an ellipsoidal geometrical shape is used to measure the static hardness of materials, where it has been calculated according to semi-axes A, B, and C of the indenter, semi-axes of the projected imprint a^-, b^-, and r, applied load F, and depth h of the imprint. Furthermore, the hardness has been theoretically calculated by using geometrical approaches for the real imprint cap.

The paper is organized as follows: First, general theoretical concepts related to the indenter theory are highlighted. Secondly, the area of the imprint is calculated as a function of its semi-axes and its depth. Finally, mathematical and geometrical assumptions have been made to reduce and give a useful expression of the hardness expression.

1. **Mathematical Concepts**.

1.1. ***Ellipse***. An ellipse is formed by cutting a three-dimensional cone with a slanted plane. It has radius that changes in between A along the x axis and B along the y axis [11].

The standard equation of an ellipse (Fig. 1) with a center at the origin of a Cartesian coordinates system and aligned with the axes is

$$\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1. \tag{1}$$

Thus, the area of the ellipse (S) is

$$S = \pi AB. \tag{2}$$

1.2. ***Ellipsoid***. The standard characteristic equation of an ellipsoid centered at the origin of a Cartesian coordinates system and aligned with the axes is [11]:

$$\left(\frac{x}{A}\right)^2 + \left(\frac{y}{B}\right)^2 + \left(\frac{z}{C}\right)^2 = 1. \tag{3}$$

For an ellipsoid of revolution of semi-axes (R, R, C) respectively, along axes Ox, Oy, and Oz (Fig. 2), its characteristic equation is written as the following:

$$\left(\frac{x}{R}\right)^2 + \left(\frac{y}{R}\right)^2 + \left(\frac{z}{C}\right)^2 = 1. \tag{4}$$

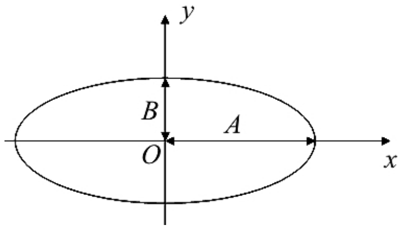

Fig. 1. The ellipse.

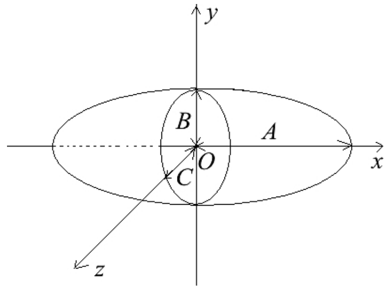

Fig. 2. The ellipsoid.

We put

$$x^2 + y^2 = r^2. \tag{5}$$

By introducing Eq. (5) into Eq. (4), we get

$$\left(\frac{r}{R}\right)^2 + \left(\frac{z}{C}\right)^2 = 1. \tag{6}$$

1.3. ***Surface of a Body of Revolution***. The side area of a body generated by a revolution of a curve of a characteristic equation $z = f(x)$, around an axis Ox and ranging between the points a and b (Fig. 3) is expressed by [11]:

$$S = \int_a^b 2\pi z dl, \tag{7}$$

where dl is the differential of the curve arc, it is given by the formula:

$$dl = \sqrt{(dz)^2 + (dx)^2}. \tag{8}$$

By introducing Eq. (8) in Eq. (7), we get

$$S = \int_a^b 2\pi z \sqrt{1 + \left(\frac{dz}{dx}\right)^2} dx \qquad \text{with} \qquad \frac{dz}{dx} = \frac{df(x)}{dx}. \tag{9}$$

Then, Eq. (7) may be written as

$$S = \int_a^b 2\pi f(x) \sqrt{1 + \left(\frac{df(x)}{dx}\right)^2} dx. \tag{10}$$

2. **Hardness Measured by an Ellipsoidal Indenter**.

2.1. ***Principle of Penetration***. In the case of hardness tests by using an ellipsoidal indenter, we used an indenter of an ellipsoidal shape of semi-axes A, B, and C, under the action of a known constant force applied perpendicular to the indenter and under defined conditions. We measure the dimensions of the imprint (transverse and depth) and deduct the hardness.

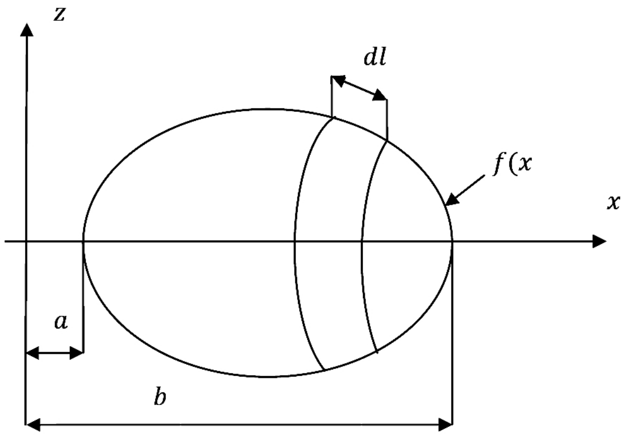

Fig. 3. A body of revolution.

2.2. ***Static Hardness***. The static hardness H_e is expressed by the ratio of the load applied F to the projected surface S of the imprint [12], as following:

$$H_e = \frac{F}{S}. \tag{11}$$

For an elliptical indenter, the mathematical expression of the area of the projected imprint of the semi-axes; a^- and b^- is given by the following relation:

$$S = \pi a^- b^-. \tag{12}$$

Introducing Eq. (12) in Eq. (11), the static hardness becomes:

$$H_e = \frac{F}{\pi a^- b^-}. \tag{13}$$

For an indenter having a geometrical form of an ellipsoidal (Figs. 4 and 5), the semi-axes a^- and b^- of the projected surface of the imprint can be written according of the penetration h and the semi-axes of the indenter A, B, and C as following:

$$a^- = A\sqrt{\frac{2h}{C} - \left(\frac{h}{C}\right)^2}, \tag{14}$$

$$b^- = B\sqrt{\frac{2h}{C} - \left(\frac{h}{C}\right)^2}. \tag{15}$$

Then the expression of hardness becomes

$$H_e = \frac{C^2 F}{\pi AB(2hC - (h)^2)}. \tag{16}$$

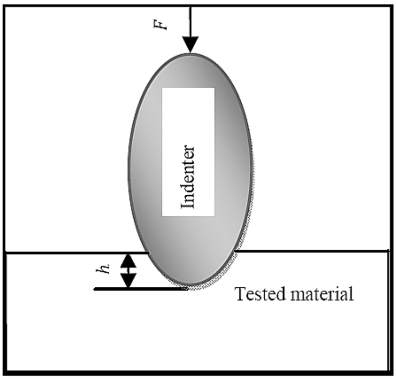

Fig. 4. Penetration of ellipsoid.

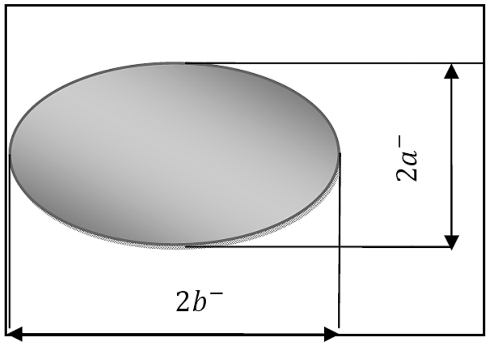

Fig. 5. Area of project imprint of ellipsoid indenter.

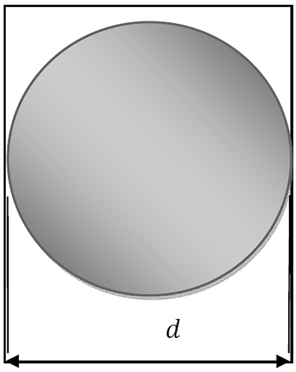

Fig. 6. Area of the project imprint of a revolution ellipsoid indenter.

2.3. ***The Hardness when the Indenter is a Revolution Ellipsoid***.

2.3.1. *A Circular Imprint*. For an ellipsoidal indenter of circular section ($C = R = B$), the projected surface of the imprint becomes a circular disk of a radius $r = d/2$ (Fig. 6). We can write the expression of the radius according of the semi-axes of the body of revolution A and R, and the imprint depth h, as follows:

$$\frac{d}{2} = r = R\sqrt{\frac{2h}{A} - \left(\frac{h}{A}\right)^2}. \tag{17}$$

Then, the hardness takes following form:

$$H_{ec} = \frac{F}{\pi r^2} = \frac{4F}{\pi d^2} = \frac{A^2 F}{\pi R^2 (2hA - (h)^2)}. \tag{18}$$

2.3.2. *A True Imprint*. For an ellipsoidal indenter (body of revolution) of a circular section ($C = B = R$), the imprint is a cap of circular basis of a diameter d and a depth h (Fig. 4). Accordingly, the surface of this imprint gets the formula:

$$S = \int_{A-h}^{A} 2\pi z \sqrt{1 + \left(\frac{dz}{dx}\right)^2}\, dx. \tag{19}$$

From the characteristic equation (1), we deduce:

$$z^2 = R^2\left(1 - \frac{x^2}{A^2}\right). \tag{20}$$

The derivative of equation (20) leads to following expression:

$$zz' = -\frac{R^2 x}{A^2}. \tag{21}$$

By introducing Eqs. (20) and (21) in Eq. (19), we get

$$S = \int_{A-h}^{A} 2\pi z \sqrt{1 + \left(\frac{dz}{dx}\right)^2}\, dx = \int_{A-h}^{A} 2\pi \sqrt{z^2 + \left(z\frac{dz}{dx}\right)^2}\, dx = 2\pi R \int_{A-h}^{A} \sqrt{1 - \frac{x^2}{A^2}\left(1 - \frac{R^2}{A^2}\right)}\, dx.$$

Let $t = \beta \frac{x}{A}$ with $\beta^2 = \frac{A^2 - R^2}{A^2}$.

Then, the area of the imprint becomes

$$S = \frac{2\pi CA}{\beta} \int_{\frac{\beta(A-h)}{A}}^{\beta} \sqrt{1 - t^2}\, dt.$$

By integration, we get

$$S = \frac{\pi RA}{\beta}[t\sqrt{1 - t^2} + \arcsin t]_{\beta\frac{(A-h)}{A}}^{\beta}.$$

Thus,

$$S = \frac{\pi RA}{\beta}[\beta\sqrt{1 - \beta^2} + \arcsin \beta] - \frac{\pi RA}{\beta}\left[\left(\beta\frac{A-h}{A}\right)\sqrt{1 - \left(\beta\frac{A-h}{A}\right)^2} + \arcsin\left(\beta\frac{A-h}{A}\right)\right] =$$

$$= \pi AR\sqrt{1 - \beta^2} + \pi AR\frac{\arcsin \beta}{\beta} - \pi R(A - h)\sqrt{1 - \left(\beta\frac{A-h}{A}\right)^2} -$$

$$-\frac{\pi(A-h)}{\beta\left(\frac{A-h}{A}\right)}\arcsin\left(\beta\frac{A-h}{A}\right). \tag{22}$$

For a modest applied load, the imprint is very small ($A >> h$) so $\frac{A-h}{A} \approx 1$

$$S = \pi Rh\left(\sqrt{1 - \beta^2} + \frac{\arcsin \beta}{\beta}\right). \tag{23}$$

Then, the hardness of a penetration (h) gets following expression:

$$H_{er} = \frac{\beta F}{\pi R h(\beta\sqrt{1-\beta^2} + \arcsin\beta)}. \tag{24}$$

From Eq. (17), the penetration expression h becomes a function of the both indenter dimensions (A, R) and the radius r of the projected imprint,

$$h = \frac{AR - A\sqrt{R^2 - r^2}}{R} \quad \text{(accepted)}, \qquad \text{and} \qquad h = \frac{AR + A\sqrt{R^2 - r^2}}{R} > A \quad \text{(rejected)}.$$

Thus,

$$h = A\left(1 - \sqrt{1 - \left(\frac{r}{R}\right)^2}\right). \tag{25}$$

For a small imprint (cases: microhardness or nanohardness), the ratio $\left(\frac{r}{R}\right)$ is negligible [12]:

$$\left(\sqrt{1 - \left(\frac{r}{R}\right)^2}\right) = 1 - \frac{1}{2}\left(\frac{r}{R}\right)^2. \tag{26}$$

By introducing Eq. (26) in Eq. (25), we get

$$h = \frac{A}{2}\left(\frac{r}{R}\right)^2. \tag{27}$$

Then, the expression of the imprint surface becomes

$$S = \pi\frac{Ar^2}{2R}\left(\sqrt{1-\beta^2} + \frac{\arcsin\beta}{\beta}\right). \tag{28}$$

Then, the hardness expression will be given by the formula

$$H_{er} = \frac{2R\beta F}{\pi A r^2(\beta\sqrt{1-\beta^2} + \arcsin\beta)} = G\frac{F}{r^2} \tag{29}$$

with the constant

$$G = \frac{2R\beta}{\pi A(\beta\sqrt{1-\beta^2} + \arcsin\beta)}. \tag{30}$$

Conclusions

1. In the present paper, the most important result of using an indenter of ellipsoidal geometric form to measure the material hardness is the derived mathematical expressions of the imprint in different geometric shapes.

2. In the determination of hardness expression, when the indenter has a geometrical form of an ellipsoidal, the considered surface of the resulting imprint has been taken as an elliptical section (projection of the real print) in the first case. The related mathematical was simple, but in the second case, where the surface of the imprint is considered as an ellipsoidal cap, the hardness expression is more complex. Thus, a geometrical approach has been made, to ease the mathematical expression of the resulting imprint and hardness.

3. As a main conclusion, the indenter of ellipsoidal shape presents theoretical and experimental interests, which will lead to widen the field of applications of various methods of the hardness tests.

4. In this theoretical study , the hardness expression of a spherical indenter can be deduced. Also, it can be seen the differences between an ellipsoid indenter and spherical indenter.

Резюме

Якщо при дослідженні твердості за індентор слугувало тверде тіло еліпсоїдної форми, вираз для статичної твердості розглядався як функція від глибини і радіусів відбитка індентора. При цьому використовувалась загальна формула, що зв’язувала статичну твердість із відношенням нормальної сили, прикладеної до індентора, до реальної площі його відбитка. При отриманні кінцевої формули для розрахунку твердості використовувались геометричний і механічний підходи.

1. D. Tabor, *The Hardness of Metals*, Clarendon Press, Oxford (1951).
2. B. Jönsson and S. Hogmark, “Hardness measurements on thin films,” *Thin Solid Films*, **114**, 257–269 (1984).
3. P. Morisset et P. Salmon, *Chromage Dur et Décoratif*, CETIM, Senlis (Oise) (1988).
4. J. P. Mercier, G. Zambelli, et W. Kurz, *Introduction à la Science des Matériaux*, PPUR, Lausanne (1999).
5. C. Barlier et L. Girardin, *Matériaux et Usinage*, Paris (1999).
6. E. S. Puchi-Cabrera, “A new model for the computation of the composite hardness of coated systems,” *Surf. Coat. Technol.*, **160**, 177–186 (2002).
7. A. M. Korsunsky, M. R. McGurk, S. J. Bull, and T. F. Page, “On the hardness of coated systems,” *Surf. Coat. Technol.*, **99**, 171–183 (1998).
8. J. V. Fernandes, A. C. Trindade, L. F. Menezes, and A. Cavaleiro, “A model for coated surface hardness,” *Surf. Coat. Technol.*, **131**, 457–461 (2000).
9. D. Chicot, L. Gil, K. Silva, et al., “Thin film hardness determination using indentation loading curve modelling,” *Thin Solid Films*, **518**, 5565–5571(2010).
10. J. Qin, Y. Huang, K. C. Hwang, et al., “The effect of indenter angle on the microindentation hardness,” *Acta Mater.*, **55**, 6127–6132 (2007).
11. N. Piskounov, Calcul Diférentiel et Intégral, Office des Publication Universitaires, Algeria (1991).
12. L. Chambadal, *Formulaire de Mathématique*, BORDAS, Paris (1985).

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