INVERSE PROBLEMS AND "LOADED" COMPOSITE TYPE EQUATIONS

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In the present article we state a new approach to study nonlinear inverse problems for partial differential equations. Our approach bases on transition to "loaded" composite type equations [16]. As an example we consider the inverse problem for a parabolic equation with two unknown coefficients. Parabolic inverse problems were studied by many authors (see, for instance, [1–15] and the bibliography therein) but the problem we address below is new.

Let D be a bounded domain in the space R^n, $x = (x_1, \dots, x_n) \in D$, $t \in (0,T)$, $0 < T < +\infty$, and Q be the cylindrical domain $D \times (0,T)$. For simplicity, we assume that the boundary Γ of the domain D is infinitely differentiable.

Consider the equation

$$Lu \equiv \rho(x)u_t - \Delta u + \lambda u + q(x)c(x,t)u = f(x,t), \tag{1}$$

where λ is a given positive constant, $f(x,t)$ and $c(x,t)$ are given functions, and $c(x,t)$ is infinitely differentiable for $(x,t) \in \overline{Q}$.

Inverse Problem I. *Find the functions $u(x,t)$, $\rho(x)$ and $q(x)$ satisfying* (1) *and the following conditions:*

$$u(x,0) = u_0(x), \quad u_t(x,0) = v_0(x), \quad u_t(x,T) = v_1(x) \ \textit{for} \ x \in D, \tag{2}$$

$$u(x,t)|_{\Gamma\in(0,T)} = \mu(x,t) \tag{3}$$

(the functions $u_0(x)$, $v_0(x)$, $v_1(x)$, and $\mu(x,t)$ are given).

We describe the method for studying Inverse Problem I.

Put $t = 0$ in the equation

$$\frac{\partial}{\partial t}Lu = f_t. \tag{*}$$

Under (1) and (2) we may find $\rho(x)$ and $q(x)$ by means of $u_{tt}(x,0)$. Further, put $t = T$. Considering (*) and (2), we derive the nonlocal boundary conditions for $u_{tt}(x,0)$ and $u_{tt}(x,T)$. Look at the equation

$$\frac{\partial^2}{\partial t^2}Lu = f_{tt}. \tag{**}$$

"Forgetting" the condition $u_t(x,0) = v_0(x)$, we may obtain the nonlocal boundary problem for "loaded" composite type equation (**). To prove the existence theorem for this problem, under integrating we come to the existence theorem for Inverse Problem I.

Suppose that the following condition is satisfied: *there is a function $U(x,t)$ in the space $C^4(\overline{Q})$ provided that*

$$U(x,0) = u_0(x), \quad U_t(x,0) = v_0(x), \quad U_t(x,T) = v_1(x) \ \textit{for} \ x \in D, \tag{4}$$

$$U(x,t)|_{\Gamma\times(0,T)} = \mu(x,t).$$

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Define the set V_3:

$$V_3 = \{v(x,t) : v \in L_2(0,T; W_2^2(D)) \bigcap L_\infty(Q),$$
$$v_t \in L_2(0,T; W_2^2(D)) \bigcap L_\infty(Q),$$
$$v_{tt} \in L_2(0,T; W_2^2(D)) \bigcap L_\infty(Q), \quad v_{ttt} \in L_2(Q)\}$$

It is obvious that V_3 is a Banach space under the norm

$$||v||_V = \operatorname{vrai}\max_{\overline{Q}} |v| + \operatorname{vrai}\max_{\overline{Q}} |v_t| + \operatorname{vrai}\max_{\overline{Q}} |v_{tt}|$$
$$+ \left(\int_Q [v_{ttt}^2 + (\Delta v)^2 + (\Delta v_t)^2 + (\Delta v_{tt})^2] dx dt \right)^{1/2}.$$

Define the functions

$$A(x) = v_0(x)[c(x,0)v_0(x) + c_t(x,0)u_0(x)], \quad B(x) = -c(x,0)u_0(x),$$
$$A_1(x) = [f(x,0) + \Delta u_0(x) - \lambda u_0(x)][c(x,0)v_0(x) + c_t(x,0)u_0(x)]$$
$$-[f_t(x,0) + \Delta v_0(x) - \lambda v_0(x)]c(x,0)u_0(x),$$
$$C_1(x) = [f_t(x,0) + \Delta v_0(x) - \lambda v_0(x)]v_0(x),$$
$$D_1(x) = -[f(x,0) + \Delta u_0(x) - \lambda u_0(x)],$$
$$A_0(x) = c(x,T)v_1(x)D_1(x) - [f_t(x,T) + \Delta v_1(x) - \lambda v_1(x)]B(x),$$
$$F_0(x) = [f_t(x,T) + \Delta v_1(x) - \lambda v_1(x)]A(x) - c(x,T)v_1(x)C_1(x);$$

and the numbers

$$a_0 = \max_{\overline{D}} u_0(x), \quad b_1 = \min_{\overline{D}} v_1(x),$$
$$\lambda_0 = \max_{\overline{D}} \left| \frac{A_1(x)}{A_0(x)} \right|, \quad k_0 = \max_{\overline{D}} \left| \frac{F_0(x)}{A_0(x)} \right|,$$
$$k_1 = \max_{\overline{Q}} |f_{tt}(x,t)| + 2|b_1| \max_{\overline{Q}} |c_t(x,t)| \cdot \max_{\overline{D}} \left| \frac{C_1(x)}{A(x)} \right|$$
$$+a_0 \max_{\overline{Q}} |c_{tt}(x,t)| \max_{\overline{D}} \left| \frac{C_1(x)}{A(x)} \right|,$$
$$k_2 = 2|b_1| \max_{\overline{Q}} |c_t(x,t)| \cdot \max_{\overline{D}} \left| \frac{D_1(x)}{A(x)} \right| + a_0 \max_{\overline{Q}} |c_{tt}(x,t)| \cdot \max_{\overline{D}} \left| \frac{D_1(x)}{A(x)} \right|,$$
$$\mu = \max_{\Gamma \in [0,T]} |\mu_{tt}(x,t)|, \quad k = \frac{\lambda}{\lambda(1-\lambda_0) - k_2} \max \left\{ \frac{k_1}{\lambda}, k_0, \mu \right\}.$$

THE MAIN THEOREM. *Assume that (4) holds and the functions $f(x,t)$, $f_t(x,t)$, and $f_{tt}(x)$ are bounded and measurable on $\overline{Q}$. Moreover, the following conditions are satisfied:*

$$f_{tt}(x,t) \leq 0, c(x,t) \geq 0, c_t(x,t) \leq 0, c_{tt}(x,t) \geq 0 \quad \textit{for} \quad (x,t) \in \overline{Q}; \tag{5}$$

$$c_t(x,T) = 0,$$
$$A(x) \geq A^0 > 0, \quad A_1(x) \geq A_1^0 > 0,$$
$$C_1(x) \geq C_1^0 > 0, \quad B(x) \leq 0, \quad D_1(x) \leq 0$$
$$\textit{for } x \in \overline{D} \ (A^0, A_1^0, \textit{ and } C_1^0 \textit{ are constants}); \tag{6}$$

$$\frac{F_0(x)}{A_0(x)} \in W_2^1(D), \quad \frac{A_1(x)}{A_0(x)} \in W_\infty^1(D),$$
$$F_0(x) \geq 0, \quad A_1(x) + A_0(x) \leq 0 \quad \textit{for} \quad x \in \overline{D}, \tag{7}$$

$$\mu_{tt}(x,t) \le 0, \quad \mu_t(x,t) \le 0, \quad \mu(x,t) \ge 0 \quad for \quad x \in \Gamma, \quad t \in [0,T]; \tag{8}$$

$$\lambda_0 < 1, \quad k_2 < \lambda(1-\lambda_0); \tag{9}$$

$$v_1(x) + kT \le v_0(x) \le 0, \quad u_0(x) + b_1 T \ge 0 \quad for \quad x \in \overline{D}; \tag{10}$$

$$A_1(x)\mu_{tt}(x,T) + A_0(x)\mu_{tt}(x,0) = F_0(x) \quad for \quad x \in \Gamma. \tag{11}$$

Then Inverse Problem I has the solution $\{u(x,t), \rho(x), q(x)\}$ *such that* $u(x,t) \in V_3$, $\rho(x)$ *and* $q(x)$ *are positive functions bounded almost everywhere on* $\overline{D}$.

Proof. Define the functions

$$\rho_v(x) = \frac{A_1(x)}{A(x) - B(x)|v_{tt}(x,0)|}, \quad q_v(x) = \frac{C_1(x) - D_1(x)|v_{tt}(x,0)|}{A(x) - B(x)|v_{tt}(x,0)|}$$

where $v(x,t) \in V_3$. Show that under (6) the functions ρ_v and q_v are positive on $\overline{D}$ and under the condition $v(x,t) \in V_3$ these functions are bounded almost everywhere on $\overline{D}$. Define the functions $G_0(\xi)$ and $G_1(\xi)$:

$$G_0(\xi) = \begin{cases} 0 & if \quad \xi < 0, \\ \xi & if \quad \xi \in [0, a_0], \\ a_0 & if \quad \xi > a_0, \end{cases}$$

$$G_1(\xi) = \begin{cases} b_1, & if \quad \xi < b_1, \\ \xi, & if \quad \xi \in [b_1, 0], \\ 0, & if \quad \xi > 0. \end{cases}$$

Finally, define the function:

$$f_v(x,t) = f_{tt}(x,t) - 2q_v(x)c_t(x,t)G_1(v_t(x,t)) - q_v(x)c_{tt}(x,t)G_0(v(x,t))$$

with $v(x,t) \in V_3$.

Consider the boundary value problem: *Find a solution to the equation*

$$\rho_v(x)u_{ttt} - \Delta u_{tt} + \lambda u_{tt} + q_v(x)c(x,t)u_{tt} = f_v(x,t) \tag{1'}$$

satisfying the following conditions:

$$A_1(x)u_{tt}(x,T) + A_0(x)u_{tt}(x,0) = F_0(x), \tag{12}$$

$$u_t(x,T) = v_1(x), u(x,0) = u_0(x) \quad \text{for} \quad \in D, \tag{13}$$

$$u_{tt}(x,t)|_{\Gamma\times(0,T)} = \mu_{tt}(x,t). \tag{14}$$

Under conditions (4), (6), (7), and (11) of the theorem, there is a solution $u_{tt}(x,t)$ of the problem (1′), (12), (14) (see V. V. Shelukhin's work [17]) belonging to the space $W_2^{2,1}(Q) \cap L_\infty(Q)$. After finding the function $u_{tt}(x,t)$ it is easy to get the functions $u_t(x,t)$ and $u(x,t)$. Moreover, this solution is as follows:

$$u_{tt}(x,t) \in W_2^{2,1}(Q) \bigcap L_\infty(Q).$$

This defines the operator Φ: $\Phi(v) = u$ that Φ transforms the space V_3 into V_3. We show that Φ has fixed points.

Let M_0, M_1, M_2, and R_0 be some positive numbers and let W be the set of functions $v(x,t)$ in V_3 satisfying (12)–(14) and the conditions

$$\operatorname{vrai}\max_{\overline{Q}} |u(x,t)| \le M_0, \quad \operatorname{vrai}\max_{\overline{Q}} |u_t(x,t)| \le M_1,$$

$$\operatorname{vrai}\max_{\overline{Q}} |u_{tt}(x,t)| \le M_2,$$

$$\int_Q [u_{ttt}^2 + (\Delta u)^2 + (\Delta u_t)^2 + (\Delta u_{tt})^2]dxdt \le R_0.$$

It is evident that W is convex, closed, and bounded. Show that there are numbers M_0, M_1, M_2, and R_0 for which the operator Φ transforms the set W into W. The estimate of the maximum principle for $(1')$ implies the inequality

$$\operatorname{vrai}\max_{\overline{Q}} |u_{tt}(x,t)| \le \max\left\{\frac{1}{\lambda}\sup_{\overline{Q}}|f_v(x,t)|, \mu, \operatorname{vrai}\max_{\overline{D}}|u_{tt}(x,0)|\right\}.$$

This inequality and (12) yield

$$\operatorname{vrai}\max_{\overline{Q}} |u_{tt}(x,t)| \le \max\left\{\frac{1}{\lambda}\sup_{\overline{Q}}|f_v(x,t)|, \mu, k_0\right\} + \lambda_0 \operatorname{vrai}\max_{\overline{Q}}|u_{tt}(x,t)|.$$

Furthermore, we have

$$\operatorname{vrai}\max_{\overline{Q}} |u_{tt}(x,t)| \le \frac{1}{1-\lambda_0}\max\left\{\frac{1}{\lambda}\sup_{\overline{Q}}|f_v(x,t)|, \mu, k_0\right\}. \tag{15}$$

The function $f_v(x,t)$ can be estimated

$$\sup_{\overline{Q}}|f_v(x,t)| \le k_1 + k_2 M_2.$$

Together with (15) we have

$$\operatorname{vrai}\max_{\overline{Q}} |u_{tt}(x,t)| \le \frac{1}{1-\lambda_0}\max\left\{\frac{k_1}{\lambda}, k_0, \mu\right\} + \frac{k_2}{\lambda(1-\lambda_0)}M_2. \tag{16}$$

Let the number M_2 satisfy the condition

$$M_2 \ge \frac{\lambda}{\lambda(1-\lambda_0) - k_2}\max\{\frac{k_1}{\lambda}, k_0, \mu\}.$$

Then (16) and (9) imply the estimate

$$\operatorname{vrai}\max_{\overline{Q}} |u_{tt}(x,t)| \le M_2. \tag{17}$$

In turn, (17) and (13) imply the estimate

$$\operatorname{vrai}\max_{\overline{Q}} |u_t(x,t)| \le \max_{\overline{D}} v_1(x) + TM_2.$$

Let M_1 satisfy the condition

$$M_1 \ge \max_{\overline{D}} v_1(x) + TM_2.$$

Then we have

$$\operatorname{vrai}\max_{\overline{Q}} |u_t(x,t)| \le M_1.$$

Analogously, M_0 satisfies the condition

$$M_0 \ge \max_{\overline{Q}} u_0(x) + TM_1.$$

Then

$$\operatorname{vrai}\max_{\overline{Q}} |u_t(x,t)| \le M_0.$$

Rewrite $(1')$ as follows:

$$u_{ttt} - \Delta u_{tt} = \varphi(x,t), \tag{18}$$

where $\varphi(x,t)$ satisfies the estimate

$$\operatorname{vrai}\max_{\overline{Q}} |\varphi(x,t)| \le R_1 \tag{19}$$

and R_1 depends only on the numbers M_0, M_1, and M_2, and the functions $c(x,t)$, $f(x,t)$, $u_0(x)$, $u_1(x)$, $v_0(x)$, $v_1(x)$, and $\mu(x,t)$. The routine calculations (see, for example, [15]) together with (19) allow us to prove the integral estimate

$$\int_Q [u_{ttt}^2 + (\Delta u)^2 + (\Delta u_t)^2 + (\Delta u_{tt})^2]dxdt \le R_2,$$

where R_2 depends only on the numbers M_0, M_1, and M_2 and the functions $c(x,t)$, $f(x,t)$, $u_0(x)$, $u_1(x)$, $v_0(x)$, $v_1(x)$, and $\mu(x,t)$. Assume that R_0 satisfies $R_0 \ge R_2$.

We have

$$\int_Q [u_m^2 + (\Delta u)^2 + (\Delta u_t)^2 + (\Delta u_{tt})^2]dxdt \le R_2.$$

So, if the numbers M_0, M_1, M_2, and R_2 satisfy the above conditions, then the operator Φ transforms the set W into W.

Show now that Φ is a compact operator.

Let the sequence $\{v_m(x,t)\}_{m=1}^{\infty}$ be bounded on the space V_3. Then $\{u_m(x,t)\}_{m=1}^{\infty}$, $u_m = \Phi(v_m)$ is also bounded on V_3. So, there exist subsequences $\{v_{m_k}(x,t)\}_{k=1}^{\infty}$ and $\{u_{m_k}(x,t)\}_{k=1}^{\infty}$ such that

$$v_{m_k}(x,t) \underset{k\to\infty}{\to} v(x,t), \;\; u_{m_k}(x,t) \underset{k\to\infty}{\to} u(x,t) \;\; \textit{almost everywhere on } \overline{Q},$$

$$v_{m_k t}(x,t) \underset{k\to\infty}{\to} v_t(x,t), \;\; u_{m_k t}(x,t) \underset{k\to\infty}{\to} u_t(x,t) \;\; \textit{almost everywhere on } \overline{Q},$$

$$v_{m_k tt}(x,t) \to v_{tt}(x,t), \;\; u_{m_k tt}(x,t) \to u_{tt}(x,t) \;\; \textit{almost everywhere on } \overline{Q},$$

$$v_{m_k tt}(x,0) \to v_{tt}(x,0), \;\; u_{m_k tt}(x,0) \to u_{tt}(x,0) \;\; \textit{almost everywhere on } \overline{D},$$

$$v_{m_k ttt}(x,t) \to v_{ttt}(x,t), \;\; u_{m_k ttt}(x,t) \to u_{ttt}(x,t) \;\; \textit{weakly in } L_2(Q),$$

$$\Delta v_{m_k tt}(x,t) \to \Delta v_{tt}(x,t), \;\; \Delta u_{m_k tt}(x,t) \to \Delta u_{tt}(x,t) \;\; \textit{weakly in } L_2(Q). \tag{20}$$

Assume that $\rho_k(x)$, $q_k(x)$, $f_k(x,t)$, $\rho(x)$, $q(x)$, and $f(x,t)$ are functions determined by the functions $v_{m_k}(x,t)$ $v(x,t)$, respectively. It follows from (20) that $u = \Phi(v)$. Hence, we have

$$\rho(u_{m_k ttt} - u_{ttt}) - \Delta(u_{m_k tt} - u_{tt}) + \lambda(u_{m_k tt} - u_{tt})$$
$$+cq(u_{m_k tt} - u_{tt}) = f_k - f + (\rho - \rho_k)u_{m_k ttt} + c(q - q_k)u_{m_k tt}. \tag{21}$$

Repeating for (21) the calculations which give the integral estimate for (1′), (12)–(14) and again using (20) for the sequences $\{u_{m_k}(x,t)\}_{k=1}^{\infty}$ and $\{v_{m_k}(x,t)\}_{k=1}^{\infty}$, we easily infer the convergence

$$\int_Q [(u_{m_k ttt} - u_{ttt})^2 + (\Delta u_{m_k} - \Delta u)^2 + (\Delta u_{m_k t} - \Delta u_t)^2 + (\Delta u_{m_k tt} - \Delta u_{tt})^2]dxdt \underset{k\to\infty}{\to} 0. \tag{22}$$

Convergences (20) (22) mean that the sequence $\{u_{m_k}(x,t)\}_{k=1}^{\infty}$ converges weakly in V_3. In other words, for every sequence $\{v_m(x,t)\}_{m=1}^{\infty}$ bounded in V_3 we may construct the sequence $\{v_{m_k}(x,t)\}_{k=1}^{\infty}$ such that $\{\Phi(v_{m_k})\}_{k=1}^{\infty}$ converges weakly in V_3. This means that Φ is a compact operator.

So, we prove that for Φ and the constructed set W all conditions of the Schauder Theorem are satisfied. By this theorem there is a function $u(x,t)$ in W satisfying the boundary value conditions (12)–(14) and solving the equation

$$\rho_u u_{ttt} - \Delta u_{tt} + \lambda u_{tt} + cq_u u_{tt} = f_u(x,t). \tag{23}$$

Note that by (5), (8) and the maximum principle the inequality $u_{tt}(x,t) \le 0$ holds at almost every point (x,t) of the cylinder $\overline{Q}$ for the function $u(x,t)$. Then the functions $\rho_u(x)$ and $q_u(x)$ have the form

$$\rho_u(x) = \frac{A_1(x)}{A(x) + B(x)u_{tt}(x,0)}, \quad q_u(x) = \frac{C_1(x) + D_1(x)u_{tt}(x,0)}{A(x) + B(x)u_{tt}(x,0)}.$$

Further, repeating the proof of (17), we easily establish the estimate

$$\operatorname*{vrai\,max}_{\overline{Q}} |u_{tt}| \le \frac{\lambda}{\lambda(1-\lambda_0) - k_2} \max\left\{\frac{k_1}{\lambda}, \mu, k_0\right\}.$$

Together with nonnegativeness of the function $u_{tt}(x,t)$, this means that we infer the inequalities

$$-k \le u_{tt}(x,t) \le 0 \tag{24}$$

at almost all (x,t) of $\overline{Q}$.

Integrating (24) from t to T and using (10), we obtain that for the function $u_t(x,t)$ the following inequalities hold

$$b_1 \le u_t(x,t) \le 0 \tag{25}$$

for almost all (x,t) of $\overline{Q}$. In turn, integrating (25) from 0 to t and once more using (10), we derive that for $u(x,t)$ the inequalities

$$0 \le u(x,t) \le a_0 \tag{26}$$

also hold at all points (x,t) of $\overline{Q}$.

Inequalities (25) and (26) imply $G_0(u) = u$ and $G_1(u_k) = u_t$. Then (23) turns into the equation

$$\rho_u u_{ttt} - \Delta u_{tt} + \lambda u_{tt} + c q_u u_{tt} + 2c_t q_u u_t + c_{tt} q_u u = f_{tt}. \tag{27}$$

Integrating (27) in the variable t from 0 to T, after routine but simple calculations we obtain

$$\Delta[u_t(x,0) - v_0(x)] - \lambda[u_t(x,0) - v_0(x)] - c(x,0)q_u[u_t(x,0) - v_0(x)] = 0. \tag{28}$$

The functions $u_t(x,0)$ and $v_0(x)$ agree for $x \in \Gamma$. Equation (28) yields that this functions agree for $x \in D$ as well. In other words, (27) meeting (12)–(14) also satisfies (2).

We then integrate (27) with respect to t from 0 to the varying point t^*. Using (2), (13), and (14) and renaming the variable t^*, once again after simple but routine calculations we arrive at the equality

$$\rho_u u_{tt} - \Delta u_t + \lambda u_t + c q_u u_t + c_t q_u u = f_t. \tag{29}$$

Integrating (29) in the variable t from 0 to t^*, using (2), (14), and (13), and renaming t^*, we obtain that a solution to equation (27) is a solution to the equation

$$\rho_u u_t - \Delta u + \lambda u + c q_u u = f,$$

i.e., (1).

The proof of the theorem is complete.

Using the above theorem and extra conditions on the initial data, we may study

INVERSE PROBLEM II. *Find the functions $u(x,t)$, $\rho(x)$, and $q(x)$ satisfying* (1), *the conditions*

$$u(x,0) = u_0(x), \quad u_t(x,0) = v_0(x), \quad u(x,T) = u_1(x) \ \ for \ \ x \in D, \tag{2'}$$

and condition (3).

Let $v_1(x)$ be the function

$$v_1(x) = -\frac{c(x,T)u_1(x)C_1(x)}{A_1(x)}.$$

Instead of Inverse Problem II we consider Inverse Problem I with given functions $u_0(x)$, $v_0(x)$, and $\mu(x,t)$ and the above-introduced function $v_1(x)$. Under all conditions of the Main Theorem, this problem has a solution $\{u(x,t), \rho(x), q(x)\}$ of the class determined in the theorem. Now, assume that the conditions

$$\Delta u_0(x) - \lambda u_0(x) = -f(x,0) \quad \text{for} \quad x \in D; \tag{30}$$

$$\Delta u_1(x) - \lambda u_1(x) = -f(x,T) \quad \text{for} \quad x \in D \tag{31}$$

hold. Integrating (29) in the variable t from 0 to T, we then lead to

$$\Delta[u(x,T) - u_1(x)] - \lambda[u(x,T) - u_1(x)] - c(x,T)q(x)[u(x,T) - u_1(x)] = 0.$$

The last equation implies that the functions $u(x,T)$ and $u_1(x)$ coincide for $x \in D$. In other words, *under the conditions of the Main Theorem and conditions* (30) *and* (31), *a solution of Inverse Problem I with a special function* $v_1(x)$ *is a sought solution of Inverse Problem II.*

In conclusion, we make a few remarks.

Firstly, instead of (1) we may consider more general equations. For instance, the number λ may be changed by the function $\lambda(x,t)$; the coefficient for $u_t(x,t)$ may be as $\rho(x)a(x,t)$ with $\rho(x)$ unknown coefficient and $a(x,t)$ the given function as before.

Secondly, existence of solutions to Inverse Problems I and II was proven in the spaces constructed by means of the L_2 and L_∞ spaces. The method, exposed in the present article, makes it possible to prove existence of solutions to Inverse Problems I and II in the spaces constructed by means of Hölder spaces.

Thirdly, the algebraic conditions of the Main Theorem are mostly sufficient but not minimal. They may be improved in many cases. Moreover, these conditions clearly describe a nonempty class of input data for Inverse Problems I and II.

Finally, the method of the present article is applicable to other inverse problems.

References

[1] Beznoshchenko N. A., *Some problems of determining coefficients for lower terms of parabolic equations*, Sibirsk. Mat. Zh., **16** (1975), no. 6, 1135–1147.

[2] Beznoshchenko N. A. *On existence of a solution to a problem of determining a coefficient q in the equation $u_t - \Delta u + qu = F$*, Diffenetsial'nye Uravneniya, **15** (1979), no. 1, 10–17 .

[3] Bubnov B. A. *Existence and uniqueness of a solution to inverse problems for parabolic and elliptic equations*, in: Nonclassical Equations of Mathematical Physics, Inst. Mat. (Novosibirsk), Novosibirsk, 1986, pp. 25–29. (Russian)

[4] Anikonov Yu. E. and Bubnov B. A. *Existence and uniqueness of a solution to an inverse problem for a parabolic equation*, Dokl. Akad. Nauk SSSR, **298** ,(1988) no. 4, 777–779.

[5] Anikonov Yu. E. and Belov Yu. Ya. *On unique solvability of a certain inverse problem for a parabolic equation*, Dokl. Akad. Nauk SSSR, **306** (1989), no. 6, 1289–1293.

[6] Bubnov B. A. *To the Question on Solvability of Multidimensional Inverse Problems for Parabolic Equations*, Preprint, No. 87–714, Vychisl. Tsentr Sibirsk. Otdel. Akad. Nauk SSSR, Novosibirsk, 1989. (Russian)

[7] Riganti R. and Savateev T. *On solution of an inverse problem for the nonlinear heat equation*, Rapporto Interno, Politecnico di Torino, Turin, no. 25 (1991).

[8] Prilepko A. I. and Kostin A. B. *On inverse problems of determining a coefficient in a parabolic equation.* I, Sibirsk. Mat. Zh., **33** (1992), no. 3, 146–155.

[9] Prilepko A. I. and Kostin A. B. *On inverse problems of determining a coefficient in a parabolic equation.* II, Sibirsk. Mat. Zh., **34** (1993), no. 5, 147–162.

[10] Vishnevskiĭ M. P. *On solvability of one inverse problem for a parabolic equation with convergence*, Sibirsk. Mat. Zh., **33** (1992), no. 3, 42–49.

[11] Solov'ëv V. V. *Existence of a solution in a problem of determining a coefficient in a semilinear equation of parabolic type*, Diffenetsial'nye Uravneniya, **28** (1992), no. 12, 2101–2111.

[12] Savateev E. G. *An inverse problem for the Burgers equation and its hyperbolic regularization*, J. Inverse Ill-Posed Probl., **1** (1993), no. 3, 231–243.
[13] Belov Yu. Ya. *Inverse problems for parabolic equations*, J. Inverse Ill-Posed Probl., **1** (1993), no. 4, 283–305.
[14] Anikonov Yu. E. and Vishnevskiĭ M. P. *Reduction of multidimensional inverse problems to initial-boundary value problems in Hilbert space*, Sibirsk. Mat. Zh., **35** (1994), no. 3, 495–514.
[15] Kozhanov A. I. *Composite Type Equations and Nonlinear Inverse Problems for Elliptic and Parabolic Equations*, Preprint, No. 54, Sobolev Inst. Mat. (Novosibirsk), Novosibirsk, 1998. (Russian)
[16] Nakhushev A. M. *Equations of Mathematical Biology*, Vysshaya Shkola, Moscow, 1995. (Russian)
[17] Shelukhin V. V. *Problems Nonlocal in Time for Equations of Hydrodynamics and Variational Principles*, Dis. Dokt. Fiz.-Mat. Nauk, Novosibirsk, 1992. (Russian)

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