Yu.V. MILMAN, S.I. CHUGUNOVA,
I.V. GONCHAROVA, and A.A. GOLUBENKO

I.M. Frantsevich Institute for Problems in Materials Science, NAS of Ukraine;
3 Academician Krzhizhanovsky Str.; UA-03142 Kyiv, Ukraine

PLASTICITY OF MATERIALS DETERMINED BY THE INDENTATION METHOD

In this review, the development of techniques for determining the plasticity of materials by the indentation is considered. The development of methods for determining the plasticity of materials by the indentation is based on the use of fundamental ideas of the physics of strength and plasticity. Significant development of these methods became possible after the introduction of a new plasticity characteristic $\delta^* = \varepsilon_p/\varepsilon_t$, where ε_p is the plastic deformation, and ε_t is the total deformation. This plasticity characteristic corresponds to the modern physical definitions of plasticity, in contrast to the widely used elongation to failure δ. The new plasticity characteristic is easily determined by standard determination of hardness by the diamond pyramidal indenters at constant load P (designated as δ_H) and by instrumental nanoindentation (designated as δ_A, and $\delta_H \approx \delta_A$). A significant advantage of the new plasticity characteristic is the ability to determine it not only for metals, but for materials, which are brittle at the standard mechanical tests (ceramics, thin layers, coatings, *etc.*), as well. In the development of ideas about theoretical strength, concepts of theoretical plasticity under the dislocation-free and dislocation deformation mechanisms are introduced. A number of studies have established a correlation of δ_H with the electronic structure of the material and its physical properties. As shown, the Tabor parameter C ($C = HM/\sigma_S$, where HM is the Meyer hardness, and σ_S is the yield stress) is easily calculated by the δ_H value. Therefore, indentation allows currently determining simply not only the hardness, but also the plasticity and yielding stress of materials. Thus, indentation became a simple method for determination of the complex of mechanical properties of materials in a wide temperature range using a sample in the form of a metallographic specimen.

Keywords: hardness, plasticity, indentation, yield stress, deformation.

Introduction

Methods of determination of the hardness by indentation with a rigid indenter (ball or pyramid) are simple and extensively used techniques for characterization the mechanical properties of materials. The hard-

ness, determined by this method, is undoubtedly a strength characteristic connected with the yield strength of the material [1]. However, attempts to determine also a plasticity characteristic by the indentation method were made for many years (see, *e.g.*, Refs. [2–6]).

An analysis of the indentation process by a ball in determination of the Brinell hardness enabled the author of Ref. [2] to introduce the notion of the modulus of plasticity of a material and calculate a certain effective transverse reduction of the area ψ. In Ref. [3], the evaluation of two plasticity characteristics, namely, the elongation δ and transverse reduction of the area ψ, by the indentation method was proposed. In this work, some local elongation δ and local transverse reduction ψ at the apex of the pileups around a hardness indent made by a spherical indenter are considered. The values of ψ determined for a number of steels by the indentation method were close to those determined in tensile tests. The author of Ref. [3] used the developed technique of determining the plasticity characteristics to optimize heat-treatment regimes of steels.

However, these investigations did not find an extensive application. This is evidently due to a substantial difference in the mechanisms of plastic deformation in indention and in uniaxial tension. Moreover, indentation by a ball cannot be used for ceramics and other high-strength materials, which are brittle in standard mechanical tests. In this connection, the possibility to determine approximately δ and ψ by the indentation method did not generate great interest among researchers.

In Ref. [7], for the first time, the notion of the plasticity index in contact of two surfaces was introduced. The surfaces are assumed to be conventionally plane, but their roughness and the crumpling of asperities on the surface by plastic deformation, which are taken to be spherical with a β radius, are taken into account. The plasticity index was determined as follows:

$$\psi = (E'/H)\sqrt{\sigma/\beta},$$

where $E' = E/(1 - \nu^2)$ when the roughness of one surface is taken into account, E is Young's modulus, ν is Poisson's ratio, H is the hardness determined by a spherical indenter, and σ are standard deviations of the asperity-height distribution.

The notion of the region of plastic crumpling of asperities A_p is used, and A is the total contact area. The value of A_p/A was considered in the range of 0.01–0.50, and, in this case, ψ belongs to the range of 0.6–1.0.

Results of this work are applicable to problems of friction and wear, but they were not used for the study of the plasticity of materials by the method of indentation with a rigid indenter.

In Refs. [4–11], the term plasticity was introduced to describe the process of plastic deformation in indentation. In these works, the in-

strumented hardness with recording the 'load on the indenter P–displacement of the indenter h' curve is predominantly considered. However, in most such works, a plasticity characteristic that enables one to compare the plasticity of different materials is not introduced. In Refs. [4, 5], the plasticity index determined in instrumented indentation $D = A_p/A_t$ (where A_p is the work of plastic deformation, and A_t is the work of total deformation) was introduced. This extremely interesting approach is discussed in more detail in the section, which deals with plasticity characteristic δ_A determined in the instrumented indentation.

In Ref. [12], the inconsistency of the extensively used plasticity characteristics (elongation to fracture δ and the transverse reduction of the area to fracture ψ) with the fundamental concepts of physics of strength was noted.

In Refs. [12, 13], it is noted that two fundamental properties determining the mechanical behaviour of materials, namely, their strength and plasticity, can be distinguished in physics of strength. The strength of a material is determined by its capability to resist an applied force. More specifically, the strength of solid bodies can be determined as the resistance to rupture body into two or several parts [14].

The strength is calculated adequately to these definitions in tensile tests as the breaking load divided by the cross-sectional area of the specimen. In this case, the material is assumed to be perfect, *i.e.*, without cracks and other stress concentrators. However, it should be noted that the hardness correlates with the yield strength rather than with the strength in the sense noted above.

The situation with the determination of the plasticity is much more difficult, if it is desired to obtain a characteristic adequate to the physical definition of this property. The word 'plasticity' comes from the Greek word 'πλαστικός', which means suitable for modelling and malleable. Since the present paper is devoted to the fundamental problems of plasticity, we present some definitions of the term 'plasticity' in physics of strength.

In physics and engineering, plasticity is defined by the susceptibility of a material to undergo residual deformations under load [15].

Plasticity is the property of the materials of solid bodies to deform irreversibly under the action of external forces and internal stresses [16].

Plasticity is the property of solid bodies to retain a part of strain after removal loads that caused it [17].

The plasticity of crystals is the property of crystalline bodies to change irreversibly their sizes and shape under the action of mechanical loads [18].

However, in practice, plasticity is usually characterized by the elongation δ to fracture in a tensile test or by the transverse reduction in area to fracture ψ. These parameters of the material are of great practical

importance but correspond weakly to the definitions of plasticity presented above because they determine the conditions of transition from plastic deformation to fracture and do not always correlate with plasticity in definitions presented above.

In Refs. [12, 13], it is noted that the parameters δ and ψ are determined by not only plasticity in the sense presented above (the malleability of the material and the capability to retain strain after removal of load), but also by at least two conditions: the relationship between the yield stress and the fracture stress as well as the strain hardening value. The parameter δ usually includes the uniform strain and the strain after formation of a stable 'neck' and localization of strain in it. The strain ε_c at which a stable 'neck' nucleates is determined by the condition [19]:

$$\sigma \geq \frac{1}{1-m}\left(\frac{\partial\sigma}{\partial e}\right)_{\dot{e}}.$$

Here, e is the true strain, $\dot{e}$ is the strain rate and the parameter m characterizes the influence of the strain rate on the yield stress according to the expression $\sigma = \text{const} \cdot \dot{e}^m$.

Therefore, δ depends not only on the compliance of the material to plastic deformation and on the degree of strain, which determines the transition from plastic deformation to fracture, but also on the strain hardening $(\partial\sigma/\partial e)_{\dot{e}}$ and parameter m, *i.e.*, on the rate sensitivity of the yield stress. It also should be taken into account that the rate of decrease of the cross-section of the 'neck; after its formation also depends on the parameter m. On the other hand, the strain hardening increases the yield stress and makes the transition to the fracture process more probable.

The fracture process, like the process of strain hardening, has a complicated multiform dependence on the test method, structure of the material, temperature and strain rate. In many cases, the dependence of the elongation to fracture δ on a large number of parameters leads to an inadequate estimate of plasticity. The plasticity of a material in its physical definition presented above must increase continuously with increasing temperature because an increase in the temperature facilitates the dislocation motion in solid bodies (except some intermetallics). However, for most materials, the parameter δ changes nonmonotonously with increasing temperature because the test temperature also influences on the conditions of fracture (transcrystallite or intercrystallite ones; brittle, quasi-brittle or ductile fracture [20, 21]) and strain hardening. For instance, in a number of dispersion-hardened aluminium alloys, δ decreases as the temperature increases above room temperature as a result of a decrease in the strain hardening and the earlier formation of a stable 'neck'. At the same time, for these alloys, an increase in the temperature leads to a decrease in the hardness and better deformability

during the metal forming (extrusion, rolling, *etc*.). In other words, objectively, the plasticity of the alloy increases.

The transverse reduction of the area ψ characterizes the cross-section of a specimen, at which the applied load turns to be sufficient for fracture. The transverse reduction of the area ψ can also consist of two summands: reduction of the area under uniform deformation and reduction of the area under deformation concentrated in the 'neck'. Since fracture occurs after plastic deformation, which occurs with strain hardening, ψ (like δ) is determined by the condition of transition from plastic deformation to fracture and depends on the strain hardening and the type of fracture of the material.

Thus, both usually used characteristics δ and ψ often inadequately reflect the physical meaning of the term plasticity, though they are convenient technological tests that characterize the capability of the material to be deformed plastically before fracture during tensile test.

It follows from the foregoing that, if the plasticity is characterized by the elongation to fracture δ, then, the notion of plasticity loses its clear physical meaning and, hence, ceases to be a fundamental characteristic of the material [12, 13].

Mechanical tensile tests of smooth specimens occupied a leading position many decades ago, when plastic materials, namely, steels and metals with f.c.c. and h.c.p. lattices, were the main structural materials. However, in subsequent years, radically new materials, which are low ductile or brittle in mechanical tensile tests at room temperature, were developed. These are ceramics, quasi-crystals, metallic glasses, intermetallics, fullerites, and different composites. Alloys based on refractory b.c.c. metals, for which the ductile-brittle temperature is usually higher than room temperature, found extensive application. The cold brittleness phenomenon, which was known for steels, but usually manifested itself below room temperature, turned to be typical for most alloys based on refractory metals with a b.c.c. lattice (Cr, Mo, and W) at room temperature as well. The efficiency of mechanical tensile tests for materials, which are brittle in tensile tests, is very low, and it is possible to determine only ultimate strength as a fracture stress. As for the plasticity of these materials in tensile tests, it can be said only that their elongation is δ = 0.

Such tests do not give any information on the comparative plasticity (or brittleness) of materials. At the same time, most materials, which are brittle in tensile tests, exhibit some plasticity in other 'softer' test methods, in particular, in determination of the hardness with a rigid indenter [12].

This situation made reasonable the introduction of a new plasticity characteristic as a fundamental property of a material that satisfies the physical definitions presented above. Such plasticity characteristic was

proposed in Ref. [12] in the form of the dimensionless parameter:

$$\delta^* = \frac{\varepsilon_p}{\varepsilon_t} = 1 - \frac{\varepsilon_e}{\varepsilon_t}, \tag{1}$$

where ε_p, ε_e, and ε_t are plastic, elastic and total strain, on the contact area specimen−indenter in the direction load P, respectively, and $\varepsilon_t = \varepsilon_p + \varepsilon_e$.

This plasticity characteristic (which, as is seen from Eq. (1), is determined by the fraction of the plastic strain in the total elastoplastic strain) corresponds fairly well to the physical definitions of plasticity presented above. Actually, the fraction of plastic strain in the total strain characterizes the malleability of the material, *i.e.*, its capability to change its shape (deform) with preservation of strain after removal of load. The considered plasticity characteristic is universal in the sense that it can be determined by any method of mechanical tests (tension, compression and bending) and, as shown in Ref. [12], in indentation.

The plastic and elastic components of strain can be determined from the curve of deformation in tension in coordinates stress σ−strain ε_t (Fig. 1), and the plasticity characteristic δ^* can be calculated with the help of these components ε_p and ε_e by Eq. (1).

In uniaxial tension or compression, as it follows from Eq. (1) and the Hooke law, the plasticity characteristic δ^* can be represented in the form [12] as follows:

$$\delta^* = 1 - \frac{\sigma_S}{\varepsilon_t E}, \tag{2}$$

where E is Young's modulus and σ_S is the yield stress for the achievement of the strain ε_t.

From expressions (1) and (2), it is seen that δ^* depends on the degree of total strain ε_t. The same can also be said about the value of δ^* determined according to Eq. (1) by other methods of mechanical tests. This dependence follows directly from the definitions of plasticity presented in the foregoing. Actually, *e.g.*, in tensile tests, in the first stages of loading, $\varepsilon_t = \varepsilon_e$, and plastic strain is absent, even in the ductile metals, *i.e.*, the material does not preserve a part of strain after removal of loads. As the critical shear stress is attained, plastic strain appears, and, subsequently, its fraction rises with increase in the load and ε_t, *i.e.*, the plastic deformability of a material, and, therefore, δ^* increases [12].

It is noted in Refs. [12, 13] that the dependence of the plasticity and its characteristic δ^* on the degree of strain ε_t and a comparison of the values of plasticity of different materials should be performed at a certain representative degree of strain ε_t = const. The condition $\varepsilon_t \approx$ ≈ const is automatically provided in indentation of materials by an indenter in the form of a pyramid, *e.g.*, a Vickers tetrahedral pyramid or a Berkovich trihedral pyramid. At the same time, the small volume

Fig. 1. Decomposition of the total strain ε_t at the point *A* into the plastic (ε_p) and the elastic (ε_e) components for the calculation of the plasticity characteristic δ^* in a tensile test [22]
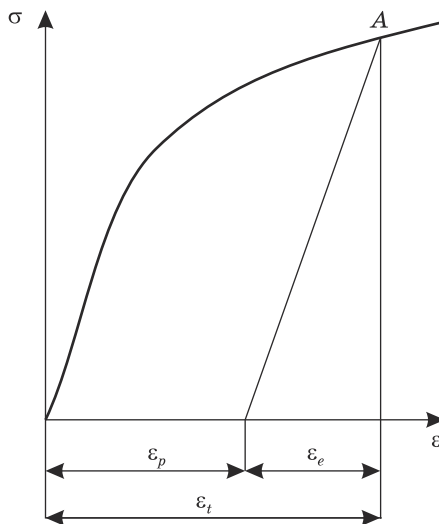
of the deformed material and the specific character of stress fields reduce the susceptibility to macroscopic fracture and decrease abruptly the ductile–brittle transition temperature T_{db}. This makes it possible to determine the hardness and plasticity characteristics of most materials even at cryogenic temperatures [23−26]. A total degree of strain $\varepsilon_t = 8{-}9\%$ observed in indentation by these indenters is sufficiently representative and convenient for comparison of values of plasticity of different materials.

The plasticity characteristic determined according to Eq. (1) by the indentation method was denoted by δ_H in Ref. [12]. A theory that makes it possible to determine ε_p and ε_e in indentation and calculate δ_H was developed [12, 13, 24], and experiments on determination of δ_H for a large number of materials, including materials brittle in standard mechanical tests, were carried out. Values of δ_H were determined for different materials: f.c.c., b.c.c., and h.c.p. metals, covalent crystals and refractory compounds with a large fraction of the covalent component in the interatomic bond, intermetallics, amorphous metallic alloys and quasi-crystals [12, 13, 24].

In view of the locality of the indentation method, it is possible to determine the value of δ_H for the thin coatings [27].

In the papers [5, 12, 22, 28, 29], the possibility to determine such plasticity characteristic in instrumented indentation was considered. In the recently published work [30], methods for determining of the plasticity characteristics by the indentation method in thin layers and coatings were considered, and the correlation of the plasticity characteristic with other mechanical properties of these materials was studied.

The work [31] reports on the possibility to calculate the Tabor parameter *C* in the relation $HM = C\sigma_S$ (where HM is the Meyer hardness, and σ_S is the yield strength) from the value of δ_H and, therefore, to determine the yield strength σ_S.

The review focused on the introduction and development of notions of the plasticity characteristic $\delta_H = (plastic\ strain)/(total\ strain)$ and on the application of δ_H for the determination of the plasticity of materials.

Plasticity Characteristic δ_H Determined by the Indentation Method from the Value of Microhardness, and Classification of Materials by the Value of δ_H

The plasticity characteristic determined by expression (1) was proposed, as has been noted, in Ref. [12]. In Ref. [12], the mean elastic strain on the indenter−specimen contact area in the direction of the applied load was obtained in the form:

$$\varepsilon_e = -\frac{HM}{E_S}(1 - \nu_S - 2\nu_S^2), \tag{3}$$

where HM is the Meyer hardness, which is considered as the average contact pressure ($HM = P/S$, where P is the load on the indenter and S is the projection area of the hardness indent on the surface of the specimen), E_S is Young's modulus of the investigated material, and ν_S is its Poisson's ratio. Expression (3) can be considered as the Hooke law for the indentation process.

The total strain ε_t was determined for pyramidal indenters as follows:

$$\varepsilon_t = -\ln(\sin\gamma), \tag{4}$$

where γ is the angle between a face and the axis of the pyramid.

Then, according to Eq. (1), for a pyramidal indenter, the plasticity characteristic, determined in indentation, has the form:

$$\delta_H = 1 - \frac{HM}{E_S \cdot \varepsilon_t}(1 - \nu_S - 2\nu_S^2). \tag{5}$$

In particular, for the Vickers indenter, with regard for the fact that $HV = HM\sin\gamma$ and $\gamma = 68°$, the following relation was obtained:

$$\delta_H = 1 - 14.3(1 - \nu_S - 2\nu_S^2)\frac{HV}{E_S}. \tag{6}$$

For the Berkovich hardness, in which a trihedral indenter with an angle $\gamma = 65°$ is used, the plasticity characteristic is described by the formula:

$$\delta_H = 1 - 10.2(1 - \nu_S - 2\nu_S^2)\frac{HM}{E_S}. \tag{7}$$

The theory of determination of the plasticity characteristics δ_H was further developed in Ref. [13]. In this work, the condition of incompressibility of a material under the indenter was used only for the calculation of the plastic part of the strain ε_p, but not for the total strain, as it was done in Ref. [12]. This is why the results obtained in Ref. [13] can be used to calculate strains and the plasticity characteristic δ_H for the hard and superhard materials with a large fraction of elastic strain in indentation.

For the plastic strain, the following relation was obtained:

$$\varepsilon_p = -\ln \sqrt{1 + \left(ctg\gamma - \frac{HM}{kE^*}\right)^2}. \tag{8}$$

Here, $k = 0.565$ for a trihedral and a tetrahedral pyramid, $k = 0.5$ for a conic indenter with an apex angle of 2γ, and E^* is the effective Young's modulus of the indenter–specimen contact pair.

$$\frac{1}{E^*} = \frac{1 - v_S^2}{E_S} + \frac{1 - v_i^2}{E_i}. \tag{9}$$

The subscripts 'S' and 'i' correspond to the specimen and indenter, respectively.

The plasticity characteristic δ_H is calculated with the use of relations (8), (3), and (1). In Refs. [13, 22], it was shown that, for metals, such calculation gives values of δ_H coinciding with results obtained by Eq. (5). Only for the hard and superhard materials, at $\delta_H < 0.3$, substantial differences are observed, and calculations should be performed with relations (1), (3), and (8).

It is seen from Eq. (1) that the plasticity characteristic δ^* is a dimensionless parameter and can change from 0 (purely elastic deformation) up to 1 (for the purely plastic behaviour of the material). In Ref. [32], it was shown that the parameter δ_H correlates to some degree with the elongation to fracture δ determined in a tensile test at a temperature higher than the ductile–brittle transition temperature T_{db}. It is natural that, at $T < T_{db}$, the elongation to fracture $\delta \to 0$, whereas δ_H has well defined values characterizing the plasticity of the material. Figure 2 shows temperature dependences of δ (obtained in a bending test as the elongation to fracture of edge stretched fibres) and of the plasticity characteristics δ_H for a WC–6 mas.% Co hard alloy [32].

In Ref. [12], it was experimentally established that there exists a critical value δ_{Hcr}. Materials having $\delta_H > \delta_{Hcr}$ are plastic in standard mechanical tensile and bending tests ($\delta > 0$), whereas for $\delta_H < \delta_{Hcr}$, the elongation in tensile tests δ usually
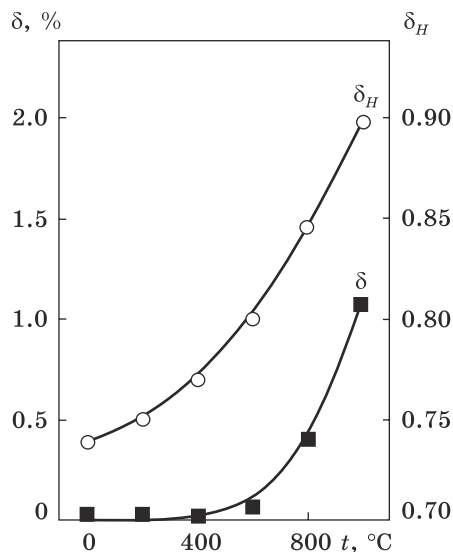

Fig. 2. Temperature dependence of the elongation to fracture δ and of the plasticity characteristic in microindentation δ_H for a WC–6 mas.% Co hard alloy with an average grain size $d = 1.3$ µm [32]

approaches to zero. For pure single-phase materials, $\delta_{Hcr} \approx 0.9$, whereas for composites and particularly coatings, δ_{Hcr} can be lower.

The introduction of the plasticity characteristic δ_H made it possible, for the first time, to classify practically all materials by their plasticity. In Table 1, which generalizes the results of Refs. [12, 24, 31, 33], such a

Table 1. **Classification of materials by the plasticity characteristic δ_H at room temperature [31]**

Materials		HM, GPa	E_s, GPa	v_s	δ_H
f.c.c. metals	Al	0.173	71	0.350	0.99
	Au	0.270	78	0.420	0.99
	Cu	0.486	130	0.343	0.98
	Ni	0.648	210	0.290	0.98
b.c.c. metals	Cr	1.404	298	0.310	0.97
	Ta	0.972	185	0.342	0.97
	V	0.864	127	0.365	0.97
	Mo(111)	1.998	324	0.293	0.96
	Nb	0.972	104	0.397	0.96
	Fe	1.512	211	0.280	0.95
	W(001)	4.320	420	0.280	0.92
h.c.p. metals	Ti	1.112	120	0.360	0.95
	Zr	1.156	98	0.380	0.95
	Re	3.024	466	0.260	0.95
	Mg	0.324	44.7	0.291	0.95
	Be	1.620	318	0.024	0.94
	Co	1.836	211	0.320	0.94
Intermetallics (IM)	$Al_{66}Mn_{11}Ti_{23}$ (IM$_3$)	2.203	168	0.190	0.87
	$Al_{61}Cr_{12}Ti_{27}$ (IM$_2$)	3.456	178	0.190	0.81
	Al_3Ti (IM$_1$)	5.335	156	0.300	0.76
Metallic glasses (MG)	$Fe_{40}Ni_{38}Mo_4B_{18}$ (MG$_2$)	7.992	152	0.300	0.62
	$Co_{50}Ni_{10}Fe_5Si_{12}B_{17}$ (MG$_3$)	9.288	167	0.300	0.60
	$Fe_{83}B_{17}$ (MG$_1$)	10.044	171	0.300	0.58
Quasi-crystals (QC)	$Al_{70}Pd_{20}Mn_{10}$ (QC$_2$)	7.560	200	0.280	0.71
	$Al_{63}Cu_{25}Fe_{12}$ (QC$_1$)	8.024	113	0.280	0.48
Refractory compounds	WC(0001)	18.036	700	0.310	0.81
	NbC(100)	25.920	550	0.210	0.54
	LaB_6(001)	23.220	439	0.200	0.50
	TiC(100)	25.920	465	0.191	0.46
	ZrC(100)	23.760	410	0.196	0.46
	Al_2O_3(0001)	22.032	323	0.230	0.41
	α-SiC(0001)	32.400	457	0.220	0.36
Covalent crystals	Ge(111)	7.776	130	0.210	0.49
	Si(111)	11.340	160	0.220	0.42

classification is presented. As seen from Table 1, f.c.c. metals have the highest plasticity, b.c.c. and h.c.p. metals ranking next in order of decreasing δ_H. For all metals in a highly pure and possible perfect state, $\delta_H > \delta_{Hcr}$, and, therefore, they are ductile in tensile tests. The next group in the Table 1 is intermetallics. Plasticity of intermetallics exceeds the plasticity of other materials presented in Table 1. However, intermetallics are brittle at room temperature ($\delta_H < \delta_{Hcr}$). Metallic glasses, quasi-crystals, refractory compounds, and purely covalent crystals usually have an even smaller plasticity.

Plasticity Characteristic δ_A Determined in the Instrumented Indentation

Note also that the plasticity characteristic δ_H can also be determined in instrumented indentation (which is more often used in determination of nanohardness), during which loading and unloading curves are recorded in coordinates 'load P–displacement of an indenter h' (Fig. 3) [22].

In this case, the plasticity characteristic has the form:

$$\delta_A = \frac{A_p}{A_t} = 1 - \frac{A_e}{A_t}, \qquad (10)$$

where A_p, A_e and A_t are the works expended on plastic, elastic and total deformations, respectively, during penetration of the indenter; $A_t = A_p + A_e$. The ratio A_p/A_t can be determined from the ratio of areas under the unloading and the loading curve. The authors of Refs. [4, 5, 29, 34–37], *etc.*, in measurement of the nanohardness, also use the ratio A_p/A_t for the characterization of the plastic behaviour of materials and denote it by *PI* (the so-called plasticity index).

In Ref. [28], it was shown that $\delta_H \approx \delta_A$ if both δ_A and δ_H are determined by identical indenters under equal loads on the indenter.

In Figure 4, results obtained in Ref. [28] (with the use of a Berkovich indenter) in the form of a dependence of δ_A and δ_H on the ratio $HM(1 - \nu_S - \nu_S^2)/E_S$ (see Eq. (7)) are shown. As seen, for all metals and most ceramic materials investigated in Ref. [28], at
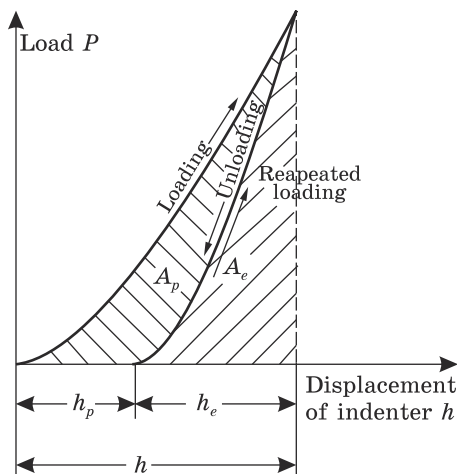

Fig. 3. Diagram of penetration of a pyramidal indenter in coordinates 'load P–displacement of the indenter h'; h_e and h_p are the elastic and plastic displacement approaching the indenter and the specimen; A_e and A_p are the elastic and plastic components of the work of deformation in instrumented indentation [22]
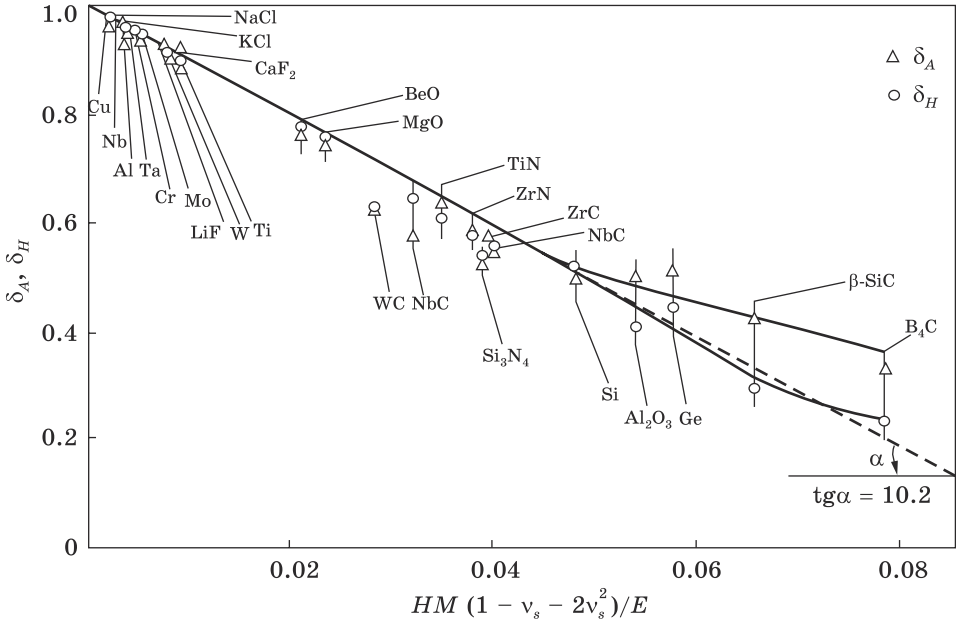
Fig. 4. Dependence of the plasticity characteristics δ_A and δ_H on the ratio $HM(1 - \nu_s - 2\nu_s^2)/E_s$. Values of δ_A and δ_H were obtained with the use of a Berkovich indenter under the same load for each material [28]

$\delta_A > 0.5$, the values of the characteristics δ_A and δ_H practically coincide (with considering the error in calculations of the values of HM and E_s).

In recent years, the plasticity parameter δ_A is often used for the determination of the plasticity characteristic by the indentation method [4, 5, 30, 36, 38]. In this case, different notation of the parameter δ_A is used. In Ref. [4], it is presented as the ductility index $D = A_p/A_t$, and, in Ref. [5], the same parameter is denoted by PI (plasticity index). In some works, the parameter $\delta_H = \varepsilon_p/\varepsilon_t$ is also denoted by PI.

For instance, formula (10) was used for the calculation of the plasicity in Refs. [4, 5]. In Ref. [5], the following values of δ_A were obtained: 0.91 for W, 0.95 for Cu, 0.98 for Al, and 0.52 for Al_2O_3, which are close to the results presented in Fig. 4 and Table 1. It should be noted that Table 1 presents data obtained for single-crystal Al_2O_3, whereas Ref. [5] reports on data for polycrystalline Al_2O_3.

An advantage of the characteristic δ_A (or PI) is that, in its calculation, the preliminary determination of the hardness, Young's modulus and Poisson's ratio is not required that increases the accuracy of determination of this characteristic.

It is important that δ_A correlates well with the plasticity parameter δ_H, which, as shown above, has a clear physical meaning coinciding with the physical definition of plasticity.

Size Dependence (ISE) of the Plasticity Characteristic δ_H

The plasticity characteristic δ_H is calculated by formulas (6) and (7) or with the use of Eqs. (3) and (8), into which the hardness *HM* enters. Since in determination of the microhardness and particularly nanohardness, HM depends on the size of the hardness indent (and, hence, on the load on the indenter, *P*), δ_H also must depend on *P* and the depth of plastic penetration of the indenter *h*. The size dependence of the hardness (indentation size effect, abbreviated as ISE) is widely discussed in the literature: see, *e.g.*, Refs. [38–41]. For the explanation of the ISE, dislocation notions [39, 40], which lead to a dependence of the type $H^2 = K(1 + K_1/h)$, where K and K_1 are constants, were developed [38].

However, in many cases, the difference in mechanisms of plastic deformation in indentation of different crystalline materials does not make it possible to use theoretical values of K and K_1. Moreover, in some cases, the dependence $H^2 \propto 1/h$ has a bilinear character.

In Ref. [38], it was shown that a large ISE in measurement of the hardness of crystalline materials is caused by the fact that, in the case of using pyramidal indenters, the relation $\varepsilon_p + \varepsilon_e \approx$ const holds. With decrease in the size of the hardness indent, plastic deformation is impeded because of the hindrance of the work of dislocation sources and

Fig. 5. Elastic ε_e, plastic ε_p, and total ε_t strains *vs.* the applied load *P* in indentation of a copper single crystal (111) [38]
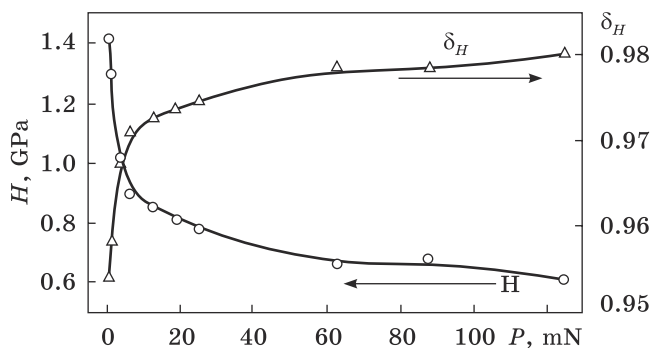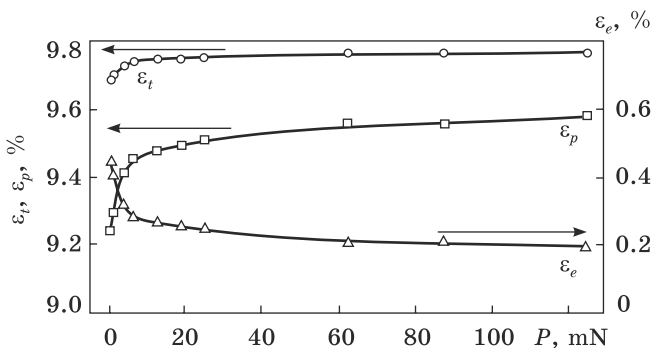

Fig. 6. Influence of the load on an indenter *P* on the nanohardness *H* and plasticity characteristic δ_H on a copper single crystal (111) [38]

a decrease in the length of mean free path of dislocations [38]. As a result, with decrease in the size of a hardness indent or in the load on the indenter, P, the fraction of the plastic strain ε_p diminishes, whereas the fraction of the elastic strain ε_e increases (Fig. 5).

The decrease in ε_p leads to a decrease in the plasticity characteristic δ_H according to Eq. (1). The increase in ε_e leads to an increase in the hardness H according to the Hooke law for indentation (3).

In Figure 6, a decrease in δ_H and a rise in the hardness H with decreasing load on an indenter P are illustrated for the case of nanoindentation of copper.

In Ref. [38], for the calculation of the ISE in crystalline materials, it was proposed to use the empirical Meyer relation in the form:

$$P = N \left(\frac{h}{h_0} \right)^m, \qquad (11)$$

where N and m are constants and h_0 is a unit of length in the used system of units. For nanohardness, it is assumed that $h_0 = 1$ nm.

Table 2. Values of the modulus of elasticity (Young's modulus) E_S, nanohardness H and constants m (see No. 11), n (see No. 12) and i (see No. 13) [38]

No.	Material	P_{max}, mN	E_S, GPa	h_{max}, nm	H, GPa	M	n	i	H_f, at h_f = 100 nm	H_f at h_f = 1000 nm
1	BeO *	10	400	181.5	12.8	1.58	−0.27	−0.42	16.5	6.2
2	TiN ***	50	440	394.3	24.6	1.72	−0.16	−0.28	36.2	18.9
3	Si$_3$N$_4$ **	50	324	415.3	24.3	1.67	−0.20	−0.33	39.0	18.2
4	NbC *	50	550	404.8	25.2	1.82	−0.10	−0.18	32.5	21.4
5	NbC *	50	550	359.3	31.3	1.65	−0.21	−0.35	48.9	21.9
6	ZrN *	50	400	400.7	24.3	1.65	−0.21	−0.35	39.7	17.6
7	TiB$_2$ **	50	540	308.2	44.1	1.63	−0.22	−0.37	66.7	28.6
8	WC *	50	700	310.6	39.8	1.59	−0.26	−0.41	63.6	24.5
9	LaB$_6$ *	50	439	336.6	38.7	1.53	−0.30	−0.46	68.0	23.3
10	β-SiC *	50	460	323.2	44.3	1.70	−0.17	−0.30	62.8	31.6
11	ZrC **	50	480	386.0	26.4	1.63	−0.22	−0.37	43.3	18.6
12	B$_4$C ***	10	500	123.3	48.9	1.64	−0.22	−0.36	52.8	22.8
13	Al$_2$O$_3$ *	10	409	144.9	33.3	1.64	−0.22	−0.36	38.0	16.6
14	MgO *	50	310	584.0	9.46	1.74	−0.15	−0.26	15.1	8.2
15	W *	10	420	301.3	6.10	1.85	−0.08	−0.15	7.2	5.1
16	Mo *	50	324	931.2	3.21	1.71	−0.17	−0.29	6.1	3.1
17	Cr *	50	279	1025.3	2.63	1.66	−0.20	−0.34	5.7	2.6
18	Nb **	50	104	1460.2	1.26	1.84	−0.08	−0.16	1.9	1.3
19	Ta **	50	185	1259.2	1.74	1.75	−0.14	−0.24	3.2	1.8
20	Cu *(111)	62.5	170	2100.8	0.66	1.72	−0.16	−0.28	1.6	0.8
21	Al **	120	70	3148.0	0.66	1.73	−0.16	−0.27	1.7	0.9

Here, * — denotes single crystal, ** — polycrystalline, and *** — individual grain.

Using Eq. (11), the authors of Ref. [38] proposed the following equation:

$$H = N_1 P^n, \tag{12}$$

where $n = 1 - 2/m$, and

$$H = N_2 h^i, \tag{13}$$

where $i = m - 2$, N_1 and N_2 are constants.

Equations (12) and (13) enable us to recalculate the hardness H_1 from the load P_1 and the depth of the indent $h = h_1$ to the hardness H_2 under the load P_2 and at $h = h_2$ according to the expressions:

$$H_2 = H_1 \left(\frac{P_2}{P_1} \right)^n, \tag{14}$$

$$H_2 = H_1 \left(\frac{h_2}{h_1} \right)^i. \tag{15}$$

The value of the parameter m can be experimentally determined by the relation $P(h)$. For some materials, values of m, n, and i are presented in Table 2 according to Ref. [38].

In Ref. [38], to prevent the influence of the ISE on the value of the hardness, it is proposed to determine the instrumented hardness at a constant value $h = h_f$, rather than at a constant value of the load P. If this is impossible, it is proposed to recalculate the values of H for the fixed value of the depth of plastic penetration, namely, $h_f = 1000$ nm for metals and $h_f = 100$ nm for ceramics, refractory compounds, and other high-strength materials, which are brittle in standard mechanical tests of materials.

Influence of Structural Factors on the Plasticity Characteristic δ_H

The complex physical meaning of the elongation to fracture δ in a tensile test made it impossible to develop a theory of dependence of δ on structural factors, temperature, and strain rate for many years of using this quantity. The theory of structural sensitivity of δ_H was developed in Ref. [12, 22, 42] with the use of the notion of the structural sensitivity of the yield strength. Since the new plasticity characteristic δ_H is proportional to $HM = C\sigma_S$ (where C is the Tabor parameter [1], and δ_S is the yield strength), these problems are easily solved for δ_H.

In Refs. [12, 22], for the case where the dependence of σ_S on the grain size d is described well by the known Hall–Petch equation $\sigma_H = \sigma_0 + K_y d^{-1/2}$, the following equation was obtained using Eq. (5):

$$\delta_H = \delta_{H_0} - K_1 d^{-1/2}, \tag{16}$$

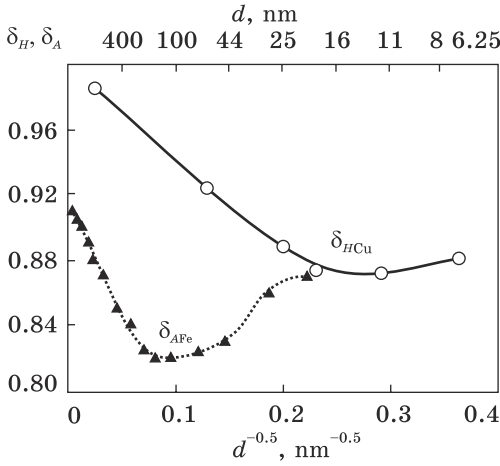where δ_{H_0} denotes the plasticity of a single crystal, and $K_1 = = CK_y / E_S \varepsilon_t (1 - v_S - 2v_S^2)$.

For example, a dependence of δ_H on the grain size for nanostructured copper (based on data of the hardness taken from Ref. [43]) and a dependence of δ_A on the grain size for nanostructured iron (calculated with Eq. (6)) [42] are shown in Fig. 7.

It is seen from Fig. 7 that Eq. (16) is well satisfied if $d \gg 25$ nm for Cu and if $d > 200$ nm for Fe. For a smaller value of d, the Hall–Petch equation is not satisfied due to slippage along the grain boundaries.

In Ref. [44], Eq. (16) was confirmed in a study of the influence of the grain size in a Ni–48.4 at.% Al alloy on the plasticity characteristic δ_H.

If the dependence of the yield strength on the density of chaotically distributed dislocations ρ is described by the reliably established relation $\sigma_S = \sigma_0 + \alpha G b \sqrt{\rho}$, where σ_0 is the yield strength of a dislocation-free crystal, G is the modulus of elasticity, b is the modulus of the Burgers vector, and α is a constant, then, according to Ref. [12]:

$$\delta_H = \delta_{H_1} - K_2 \sqrt{\rho}, \tag{17}$$

where δ_{H_1} is the plasticity of the crystal at $\rho = 0$, and

$$K_2 = \frac{C \alpha G b}{E_S \varepsilon_t}(1 - \nu_S - 2\nu_S^2).$$

It is seen from the presented relations that the plasticity characteristic δ_H decreases with increasing dislocation density and decreasing grain size. It follows from Eq. (5) that δ_H decreases also in the case of any other hardening that leads to a rise in *HM*.

Influence of Temperature on the Plasticity Characteristic δ_H

An experimental determination of the temperature dependence of the hardness of materials enables one to calculate and analyze the change in the plasticity characteristic δ_H with temperature [12, 22–24].

Since the hardness measured by pyramidal indenters corresponds to the yield stress at a certain fixed degree of strain, for the description of the temperature dependence of the hardness, it is reasonable to use the

theory that describes the conventional yield strength rather than the critical shear stress or the lower yield point.

In Refs. [23, 24], the theory of conventional yield strength developed in Ref. [45, 46] was used for description the dependence $\delta_H(T)$.

In particular, for the interval of cold deformation, at a temperature $T < T^*$ (where T^* is the characteristic temperature of deformation, below which an intensive rise in the yield strength is observed, and the cold-brittleness is possible [47]), the following relation was obtained [22]:

$$\delta_H = \delta_H(0) + \frac{AT}{VE} \ln \frac{M}{\dot{\varepsilon}},$$ (18)

where $\delta_H(0)$ is the plasticity at 0 K,

$$\delta_H(0) = 1 - \frac{HM(0)}{E_S\,\varepsilon_t}(1 - \nu_S - 2\nu_S^2) = 1 - \frac{C\sigma_S(0)}{E_S\,\varepsilon_t}(1 - \nu_S - 2\nu_S^2),$$

(18a)

$$A = \frac{C\,k}{\varepsilon_t}(1 - \nu_S - 2\nu_S^2),$$

V is the activation volume, $\dot{\varepsilon}$ is the strain rate, k is the Boltzmann constant, and M is a material constant.

Temperature dependences of δ_H for different materials are shown in Figs. 8–11. As is seen in these figures, a linear dependence $\delta_H(T)$ is observed for most materials at low temperatures according to formula (18).

The strong dependence of the yield stress, hardness and plasticity characteristic δ_H on temperature is observed for crystals with a high Peierls–Nabarro stress (covalent crystals, b.c.c. metals, *etc.*). It is seen
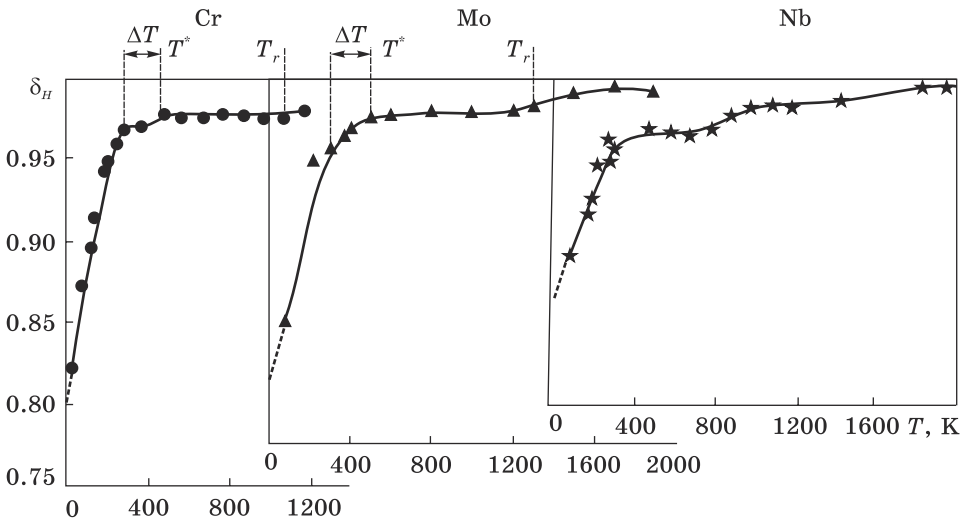

Fig. 8. Temperature dependence of the plasticity characteristic δ_H for Cr, Mo and Nb. For Mo and Cr, the values of the characteristic temperature T^* and recrystallization temperature T_r are marked [23]
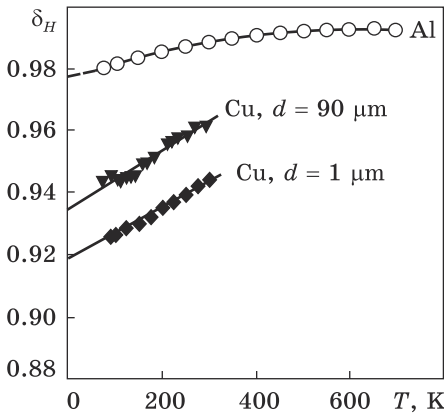
Fig. 9. Temperature dependence of the plasticity characteristic for aluminium and copper [23]. Data for the calculation of δ_H were taken from Ref. [49] for aluminium and from Refs. [50, 51] for copper
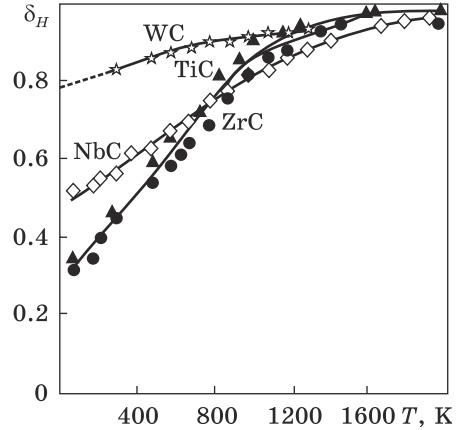
Fig. 10. Temperature dependence of the plasticity characteristic δ_H for refractory compounds WC, NbC, ZrC and TiC [23]

in Figs. 8–11 that, in contrast to elongation to fracture δ, δ_H always rises with increasing temperature.

In covalent and partially covalent crystals, the linear dependence $\delta_H(T)$, which follows from Eq. (18) and, as is seen in Fig. 11, can be broken in the temperature range adjacent to 0 K due to the semiconductor–metal phase transition under the pressure of an indenter (Si, Ge) or due to the fact that the formation of a hardness indent occurs as a result of brittle fracture rather than through plastic deformation [48].

On the curves of the dependence $\delta_H(T)$, three temperature regions with different character of the dependence $\delta_H(T)$ can be distinguished. These are regions of cold, warm, and hot deformation [52, 53]. On the $\delta_H(T)$ curves, these three regions manifest themselves most clearly for refractory metals with a b.c.c. lattice (Fig. 9). The characteristic temperature of deformation T^* is the boundary between the temperature intervals of cold and warm deformation, and the recrystallization temperature T_r is the boundary between the temperature intervals of warm and hot deformation. Characteristic features of the behaviour of $\delta_H(T)$ in different temperature intervals were discussed in Refs. [23, 24, 33]. The behaviour of the plasticity in the interval of cold deformation ($T < T^*$), studied for the first time, turned to be particularly important because, in this temperature interval, the elongation to fracture tends to zero ($\delta \rightarrow 0$) for many materials, whereas δ_H has well-defined values, which characterize the plasticity of the material.

As seen in Figs. 8–11, for all materials, except covalent crystals shown in Fig. 12, in this temperature interval, an abrupt decrease in δ_H is
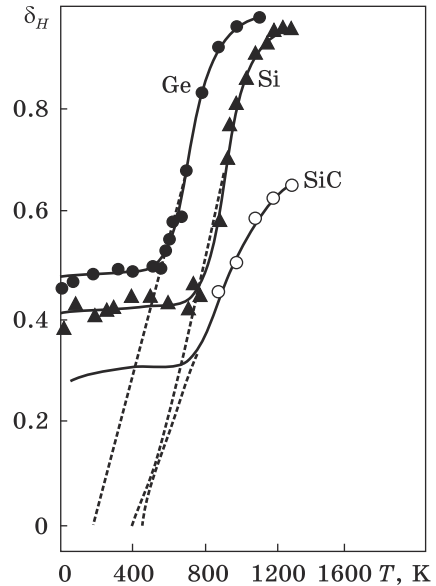
Fig. 11. Temperature dependence of the plasticity characteristic δ_H for Si (the plane (111)), Ge (the plane (111)), and SiC (the plane (0001)) single crystals [23]. The extrapolation of $\delta_H(T)$ from the region of the dislocation mechanism of deformation was performed to determine temperatures, at which $\delta_H = 0$ in the absence of a phase transition in Si and Ge in the indentation process. For SiC, the low-temperature region, on which fracture is a leading mechanism of formation of an indent, is not taken into account

observed as the temperature decreases. Note that, in the temperature region adjacent to 0 K, the dependence $\delta_H(T)$ has a linear character.

According to Refs. [52, 54], in the interval of cold deformation, a chaotic distribution of dislocations is typical. Here, dislocation stoppers are unstable due to a high level of external stresses, and dislocation boundaries practically do not form. The strain hardening is caused by the interaction of mobile dislocations with forest dislocations and is the strongest in this interval. The mobility of dislocation in crystals with a substantial fraction of the covalent component in the interatomic bond is predominantly determined by the thermally activated overcoming of Peierls barriers and diminishes with decreasing temperature. This is why,
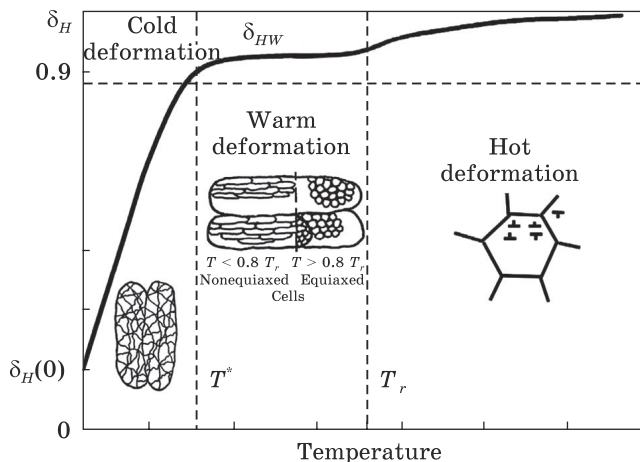

Fig. 12. Scheme of the temperature dependence of the plasticity δ_H in the regions of cold, warm, and hot deformation. The change in the granular and dislocation structure under deformation is shown schematically [55]

with decrease in the temperature, plastic deformation decreases, the hardness HM increases, and the plasticity characteristic δ_H decreases.

From equation (18), it follows that, near low temperatures, the characteristic δ_H must rise linearly with increasing temperature, which is observed in practice. In this case, $d\delta_H/dT \propto A/VE$, *i.e.*, Young's modulus E and the activation volume V are the most important parameters, which cause the rise in δ_H with increasing temperature.

At higher temperatures, near the characteristic temperature of deformation T^*, σ_S decreases exponentially as the temperature increases, whereas δ_H exponentially rises with increasing temperature T [45, 55]. In the interval of warm deformation, a cellular dislocation structure forms, and the dependence $\delta_H(T)$ is very weak in this interval. For most crystalline single-phase materials, δ_H attains the value 0.9 and somewhat exceeds it in this temperature interval [55]. In the temperature interval of hot deformation, δ_H increases, attaining the maximum value $\delta_H = 1$, but, even for most plastic metals, remains slightly smaller than 1 [55].

Theoretical Plasticity of Materials

The notion of theoretical plasticity was introduced into the development of the knowledge of theoretical strength [55]. Theoretical plasticity is considered as the plasticity of an ideal crystal upon attainment of the theoretical shear strength, and the dislocation-free mechanism of shear deformation is assumed. The introduction of the notion of theoretical plasticity seems to be rational because both characteristics, namely, strength and plasticity, can adequately characterize the mechanical behaviour of materials. The insufficient development of the physical knowledge of plasticity at cryogenic temperatures and the absence of the notion of theoretical plasticity until the publication of Ref. [55] are explained by the imperfection of the extensively used plasticity characteristics δ and ψ.

In Ref. [55], theoretical plasticity was considered with regard for the fact that a solid body deforms purely elastically until the attainment of the theoretical strength σ_{theor}, and then plastic deformation develops without strain hardening.

In Ref. [56], for the calculation of the theoretical plasticity $\delta_{H\text{theor}}$, the following expression was obtained:

$$\delta_{H\,\text{theor}} = 1 - \frac{22.8\,C\,\tau_t\,(1 - \nu_S - 2\nu_S^2)}{E_S}.\tag{19}$$

Here, C is the Tabor parameter: $C = HM/\delta_S$. The value of C can be calculated according to the Johnson theory (see section about relationship between the plasticity characteristic δ_H and Tabor parameter C). Values of the theoretical shear strength τ_t for different materials were taken from the book [57].

Values of the theoretical plasticity $\delta_{H\text{theor}}$ are presented in Table 3 in comparison with values of δ_H at 20 °C and δ_H at 0 K at the dislocation mechanism of deformation (see section on plasticity at 0 K) if δ_H is determined with the use of a Vickers indenter.

As seen from Eq. (19), the theoretical plasticity is determined by three parameters: the value of τ_t, Young's modulus E_S and Poisson's ratio ν_S. In this case, a reduction in τ_t and an increase in E_S and ν_S lead to a rise in the theoretical plasticity.

The results presented in Table 3 show that, for all studied crystals $\delta_{H\text{theor}} < \delta_H(0)$, which is natural, because, in the case of the dislocation mechanism of deformation even without the help of thermal oscillations of atoms, the plasticity must be larger than that in the case of the dislocation-free mechanism (see the physical understanding of $\delta_H(0)$ in the section about plasticity at 0 K).

It is seen from Table 3 that f.c.c. metals have the largest theoretical plasticity. However, even for these metals, $\delta_{H\text{theor}} < \delta_{Hcr}$ (except gold), *i.e.*, in tensile tests, in the case of the dislocation-free deformation mechanism, their elongation to fracture δ will be equal to zero at 0 K.

The theoretical strength of crystals decreases as the temperature of deformation rises, and, at room temperature, it can be less by a factor of two than the presented estimates [57]. This is why, in Ref. [55], it was suggested that f.c.c. crystals presented in Table 3 would have plasticity in tensile tests at room temperature in the case of the dislocation-free mechanism of deformation. For b.c.c. metals (Fe and W), $\delta_{H\text{theor}}$ is substantially lower than δ_{Hcr} and, in the case of the dislocation-free deformation mechanism in tensile tests, δ for them will be equal to zero.

All the more, this situation extends to covalent crystals.

It is seen from the analysis of expression (19) that an increase in the theoretical strength and τ_t leads to a decrease in the theoretical plasticity

Table 3. **Hardness *HM*, Young's modulus E_S, Poisson's ratio ν_S, plasticity characteristic δ_H (at temperatures of 20 °C and 0 K), theoretical strength τ_t and theoretical plasticity $\delta_{H\text{theor}}$**

Material	HM at 20 °C, GPa	E_S, GPa	ν_S	δ_H at 20 °C	δ_H at 0 K	τ_t, GPa	$\delta_{H\text{theor}}$
Al	0.16	70	0.35	0.988	0.976	0.90	0.78
Cu	0.90	130	0.343	0.961	0.935	1.20	0.85
Ag	0.03	29.5	0.38	0.995	–	0.77	0.76
Au	0.05	78	0.42	0.998	–	0.74	0.91
Zn	0.06	110	0.231	0.995	–	2.30	0.44
W	5.63	420	0.28	0.900	0.850	16.50	0.50
Fe	1.34	200	0.28	0.950	0.880	6.60	0.58
Al_2O_3	22.04	325	0.23	0.400	0.250	16.90	0.21

$\delta_{H\text{theor}}$. This is why all formulated conditions of choosing high-strength materials with a high theoretical strength (see, *e.g.*, Ref. [57]) should be discussed with regard for a decrease in the theoretical plasticity $\delta_{H\text{theor}}$ with increase in τ_t.

Plasticity at 0 K, $\delta_H(0)$

In Ref. [23], it was shown that the plasticity characteristic at 0 K $\delta_H(0)$ is also a fundamental characteristic of materials. The $\delta_H(0)$ is determined under the same conditions of plastic deformation as the Peierls–Nabarro stress σ_{P-N}, which is required for a dislocation to overcome potential barriers of the crystal lattice without the help of thermal oscillations of atoms, *i.e.*, at 0 K. σ_{P-N} is practically the theoretical strength in the case of the dislocation mechanism of deformation, and $\delta_H(0)$ can be considered as the theoretical plasticity in the case of the dislocation mechanism of deformation. As has been noted, for most real materials, the value of $\delta_H(0)$ can be determined by extrapolation of the temperature dependence of δ_H to 0 K, because the linear character of the dependence $\delta_H(T)$ [23] was shown theoretically (Eq. (18)) and experimentally (Figs. 8–11).

It is very important to note that Young's modulus E_S, which is the most important parameter that determines the theoretical strength of crystals [57], enters also into expression (18a), and an increase in E_S leads not only to an increase in the theoretical strength, but also to a rise in the plasticity $\delta_H(0)$.

It is seen from Table 1 and in Fig. 9 that, in f.c.c. metals (Al and Cu), the value of δ_H at room temperature exceed substantially the critical value of δ_{Hcr}, and, as is known, these metals have a high plasticity to fracture δ not only at room temperature, but also at cryogenic tempe-ratures, including the temperature of liquid hydrogen and even the temperature of liquid helium [49, 50]. It is interesting that the values of δ in these metals usually even increases as the temperature decreases below room temperature. This can be explained by an increase in the strain hardening with decreasing temperature. The increase in the strain hardening, as has been noted, extends the stage of uniform deformation before formation of a stable 'neck', but does not cause the transition to brittle fracture in these metals because the yield strength in them in-creases fairly weakly with decreasing temperature and remains lower than the fracture stress. Estimates show that, for these metals, $\delta_H(0) > \delta_{Hcr}$ also at 0 K. The plasticity reserve in these metals is so substantial that, even in the case of grain refinement down to 1 μm, $\delta_H(0) > \delta_{Hcr}$ not only at room temperature, but also at 0 K. However, in nanostructured copper, at a grain size $d \approx 0.25$ nm, already at room temperature, $\delta_H < \delta_{Hcr}$ (see Fig. 7), and the plasticity to fracture δ is only 1–2% [50].

In highly pure single crystals of b.c.c. metals, the inequality $\delta_H > \delta_{Hcr}$ also holds at room temperature, *i.e.*, they have some plasticity also in tensile tests. However, with decrease in the temperature, due to the substantial fraction of the covalent component in the interatomic bond and a high Peierls–Nabarro stress, in these metals, the yield strength and hardness increase abruptly [25], whereas the plasticity characteristic δ_H decreases sharply (Fig. 8), and at 0 K, $\delta_H(0) < \delta_{Hcr}$, *i.e.*, in these metals, as the temperature decreases below room temperature, the ductile–brittle transition occurs. In commercially pure b.c.c. metals and alloys based on them, the ductile–brittle transition temperature can also be higher than room temperature [25] (particularly, in the group VIA metals: Cr, Mo and W) and, hence, at room temperature, $\delta_H < \delta_{Hcr}$.

In covalent silicon and germanium crystals, already at room temperature, $\delta_H < \delta_{Hcr}$. In these crystals, the sharpest increase in the yield strength is observed as the temperature decreases. However, in a wide temperature range adjacent to 0 K, in these crystals, indentation is accompanied by the semiconductor–metal phase transition, and the hardness has a nearly constant value and does not reflect the yield stress any longer [58–60]. This is why, to evaluate δ_H in these crystals in the case of the dislocation mechanism of deformation, the extrapolation of the dependence $\delta_H(T)$ can be performed only from the more high-temperature region, where the mechanism of deformation has a dislocation character. Such extrapolation showed that, in these crystals, the value $\delta_H = 0$ is attained at a certain temperature T_C that is much higher than 0 K (see Fig. 11). It follows from Fig. 11 that T_C is 200 °C for germanium and 400 °C for silicon. In the considered covalent crystals, $\delta_H(0)$ will be equal to 0 in the case of the dislocation mechanism of deformation, but, actually, because of the phase transition, $\delta_H(0)$ is substantially higher. In refractory compounds with a substantial fraction of the covalent component of the interatomic bond (Al_2O_3, TiC, ZrC, NbC and WC), the inequality $\delta_H < \delta_{Hcr}$ holds already at 20 °C, and a further reduction in δ_H is observed with decrease in the temperature (Fig. 10), so that $\delta_H(0)$ is very low for most of these crystals. Note that tungsten carbide WC has a higher value of δ_H than the other considered carbides both at room temperature and at 0 K that can be explained by a large value of Young's modulus E_S and a smaller ratio HM/E_S.

In silicon carbide SiC, the low-temperature athermal segment on the dependence $H(T)$ and $\delta_H(T)$ is caused by a change in the deformation mechanism, namely, at low temperatures, fracture rather than plastic deformation becomes the leading mechanism of formation of an indent in indentation [48, 61].

As is seen from the presented results, the plasticity $\delta_H(0)$ differs substantially for materials with different types of the interatomic bond and different atomic structures: from extremely high values for f.c.c.

metals to zero value for covalent crystals. The $\delta_H(0)$ characterizes the plasticity in the case of the dislocation mechanism of deformation, but, in the absence of thermal oscillations of atoms, it can be considered, as has been noted, as a fundamental characteristic of materials [23, 55].

In Ref. [23], for the consideration of the dependence of the plasticity at 0 K on the parameters of the thermally activated dislocation motion, melting point, and Young's modulus, on the basis of Eq. (18), Fig. 12, and results obtained in Ref. [46], the following relation was obtained:

$$\delta_H(0) \approx \delta_{HW} - \frac{U^{1/2}T_m^{1/2}\ln(M/\dot{\varepsilon})}{VE_S}B, \qquad (20)$$

where U is the activation energy of dislocation motion, T_m is the melting point, δ_{HW} is the plasticity in the temperature interval of warm deformation (see Fig. 12), $\delta_{HW} \approx$ const, M is a material constant (see Eq. (18) and Ref. [46]), and

$$B = \frac{C\,k\,(1 - \nu_S - 2\nu_S^2)}{\varepsilon_t\sqrt{21}\,k^{1/2}}.$$

Expression (20) relates $\delta_H(0)$ to the parameters of thermoactivated dislocation motion U and V. From this expression, it follows that an increase in height of potential barriers U and a reduction in their width V leads to a decrease in $\delta_H(0)$. An increase in the melting point T_m also decreases $\delta_H(0)$. A rise in the modulus of elasticity E_S leads to an increase in $\delta_H(0)$.

Relationship between the Plasticity Characteristic δ_H and Tabor Parameter *C*

At present, the physical relationship between the hardness *HM* and the yield stress σ_S rather than the relationship between the hardness *HM* and the strength of the material can be thought to be justified [1]. This relationship is usually investigated in the form of the simple relation:

$$HM = C\sigma_S, \qquad (21)$$

where *C* is the Tabor parameter.

For steel and a number of other structural alloys, the Tabor parameter lies in the rather narrow interval $C = 2.8$–3.1. However, for pure f.c.c. metals, *C* can be much higher, whereas for ceramics, *C* approaches to unit.

The physical meaning of the parameter *C* was revealed in Ref. [31]. In this work, the improved Johnson inclusion core model of indentation was used [62, 63]. The scheme of interaction of the indenter under a load *P* with the surface of a specimen in the improved inclusion core model is shown in Fig. 13 [31].

As we can see in Fig. 13, in the inclusion core model, a core of deformation with a radius *c*, in which purely plastic deformation occurs, and a zone of elastoplastic deformation with a radius b_S are considered.
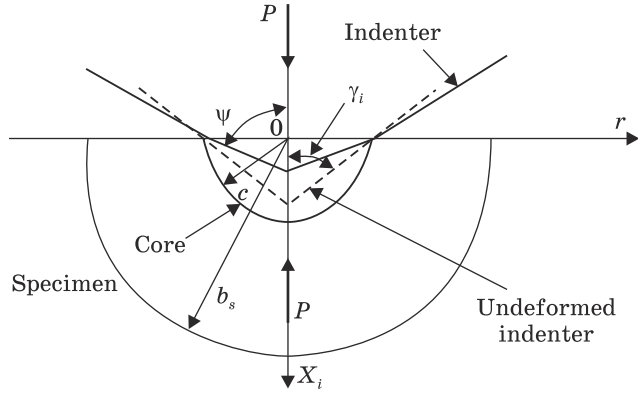
Fig. 13. Scheme of inter-action of an indenter and a specimen under a load P in a spherical coordi-nate system $\{0r\theta\psi\}$, $HM = = P/(\pi c^2)$ [31]

In this model, three transcendental equations with three unknowns were obtained. These are the yield stress σ_S, the relative size of the elastoplastic zone $x = b_S/c$ and $z \approx \mathrm{ctg}\psi$, where 2ψ is the angle at the apex of the conic indenter under the load P.

$$\begin{cases} z = \ \cot \psi = \cot \gamma_i - 2HM/E_i^*, & (22a) \\ (1 - \theta_S \sigma_S)(x^3 - \alpha_S) = z\beta_S/\sigma_S, & (22b) \\ (2/3 + 2\ln x) - HM/\sigma_S = 0. & (22c) \end{cases}$$

In these equations, $2\gamma_i$ is the apex angle of the conic indenter without load,

$$\alpha_S = \frac{2(1 - 2\nu_S)}{3(1 - \nu_S)}, \quad \beta_S = \frac{E_S}{6(1 - \nu_S)} \quad \text{and} \quad \theta_S = \frac{2(1 - 2\nu_S)}{E_S}, \quad E_i^* = \frac{E_i}{1 - \nu_i^2}.$$

The equations of system (22) enable to calculate all three indicated un-knowns from the value of the hardness HM and the elastic characteris-tics E_S and ν_S.

In Ref. [31], using the system of Eqs. (22) and Eq. (6) for the deter-mination of the plasticity characteristic δ_H, the authors obtained the equations, which relate the size of the zone of elastoplastic deformation x and Tabor parameter C to the plasticity characteristic δ_H (formulas were obtained under the condition $\theta_S \sigma_S \ll 1$, which is satisfied well for metals):

$$\delta_H = 1 - \frac{2.21\,z\,(2/3 + 2\ln x)}{x^3 - \alpha_S}\lambda_S, \qquad (23)$$

where

$$\lambda_S = \frac{1 - \nu_S - 2\nu_S^2}{1 - \nu_S} = 1 - 2\frac{\nu_S^2}{1 - \nu_S} \quad \text{and} \quad \delta_H = 1 - \frac{2.21\,z\,C\,\lambda_S}{e^{1,5\,C-1} - \alpha_S}. \qquad (24)$$

In Figs. 14 and 15, experimental data obtained in Ref. [31] and theoretical dependences $\delta_H = f(x)$ and $\delta_H = f(C)$ calculated by formulas (23) and (24) for $z = 0.38$ and $\nu_S = 0.27$ are presented. For these values
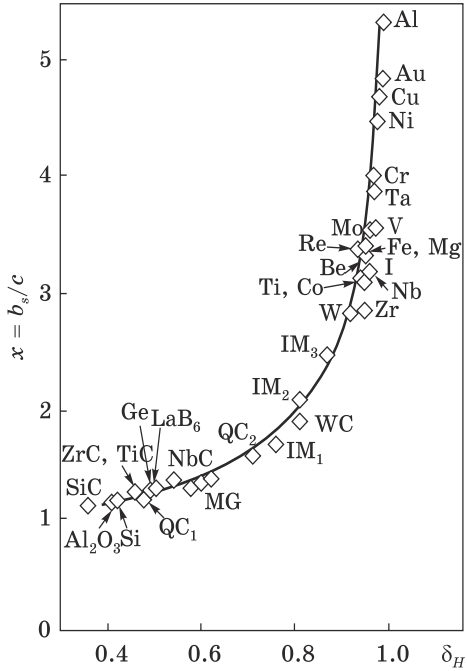
Fig. 14. Relation between the Tabor parameter $C = HM/\sigma_S$ and the plasticity characteristic δ_H. Experimental results and theoretical curve based on Eq. (24) [31]

Fig. 15. Relation between the Tabor parameter $C = HM/\sigma_S$ and the plasticity characteristic σ_H. Experimental results and theoretical curve based on Eq. (24) [31]
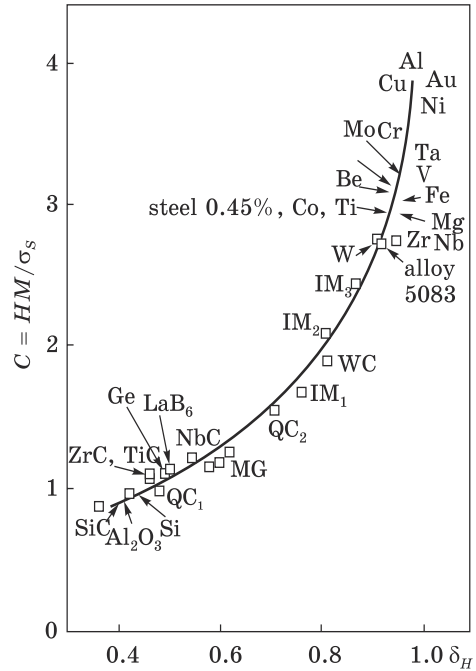
of the parameters ν_S and z, the standard deviations of experimental results from theoretical curves have a minimum value.

As we can see from Eqs. (23) and (24), along with Figs. 14 and 15, there exists a clear correlation between the plasticity characteristic δ_H, relative size of the zone of elastoplastic deformation x, and Tabor parameter C.

The larger the plasticity δ_H, the greater the values of x and C. Thus, in Ref. [31], it was shown that the values of x and C are determined by the value of the plasticity characteristic δ_H according to universal regularities (23) and (24), which hold for all homogeneous materials with different types of crystal lattices and different character of interatomic bonds.

The presented results of Ref. [31] enabled to explain for the first time the physical nature of the Tabor parameter C.

The results of Ref. [31] show that as the plasticity of a material rises, the size of the zone of elastoplastic deformation x increases, and, therefore, the pressure on the indenter $p = HM$ must provide plastic deformation in the increasing zone of elas-toplastic deformation with a radius b_S.

In this connection, the excess of the pressure p above the yield strength σ_S must rise with increasing x, *i.e.*, with increasing plasticity δ_H.

Thus, the plasticity characteristic δ_H acquires universal meaning. The value of the parameter δ_H enables one to explain not only the plastic characteristics of a material, but also the relationship between the hardness and the yield strength.

Since the parameter C is unambiguously related to the plasticity characteristic δ_H, its value, as is seen from Ref. [31], can be used as plasticity characteristic. In this case, the greater parameter C, the higher plasticity of the material. In Figure 15, it is seen that a critical value $C_{cr} = 2.5$ (which corresponds to $\delta_{Hcr} = 0.9$) can be introduced, and, therefore, materials with $C < C_{cr}$ are brittle in standard mechanical tensile tests.

Thus, it is possible to formulate clearly the dual nature of the hardness HM, which was assumed in a number of works: [2, 3, 5], *etc*. The HM is proportional to the yield strength σ_S, which is a strength characteristic of the material. However, the proportionality coefficient, namely, the Tabor parameter $C = HM/\sigma_S$, is determined by the plasticity of the material and can even be used as a plasticity characteristic.

Note also that, in practical terms, if the plasticity characteristic δ_H has been determined by formula (6), then it is possible to determine the Tabor parameter C by expression (24) or from Fig. 15, and calculate the yield strength σ_S by formula (21) [31].

The knowledge presented in Ref. [31] makes it possible to raise the efficiency of study of the hardness of materials. The indentation method enable one not only to determine the hardness of the material (which is an important strength characteristic of the material), but also to determine easily the plasticity characteristic δ_H, Tabor parameter C (from Fig. 15 or Eq. (24)) and yield stress of the material σ_S.

Examples of Using the Plasticity Characteristic Obtained by Indentation

In Refs. [12, 64], it is noted that it is reasonable to perform the calculation of the characteristic δ_H in any investigations of the influence of the chemical composition, heat treatment, metal forming and structural state on the hardness and mechanical properties of materials. The use of the combination of the strength characteristic H and plasticity characteristic δ_H makes it possible to characterize more completely the mechanical behaviour of the material than the use of only the hardness H. Moreover, as is seen from the section deals with plasticity at 0 K, it turns out that, from the known value of δ_H, it is easy to calculate the yield stress σ_S. It is also important that modern techniques of measuring
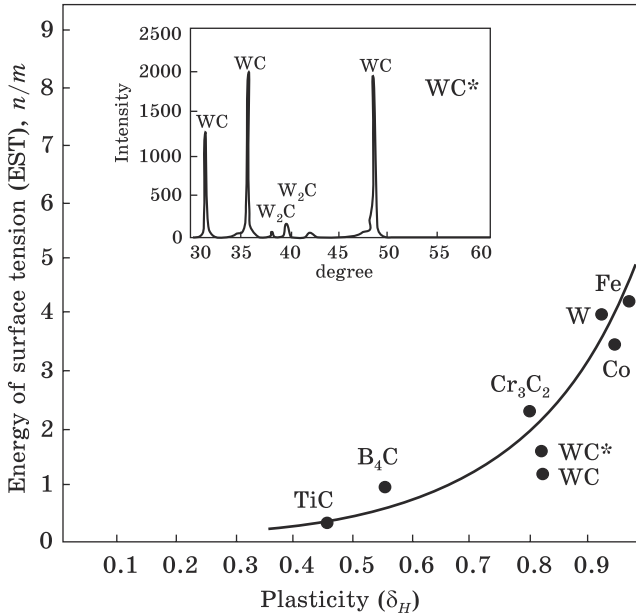
Fig. 16. Dependence of the plasticity of metals and carbides (δ_H) on their energy of surface tension (EST) [68]

the hardness make it possible to determine the microhardness and plasticity characteristic δ_H in a wide temperature range, from cryogenic temperatures close to 0 K to a temperature of 1000 °C with the use of a diamond indenter and to 1350 °C with the use of an Al_2O_3 indenter.

The use of the characteristic δ_H allows one to explain the phenomenal properties of WC–Co hard alloys because, as has been noted, WC is characterized by hardness sufficiently high for tool materials in combination with a plasticity δ_H very high for refractory compounds. Repeated attempts to replace WC by the harder carbides ZrC and TiC were not successful because these carbides have a substantially lower plasticity δ_H.

An analysis of δ_H values (see Table 1) allows classifying groups of materials (including new low-plasticity and brittle materials) by plasticity δ_H. The value of δ_H decreases in the order as follows: metals–intermetallics–metallic glasses–ceramics–quasi-crystals–covalent crystals.

As shown in Ref. [65], for superplastic materials, the determination of the temperature dependence of δ_H enables one to determine the optimal temperature of superplastic deformation.

It is well known that the deformed (work-hardening) metals usually have a smaller plasticity than annealed metals (except for the case where deformation reduces the ductile–brittle transition temperature T_{db}).

In contrast to metals, for quasi-crystals, it was shown (with the use of the plasticity characteristic) that plastic deformation reduces the hardness H and increases the plasticity characteristic δ_H, whereas annealing increases H and decreases δ_H [66].

Fig. 17. Variation in relative wear rate in high-performance stamping of magnetic steels with plasticity index (δ_H) for different cemented carbide grades [30]
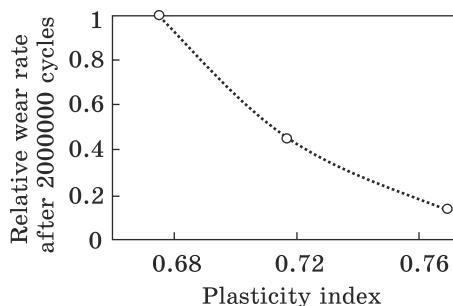
After publication of Ref. [12] and introduction of the plasticity index $PI = \delta_A$ [4, 5], the plasticity characteristics δ_H or $\delta_A = PI$ were extensively used in works of many researchers.

For instance, in Ref. [67], the character of plastic flow in two slip systems of a $MoSi_2$ single crystal ($\{110\}\langle 001\rangle$ and $\{101\}\langle 010\rangle$) was studied. The plasticity characteristic δ_H was determined, and it was shown that $\delta_H = 0.72$ and 0.80 and that the larger value $\delta_H = 0.80$ corresponds to the 'soft' orientation $\{101\}\langle 010\rangle$, which agrees well with data obtained in a study of the slip systems by the transmission electron microscopy (TEM) method.

It is interesting to note that, in Ref. [68], a correlation dependence of the surface energy of metals and carbides on their plasticity δ_H was established.

With increase in the plasticity of materials δ_H, an increase in the energy of surface tension is observed (Fig. 16). This regularity is observed for metals and refractory compounds.

In Ref. [30], it was shown that the plasticity characteristic δ_H correlates with the wear rate of magnetic steels with different sizes of cementite grains (Fig. 17).

According to Ref. [30], the value of δ_H correlates with the fracture probability of Al–Ti–N intermetallic coatings. The fracture probability decreases with increase in δ_H.

It is known that, in metals with a b.c.c. lattice, a correlation between the electronic structure and plasticity of the materials is observed: a decrease in the covalent component in the interatomic bond leads to an increase in the plasticity [25, 26, 69].

For covalent crystals with the diamond lattice, it was impossible to perform an experimental investigation of the relationship between the electronic structure and plasticity δ because, for all these crystals, the plasticity δ determined in tension is equal to zero. The introduction of the plasticity characteristic δ_H, determined by the indentation method, made it possible to perform such investigations. In Ref. [70], it was shown that the plasticity δ_H rises for these crystals with increase in the concentration of free electrons (Fig. 18).

In Ref. [71], it was also shown that the increase in the value of the pseudopotential W_{111} for covalent crystals with the diamond lattice correlates with the plasticity characteristic δ_H: the greater the value of W_{111}, the

less is the value of the plasticity (Fig. 19). These results agree well with the modern knowledge of the influence of the electronic structure on the mobility of dislocations in these crystals [72–74].

The plasticity characteristic δ_H turned to be sensitive and informative in the investigation of the mechanical properties of microlaminates [75]. It was shown that, for Nb_5Si_3/Nb microlaminates, depending on the method of their preparation, the values of δ_H vary in the wide range from 0.373 to 0.824.

The authors of Refs. [50, 51, 76] used extensively the plasticity characteristic δ_H in the investigation of the micromechanical properties of nanocrystalline materials at cryogenic temperatures. For instance, for nanocrystalline titanium, $\delta_H = 0.76$ and $\delta_H = 0.83$ at 77 K and 300 K, respectively [50]. The value of δ_H for nanocrystalline titanium is much smaller than that for Ti in the polycrystalline state ($\delta_H = 0.973$).

Fig. 18. Plasticity index (δ_H) *vs.* room-temperature concentration of free electrons for covalent crystals [70]
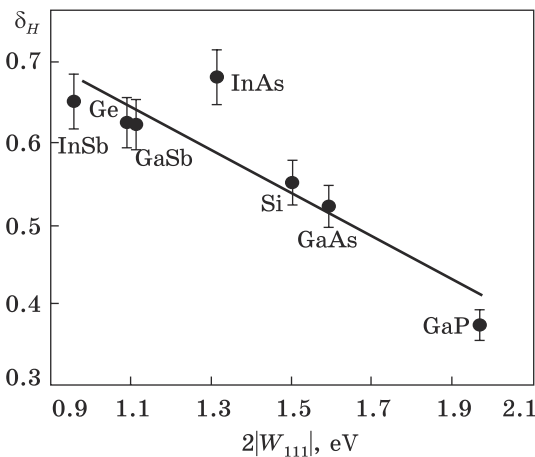
Fig. 19. Plasticity index (δ_H) *vs.* pseudopotential, W_{111}, of covalent crystals [71]

In investigation of the effect of the microstructure on the plastic deformation of copper at a temperature of 295 and 77 K, the authors of Ref. [50] calculated the plasticity characteristic δ_H and showed that, independently of the grain size and temperature, $\delta_H = 0.93–0.96$ that testifies to the high plasticity of copper under an indenter at low temperatures. These values of δ_H are typical of metals with an h.c.p. lattice [23].

In Ref. [77], the temperature dependence of the hardness and plasticity characteristic δ_H of the Fe−28Al−3Cr intermetallic in the temperature range 300−1273 K was investigated. At 300 K, $\delta_H = 0.85$, δ_H rises with increasing temperature, and only at a temperature above 800 K, $\delta_H > 0.9$, *i.e.*, the material becomes ductile.
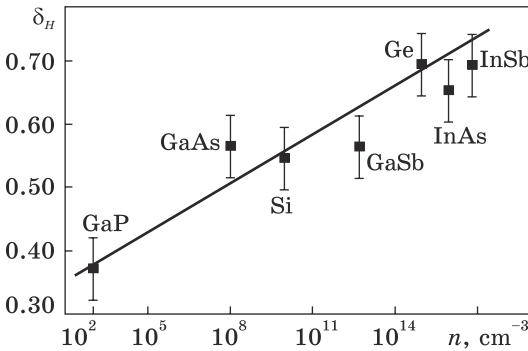
In Refs. [78, 79], by the example of a group of ceramic coatings, the efficiency of using the developed technique of determining the plasticity characteristic δ_H of coatings for application in engineering practice was shown.

It was shown that the investigated coatings could be arranged in the order of increasing plasticity characteristic δ_H as follows: carbides of group IVA metals, iron borides, carbides of the group VA and VIA metals, titanium nitride TiN, and silicides of refractory metals.

On carbide coatings, the influence of the temperature on the plasticity characteristic δ_H was studied. In Refs. [78, 79], differences in the change of the plasticity characteristic δ_H for coatings with increasing temperature were associated with different mobility of dislocations in carbides of the group VA and VIA metals. It was shown that, for ceramic coatings with a high hardness $HV \approx 20$ GPa, which are used in cutting and upsetting tools, the plasticity characteristic must be at a level $\delta_H \geq 0.45$.

Thus, thin coating can be efficiently used at values of the plasticity characteristic much lesser than the critical value $\delta_H = 0.9$ for massive materials. This is explained by the small thickness of coatings, at which bending deformation has a purely elastic character that reduces the risk of brittle fracture of coatings.

In all works where the values of δ_H or $PI = \delta_A$ were determined, their values agreed well with the values of δ_H presented in Table 1 and Refs. [12, 22, 31].

In the development and application of a wide class of functional materials (thin coatings, films, gradient materials, *etc*.), the necessity of determining the mechanical properties of these new materials arose. These problems can be successfully solved with the use of the nanoindentation method, which enables one not only to determine the hardness, Young's modulus, *etc*., but also to calculate the plasticity characteristic δ_A (plasticity index) by the technique proposed in Refs. [5, 22, 29] (see section covers plasticity characteristic δ_A determined in the instrumented indentation).

Note also that the operation and the control of hardness-testing instruments for instrumented indentation can be performed remotely, which makes it possible to determine the plasticity characteristic, hardness, and yield stress for the purposes of nuclear power engineering with the use of simple specimens in the form of metallographic specimens practically without damaging them in the process of measurement [64].

Conclusion

The development of the technique of determination of plasticity by the indentation method for two last decades [5, 12, 13, 22–24, 28, 29, 31, 55, 64] extended substantially the possibilities to characterize the mechanical properties of materials by the simple highly efficient indentation method.

A new technique of determination of plasticity by the indentation method became possible due to the successful solution of the more general problem, namely, the introduction of the universal dimensionless plasticity characteristic of materials $\delta^* = \varepsilon_p/\varepsilon_t$, *i.e.*, the ratio of the plastic part of strain ε_p to the total strain ε_t. This plasticity characteristic corresponds to the physical understanding of the plasticity of materials in the physics of strength. In the present review, the technique of determination of the plasticity δ^* by the indentation method, which has been developed in recent years, is considered (in this case, the notation of the new plasticity characteristic by δ_H was adopted). Undoubtedly, in the future, the development of a technique of determination of δ^* with the use of other methods of determination of mechanical properties, in particular, in tensile tests, will be possible.

The following developments may be the main results of the introduction of the plasticity characteristic δ_H or plasticity index $PI = \delta_A$.

The characterization of all materials (including metallic glasses, quasi-crystals, and other materials brittle in standard mechanical tests) by their plasticity is possible in a wide temperature range, including cryogenic and elevated temperatures (up to 1000 °C with the use of a diamond indenter and up to 1350 °C with the use of an Al_2O_3 indenter).

There is the possibility to characterize the plasticity of coatings and thin layers of different materials, including ceramic and other brittle coatings.

The theory of the influence of structural factors (grain size and dislocation density), the temperature and strain rate on the plasticity δ_H has been developed.

The notion of theoretical plasticity has been introduced, and a technique of its determination has been developed. In this case, the theoretical plasticity, like the theoretical strength, is considered for the case of dislocation-free deformation of a perfect crystal in the absence of thermal oscillations of atoms, *i.e.*, at 0 K.

The notion of theoretical plasticity in the case of the dislocation mechanism of deformation, but without the help of thermal oscillations of atoms has been introduced. This plasticity characteristic has been defined as the plasticity δ_H at 0 K. The $\delta_H(0)$ is determined under the same conditions as the Peierls–Nabarro stress (stress required for the motion of dislocations through potential barriers of a crystal lattice

without the help of thermal oscillations, that is practically the theoretical strength in the dislocation mechanism of deformation) and, hence, gives additional necessary data on the dislocation mechanism of deformation under the indicated conditions.

In choosing the basic material for operation under specific conditions, the possibility to take into account not only its high theoretical strength, but also the necessity of a combination of a high theoretical strength with a sufficient theoretical plasticity appears. The choice of a basic element with a high Peierls–Nabarro stress for high-strength materials is more practice-oriented; in this case, the combination of a high Peierls–Nabarro stress with a sufficient plasticity characteristic $\delta_H(0)$ is necessary. The use of the characteristic δ_H in development of real high-strength alloys and coatings, which must combine a high strength with a sufficient plasticity in a wide temperature range, is even more practice-oriented.

It has been shown that the plasticity characteristic δ_A (Plasticity Index) that can be determined in instrumented indentation is approximately equal to δ_H ($\delta_A \approx \delta_H$), if indentation is carried out by identical indenters and under equal loads on an indenter. In this case, for the determination of δ_A, only areas under loading and unloading curves of an indenter in coordinates 'load on an indenter P–displacement of the indenter h' are used, and the necessity of determining Young's modulus and Poisson's ratio ν_S disappears. It seems reasonable to introduce the definition of the plasticity characteristic in nanoindentation into the standard of nanohardness testing method [80].

It has turned to be possible to understand the dual nature of the hardness HM, which depends both on the strength characteristic, namely, the yield strength σ_S, and on the plasticity of the material. The hardness HM is proportional to the yield strength σ_S, but the proportionality coefficient, namely, the Tabor parameter C, is determined by the plasticity characteristic δ_H of the material.

For the first time, the possibility to establish the correlation between the plasticity of materials, which fracture in a brittle manner in standard mechanical tests, with their electronic structure and different physical properties has appeared.

The introduction of the plasticity characteristic δ_H made it possible for the first time to determine easily the Tabor parameter C for single-phase materials and calculate the yield strength σ_S from the simple expression $\sigma_S = HM/C$. As a result, the possibilities and efficiency of the indentation method, which now includes the determination of the hardness, plasticity, and yield strength, have been substantially extended.

Remotely controlled instrumented hardness with the determination of the hardness H, plasticity, Young's modulus E and yield strength σ_S can be used in the nuclear power industry and space research.

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Ю. В. Мільман, С. І. Чугунова, І. В. Гончарова, О. А. Голубенко
Інститут проблем матеріалознавства ім. І. М. Францевича НАН України;
вул. Академіка Кржижановського, 3; 03142 Київ, Україна

ПЛАСТИЧНІСТЬ МАТЕРІЯЛІВ,
ЩО ВИЗНАЧАЄТЬСЯ МЕТОДОЮ ІНДЕНТУВАННЯ

У даному огляді розглянуто розвиток метод визначення пластичности матеріялів індентуванням. Розроблення способів визначення пластичности матеріялів за допомогою методи індентування засновано на використанні фундаментальних уявлень фізики міцности та пластичности. Істотний розвиток цих способів став можливим після введення нової характеристики пластичности $\delta^* = \varepsilon_p/\varepsilon_t$, де ε_p — пластична деформація, а ε_t — загальна деформація. Ця характеристика пластичности відповідає сучасним фізичним визначенням пластичности на відміну від подовження до руйнування δ, яке широко використовується. Нова характеристика пластичности легко визначається при стандартному визначенні твердости алмазними пірамідальними інденторами за сталого навантаження P (позначається δ_H) і при інструментальному наноіндентуванні (позначається δ_A); при цьому $\delta_H \approx \delta_A$. Істотною перевагою нової характеристики пластичности є можливість визначення її як для металів, так і для крихких при стандартних механічних випробуваннях матеріялів, включаючи кераміку, тонкі шари та покриття. У розвиток уявлень про теоретичну міцність введено уявлення про теоретичну пластичність при бездислокаційному та дислокаційному механізмах деформації. У ряді робіт встановлено кореляцію δ_H з електронною будовою матеріялу та його фізичними властивостями. Показано, що параметр Тейбора C ($C = HM/\sigma_S$, де HM — твердість за Мейєром, σ_S — межа плинности) легко розраховується за δ_H. Тому індентування уможливлює нині достатньо просто визначити не тільки твердість, а й пластичність і межу плинности матеріялів. Таким чином, індентування стало простою методою визначення комплексу механічних властивостей матеріялів у широкому температурному інтервалі з використанням зразка у вигляді металографічного шліфа.

Ключові слова: твердість, пластичність, індентування, межа плинности, деформація.

Ю. В. Мильман, С. И. Чугунова, И. В. Гончарова, А. А. Голубенко
Институт проблем материаловедения им. И. Н. Францевича НАН Украины;
ул. Академика Кржижановского, 3; 03142 Киев, Украина

ПЛАСТИЧНОСТЬ МАТЕРИАЛОВ, ОПРЕДЕЛЯЕМАЯ МЕТОДОМ ИНДЕНТИРОВАНИЯ

В данном обзоре рассмотрено развитие методик определения пластичности материалов индентированием. Разработка способов определения пластичности материалов методом индентирования основана на использовании фундаментальных представлений физики прочности и пластичности. Существенное развитие этих способов стало возможным после введения новой характеристики пластичности $\delta^* = \varepsilon_p/\varepsilon_t$, где ε_p — пластическая деформация, а ε_t — общая деформация. Эта характеристика пластичности соответствует современным физическим определениям пластичности, в отличие от широко используемого удлинения до разрушения δ. Новая характеристика пластичности легко определяется при стандартном определении твёрдости алмазными пирамидальными инденторами при постоянной нагрузке P (получила обозначение δ_H) и при инструментальном наноиндентировании (обозначение δ_A); при этом $\delta_H \approx \delta_A$. Существенным преимуществом новой характеристики пластичности является возможность её определения как для металлов, так и для хрупких при стандартных механических испытаниях материалов, включая керамику, тонкие слои и покрытия. В развитие представлений о теоретической прочности введены представления о теоретической пластичности при бездислокационном и дислокационном механизмах деформации. В ряде работ установлена корреляция δ_H с электронным строением материала и его физическими свойствами. Показано, что параметр Тейбора C ($C = HM/\sigma_S$, где HM — твёрдость по Мейеру, а σ_S — предел текучести) легко рассчитывается по δ_H. Поэтому индентирование позволяет в настоящее время достаточно просто определить не только твёрдость, но и пластичность, и предел текучести материалов. Таким образом, индентирование стало простым методом определения комплекса механических свойств материалов в широком температурном интервале с использованием образца в виде металлографического шлифа.

Ключевые слова: твёрдость, пластичность, индентирование, предел текучести, деформация.