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Tran Thi Loan, Duong Anh Tuan (Hanoi Nat. Univ. Education, Vietnam)

GLOBAL EXPONENTIAL STABILITY OF A CLASS OF NEURAL NETWORKS WITH UNBOUNDED DELAYS

ГЛОБАЛЬНА ЕКСПОНЕНЦІАЛЬНА СТІЙКІСТЬ ОДНОГО КЛАСУ НЕЙРОННИХ СІТОК З НЕОБМЕЖЕНИМИ ЗАГАЮВАННЯМИ

In this paper, the global exponential stability of a class of neural networks is investigated. The neural networks contain variable and unbounded delays. By constructing a suitable Lyapunov function and using the technique of matrix analysis, some new sufficient conditions on the global exponential stability are obtained.

Досліджено глобальну експоненціальну стійкість одного класу нейронних сіток. Нейронні сітки містять змінні та необмежені загаювання. На основі побудови відповідної функції Ляпунова та техніки матричного аналізу отримано нові достатні умови глобальної експоненціальної стійкості.

1. Introduction. It is well-known that cellular neural networks (CNNs) proposed by L. O. Chua and L. Yang in 1988 have been extensively studied both in theory and applications. We refer the reader to [1 – 8] for more details on this matter. The CNNs have been successfully applied in signal processing, pattern recognition, associative memories and especially in static image treatments. Such applications rely on the qualitative properties of the neural networks. In hardware implementation, time delays occur due to finite switching speeds of the amplifiers and communication time. Time delays may lead to an oscillation and furthermore, to an instability of networks [1]. On the other hand, the process of moving images requires the occurrence of delays in the signal transmission among the networks [5]. Therefore, the study of stability of neural networks with delay is practically required.

It is known that fixed time delays in model of delayed feedback systems serve as a good approximation of a simple circuit having a small number of cells. The neural network usually has spatial nature due to the presence of various parallel pathways. Thus, it is desirable to model them by introducing unbounded delays.

Usually, the Lyapunov functional method is used to study qualitative properties of CNNs. Such a method is performed in three steps. In step 1, we construct a Lyapunov function $V(t)$. In step 2, we get rid of time delays in $V(t)$ to estimate $V(t)$. In step 3, we require some conditions on CNNs so that the function $V(t)$ satisfies necessary properties. Therefore, we obtain the qualitative properties of CNNs.

For autonomous CNNs, the study of the existence, uniqueness and stability of the equilibrium point of neural networks have been carried out. Some useful results have already been obtained in [3, 4, 6 – 11].

The stability of nonautonomous CNNs has been studied in [2, 3, 12, 13]. However, results about the stability of the neural networks with variable, unbounded delays and time varying coefficients have not been widely studied. In particular other authors have not used scale Lyapunov functions in their studies (see [3, 7, 8, 14]). In this paper, we study the exponential stability of CNNs with variable, unbounded delay and time varying coefficients by constructing proper scale Lyapunov functions. Some new sufficient conditions for global exponential stability are obtained.

This paper is organized as follows. In Section 2, we introduce some definitions and assumptions. In Section 3, the global exponential stability is obtained. In Section 4, we obtain a series of corollaries.

2. Definitions and assumptions. In this paper we consider the general neural networks with variable and unbounded time delays of the form

$$\frac{dx_i(t)}{dt} = -d_i(t)x_i(t) + \sum_{j=1}^{n} a_{ij}(t) f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij}(t) g_j(x_j(t-\tau_{ij}(t))) +$$

$$+ \sum_{j=1}^{n} c_{ij}(t) \int_{-\infty}^{t} k_{ij}(t-s) h_j(x_j(s))\,ds + I_i(t), \qquad i = \overline{1, n}, \tag{1}$$

where x_i is the state of neuron i, $i = 1, \ldots, n$; n is the number of neuron; $A(t) = (a_{ij}(t))_{n\times n}$, $B(t) = (b_{ij}(t))_{n\times n}$, $C(t) = (c_{ij}(t))_{n\times n}$ are connection matrices; $I(t) = (I_1(t), \ldots, I_n(t))^T$ is the input vector;

$$f(u) = (f_1(u), \ldots, f_n(u))^T, \qquad g(u) = (g_1(u), \ldots, g_n(u))^T,$$

$$h(u) = (h_1(u), \ldots, h_n(u))^T$$

are the activation functions of the neurons. $D(t) = \operatorname{diag}(d_1(t), \ldots, d_n(t))$, $d_i(t)$ represents the rate in which the ith unit will reset its potential to the resting state in isolation when disconnected from the network; $k_{ij}(t)$, $i, j = 1, \ldots, n$, are the kernel functions; $\tau_{ij}(t)$, $i, j = 1, \ldots, n$, are the delays.

For convenience, we introduce some notations: $x = (x_1, \ldots, x_n)^T \in \mathbb{R}^n$ denotes a column vector (the symbol $(^T)$ denotes the transpose) with norm $|x| = \left(\sum_{i=1}^{n} x_i^2\right)^{1/2}$. For matrix $A = (a_{ij})_{n\times n}$, A^T denotes the transpose of A, A^{-1} denotes the inverse of A. If A, B are symmetric matrices, $A > B$ $(A \geq B)$ means that $A - B$ is positive definite (positive semidefinite). The minimum and maximum real eigenvalue of a symmetric matrix A are denoted by $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$, respectively.

We consider system (1) under some following assumptions.

(H_1) Functions $d_i(t)$, $a_{ij}(t)$, $b_{ij}(t)$, $c_{ij}(t)$ and $I_i(t)$, $i, j = 1, \ldots, n$, are defined, bounded and continuous on $\mathbb{R}^+$. Functions $\tau_{ij}(t)$, $i, j = 1, \ldots, n$, are defined nonnegative, bounded and continuously differentiable on $\mathbb{R}^+$, $\inf_{t\in\mathbb{R}^+}(1 - \dot{\tau}_{ij}(t)) > 0$, where $\dot{\tau}_{ij}(t)$ is the derivative of $\tau_{ij}(t)$ with respect to t. Functions $k_{ij}\colon [0, \infty) \to [0, \infty)$, $i, j = 1, \ldots, n$, are piecewise continuous on $[0, \infty)$ and satisfy $\int_0^\infty e^{\varepsilon s} k_{ij}(s)\,ds = p_{ij}(\varepsilon)$, where $p_{ij}(\varepsilon)$ are continuous functions in $[0, \delta)$, $\delta > 0$, $p_{ij}(0) = 1$.

(H_2) There are positive constants H_i, K_i, L_i, $i = 1, \ldots, n$, such that

$$0 \leq \frac{f_i(u) - f_i(u^*)}{u - u^*} \leq H_i, \qquad |g_i(u) - g_i(u^*)| \leq K_i |u - u^*|,$$

$$|h_i(u) - h_i(u^*)| \leq L_i |u - u^*|$$

for all $u, u^* \in \mathbb{R}$ and $i = 1, \ldots, n$.

(H_3) There are a positive definite symmetric matrix S, diagonal matrices

$$\alpha = \operatorname{diag}(\alpha_1, \ldots, \alpha_n) > 0, \qquad \beta = \operatorname{diag}(\beta_1, \ldots, \beta_n) > 0,$$

$$\omega = \operatorname{diag}(\omega_1, \ldots, \omega_n) > 0, \quad \sigma = \operatorname{diag}(\sigma_1, \ldots, \sigma_n) > 0,$$

$$\gamma = \operatorname{diag}(\gamma_1, \ldots, \gamma_n) \geq 0$$

and a constant $a > 0$ such that $\lambda_{\min}(D_1(t, \eta)) \geq a$ for all $t \in \mathbb{R}^+$, $0 \leq \eta \leq H$, where

$$D_1(t, \eta) = SD(t) + D(t)S - nS(\alpha^{-1} + \omega^{-1})S -$$
$$- n\eta\gamma(\beta^{-1} + \sigma^{-1})\gamma\eta - (SA(t) - D(t)\gamma)\eta -$$
$$- \eta(A^T(t)S - \gamma D) - \eta(\gamma A(t) + A^T(t)\gamma)\eta -$$
$$- \sum_{i=1}^{n} [(\alpha_i + \beta_i)\overline{B}_i(t) + (\omega_i + \sigma_i)\overline{C}_i(t)],$$

$$\eta = \operatorname{diag}(\eta_1, \ldots, \eta_n), \quad H = \operatorname{diag}(H_1, \ldots, H_n),$$

$$\overline{C}_i(t) = \operatorname{diag}\left(L_1^2 \int_{-\infty}^{0} k_{i1}(-s)c_{i1}^2(t-s)\,ds, \ldots, L_n^2 \int_{-\infty}^{0} k_{in}(-s)c_{in}^2(t-s)\,ds \right),$$

$$\overline{B}_i(t) = \operatorname{diag}\left(K_1^2 \frac{b_{i1}^2(\psi_{i1}^{-1}(t))}{1 - \dot{\tau}_{i1}(\psi_{i1}^{-1}(t))}, \ldots, K_n^2 \frac{b_{in}^2(\psi_{in}^{-1}(t))}{1 - \dot{\tau}_{in}(\psi_{in}^{-1}(t))} \right), \quad i = 1, \ldots, n,$$

here $\psi_{ij}^{-1}(t)$ is inverse function of $\psi_{ij}(t) = t - \tau_{ij}(t)$.

(H_4) There are a positive definite symmetric matrix S, diagonal matrices

$$\beta = \operatorname{diag}(\beta_1, \ldots, \beta_n) > 0, \quad \sigma = \operatorname{diag}(\sigma_1, \ldots, \sigma_n) > 0,$$

$$\gamma = \operatorname{diag}(\gamma_1, \ldots, \gamma_n) > 0$$

and a constant $a > 0$ such that $\gamma A(t) + A^T(t)\gamma < 0$, $\lambda_{\min}(D_2(t, \eta)) \geq a$ for all $t \in \mathbb{R}^+$, $0 \leq \eta \leq H$, where

$$D_2(t, \eta) = SD(t) + D(t)S - nS(\beta^{-1} + \sigma^{-1})S -$$
$$- (SA(t) - D(t)\gamma)\eta - \eta(A^T(t)S - \gamma D(t)) -$$
$$- \sum_{i=1}^{n} [(\beta_i - \alpha_i^*)\overline{B}_i(t) + (\sigma_i - \omega_i^*)\overline{C}_i(t)],$$

$$\alpha^* = \omega^* = n \inf_{t \in \mathbb{R}^+} \left[\lambda_{\min}\left(\gamma \left(\frac{\gamma A(t) + A^T(t)\gamma}{2} \right)^{-1} \gamma \right) \right] E,$$

E is unit matrix.

We denote by BC the Banach space of bounded continuous functions $\varphi : (-\infty, 0] \to \mathbb{R}^n$ with norm $\|\varphi\| = \sup_{-\infty < s \leq 0} |\varphi(s)|$.

The initial condition associated with (1) is of the form

$$x(\theta) = \varphi(\theta), \quad \theta \in (-\infty, 0], \quad \text{where} \quad \varphi \in BC. \tag{2}$$

It is well-known that if hypotheses (H_1), (H_2) are satisfied, then the system (1) has a unique solution $x(t) = (x_1(t), \ldots, x_n(t))^T$ satisfying the initial condition (2) (see [15]).

Definition 1. *The equilibrium point* x^* *of the system* (1) *is said to be globally exponentially stable* (*GES*) *if there exist constants* $\lambda > 0$ *and* $M > 0$ *such that for any solution* $x(t)$ *of the system* (1) *with initial function* φ, *we have*

$$\left|x(t) - x^*\right| \le M\left\|\varphi - x^*\right\|e^{-\lambda t} \quad \text{for all} \quad t \in \mathbb{R}^+.$$

Definition 2. *The system* (1) *is said to be globally exponentially stable, if there are constants* $\varepsilon > 0$ *and* $M \ge 1$ *such that for any two solutions* $x(t)$, $y(t)$ *of the system* (1) *with the initial functions* φ, ψ, *respectively, one has*

$$|x(t) - y(t)| \le M\|\psi - \varphi\|e^{-\varepsilon t} \quad \text{for all} \quad t \in \mathbb{R}^+.$$

The activation function f *is said to belong to the class* PLI (*denoted* $f \in PLI$) *if for each* $j \in \{1, 2, \ldots, n\}$, $g_j : \mathbb{R} \to \mathbb{R}$ *is a partially Lipschitz continuous and monotone increasing function. A function* f *is said to be partially Lipschitz continuous in* $\mathbb{R}$ *if for any* $\rho \in \mathbb{R}$ *there exists a positive number* l_ρ *such that*

$$|f(\theta) - f(\rho)| \le l_\rho|\theta - \rho| \quad \text{for all} \quad \theta \in \mathbb{R}.$$

Definition 3. *The system* (1) *is said to be absolutely exponentially stable with respect to the class* PLI *if it possesses a unique GES equilibrium point for every functions* $g, f, h \in PLI$ *and every input vector* I.

Definition 4. *A matrix* A *is said to belong to the class* P_0 *if* A *satisfies that all principal minors of* A *are nonnegative* (*denoted* $A \in P_0$).

3. Global exponential stability of CNNs. In this section, by constructing a suitable Lyapunov function and using the technique of matrix analysis we give some sufficient conditions for the global exponential stability of solutions of the system (1).

The main results in this article are content of Theorem 1.

Theorem 1. *If the hypotheses* (H_1), (H_2) *and* (H_3) *or* (H_4) *are satisfied then the system* (1) *is globally exponentially stable.*

Proof. Let $x(t)$, $y(t)$ be two arbitrary solutions of the system (1) with initial value ψ, φ, respectively. Setting $z(t) = x(t) - y(t)$, we have

$$\dot{z}(t) = -D(t)z(t) + A(t)\Phi(z(t)) + G(z(t - \tau(t))) + F(z(t)), \tag{3}$$

where

$$\Phi(z(t)) = (\Phi_1(z_1(t)), \ldots, \Phi_n(z_n(t))),$$

$$G(z(t - \tau(t))) = (G_1(z_1(t - \tau_1(t))), \ldots, G_n(z_n(t - \tau_n(t))))^T,$$

$$F(z(t)) = (F_1(z_1(t)), \ldots, F_n(z_n(t)))^T,$$

$$\Phi_i(z_i(t)) = f_i(x_i(t)) - f_i(y_i(t)),$$

$$G_i(z_i(t - \tau_i(t))) = \sum_{j=1}^{n} b_{ij}(t)\left[g_j(x_j(t - \tau_{ij}(t))) - g_j(y_j(t - \tau_{ij}(t)))\right],$$

$$F_i(z_i(t)) = \sum_{j=1}^{n} c_{ij}(t) \int_{-\infty}^{t} k_{ij}(t - s)\left[h_j(x_j(t)) - h_j(y_j(t))\right]ds, \quad i = 1, \ldots, n.$$

Define a Lyapunov function as follows

$$V(t, z_t) = z^T(t)Sz(t)e^{\varepsilon t} + 2\sum_{i=1}^{n}\int_0^{z_i(t)}\gamma_i\Phi_i(s)\,ds\,e^{\varepsilon t} +$$

$$+ \sum_{i=1}^{n}(\alpha_i+\beta_i)\sum_{j=1}^{n}\int_{t-\tau_{ij}(t)}^{t}\frac{b_{ij}^2(\psi_{ij}^{-1}(s))}{1-\dot{\tau}_{ij}(\psi_{ij}^{-1}(s))}\big[g_j(x_j(s))-g_j(y_j(s))\big]^2 e^{\varepsilon(s+\tau_{ij}(\psi_{ij}^{-1}(s)))}\,ds +$$

$$+ \sum_{i=1}^{n}(\omega_i+\sigma_i)\sum_{j=1}^{n}\int_{-\infty}^{0}k_{ij}(-s)\int_{t+s}^{t}c_{ij}^2(u-s)\big[h_j(x_j(u))-h_j(y_j(u))\big]^2 e^{\varepsilon(u-s)}\,du\,ds, \quad (4)$$

where $S, \alpha, \beta, \gamma, \omega, \sigma$ are given by (H_3), $\varepsilon > 0$ be a constant which will be determined later on. Caculating the derivative of $V(t, z_t)$ along the solutions of equation (3), we get

$$\frac{dV(t,z_t)}{dt} = e^{\varepsilon t}\Big[-2z^T(t)SD(t)z(t) + 2z^T(t)SA(t)\Phi(z(t)) + 2z^T(t)SG(z(t-\tau(t))) +$$

$$+ 2z^T(t)SF(z(t)) - 2\Phi^T(z(t))\gamma D(t)z(t) + 2\Phi^T(z(t))\gamma A(t)\Phi(z(t)) +$$

$$+ 2\Phi^T(z(t))\gamma G(z(t-\tau(t))) + 2\Phi(z(t))\gamma F(z(t))\Big] +$$

$$+ \sum_{i=1}^{n}(\alpha_i+\beta_i)\sum_{j=1}^{n}\Bigg[\frac{b_{ij}^2(\psi_{ij}^{-1}(t))}{1-\dot{\tau}_{ij}(\psi_{ij}^{-1}(t))}\big(g_j(x_j(t))-g_j(y_j(t))\big)^2 e^{\varepsilon(t+\tau_{ij}(\psi_{ij}^{-1}(t)))} -$$

$$- b_{ij}^2(t)\big[g_j(x_j(t-\tau_{ij}(t)))-g_j(y_j(t-\tau_{ij}(t)))\big]^2 e^{\varepsilon t}\Bigg] +$$

$$+ \sum_{i=1}^{n}(\omega_i+\sigma_i)\sum_{j=1}^{n}\Bigg[\int_{-\infty}^{0}k_{ij}(-s)c_{ij}^2(t-s)\big[h_j(x_j(t))-h_j(y_j(t))\big]^2 e^{\varepsilon(t-s)}\,ds -$$

$$- \int_{-\infty}^{0}k_{ij}(-s)c_{ij}^2(t)\big[h_j(x_j(t+s))-h_j(y_j(t+s))\big]^2 ds\,e^{\varepsilon t}\Bigg] +$$

$$+ \varepsilon z^T(t)Sz(t)e^{\varepsilon t} + 2\varepsilon\sum_{i=1}^{n}\int_0^{z_i(t)}\gamma_i\Phi_i(s)\,ds\,e^{\varepsilon t}. \quad (5)$$

It follows by using Cauchy – Schwarz inequality that

$$n\sum_{i=1}^{n}b_{ij}^2(t)\big[g_j(x_j(t-\tau_{ij}(t)))-g_j(y_j(t-\tau_{ij}(t)))\big]^2 \geq$$

$$\geq \left[\sum_{j=1}^{n}b_{ij}(t)\big[g_j(x_j(t-\tau_{ij}(t)))-g_j(y_j(t-\tau_{ij}(t)))\big]\right]^2.$$

Since $\int_0^{\infty}k_{ij}(s)\,ds = 1$, we obtain

$$\int_{-\infty}^{0}k_{ij}(-s)\big[h_j(x_j(t+s))-h_j(y_j(t+s))\big]^2 ds =$$

$$= \int_{-\infty}^{0} k_{ij}(-s)\big[h_j(x_j(t+s)) - h_j(y_j(t+s))\big]^2 ds \int_{-\infty}^{0} k_{ij}(-s)\,ds \;\geq$$

$$\geq \left(\int_{-\infty}^{0} k_{ij}(-s)\big[h_j(x_j(t+s)) - h_j(y_j(t+s))\big] ds \right)^2$$

for all $i, j = 1, \ldots, n$. Hence, we have

$$\sum_{j=1}^{n} c_{ij}^2(t) \int_{-\infty}^{0} k_{ij}(-s)\big[h_j(x_j(t+s)) - h_j(y_j(t+s))\big]^2 ds \;\geq$$

$$\geq \sum_{j=1}^{n} c_{ij}^2(t) \left(\int_{-\infty}^{t} k_{ij}(t-s)\big[h_j(x_j(s)) - h_j(y_j(s))\big] ds \right)^2 \geq$$

$$\geq \frac{1}{n} \left(\sum_{j=1}^{n} c_{ij}(t) \int_{-\infty}^{t} k_{ij}(t-s)\big[h_j(x_j(s)) - h_j(y_j(s))\big] ds \right)^2.$$

Firstly, we assume that (H_3) holds. From (H_1), (H_2), we have

$$\frac{dV(t, z_t)}{dt} \leq e^{\varepsilon t} \Big\{ -2z^T(t)SD(t)z(t) + 2z^T(t)SA(t)\Phi(z(t)) + 2z^T(t)SG(z(t-\tau(t))) +$$

$$+\, 2z^T(t)SF(z(t)) - 2\Phi^T(z(t))\gamma D(t)z(t) + 2\Phi^T(z(t))\gamma A(t)\Phi(z(t)) +$$

$$+\, 2\Phi^T(z(t))\gamma G(z(t-\tau(t))) + 2\Phi(z(t))\gamma F(z(t)) +$$

$$+ \sum_{i=1}^{n} (\alpha_i + \beta_i) \sum_{j=1}^{n} K_j^2 \frac{b_{ij}^2(\psi_{ij}^{-1}(t))}{1 - \dot{\tau}_{ij}(\psi_{ij}^{-1}(t))} e^{\varepsilon \tau_{ij}(\psi_{ij}^{-1}(t))} z_j^2(t) -$$

$$- \sum_{i=1}^{n} (\alpha_i + \beta_i) \frac{1}{n} \left[\sum_{j=1}^{n} b_{ij}(t)\big[g_j(x_j(t - \tau_{ij}(t))) - g_j(y_j(t - \tau_{ij}(t)))\big] \right]^2 +$$

$$+ \sum_{i=1}^{n} (\omega_i + \sigma_i) \sum_{j=1}^{n} \int_{-\infty}^{0} k_{ij}(-s) c_{ij}^2(t-s) e^{-\varepsilon s}\, ds\, L_j^2 z_j^2(t) -$$

$$- \sum_{i=1}^{n} (\omega_i + \sigma_i) \frac{1}{n} \left(\sum_{j=1}^{n} c_{ij}(t) \int_{-\infty}^{t} k_{ij}(t-s)\big[h_j(x_j(s)) - h_j(y_j(s))\big] ds \right)^2 +$$

$$+\, \varepsilon z^T(t)Sz(t) + 2\varepsilon \sum_{i=1}^{n} \int_{0}^{z_i(t)} \gamma_i \Phi_i(s)\, ds \Big\} =$$

$$= e^{\varepsilon t} \Big\{ -2z^T(t)SD(t)z(t) + 2z^T(t)SA(t)\Phi(z(t)) + 2z^T(t)SG(z(t-\tau(t))) +$$

$$+\, 2z^T(t)SF(z(t)) - 2\Phi^T(z(t))\gamma D(t)z(t) + 2\Phi^T(z(t))\gamma A(t)\Phi(z(t))) +$$

$$+\, 2\Phi^T(z(t))\gamma G(z(t-\tau(t))) + 2\Phi(z(t))\gamma F(z(t)) + \sum_{i=1}^{n} (\alpha_i + \beta_i) z^T(t)\overline{B}_i(t, \varepsilon)z(t) -$$

$$- \frac{1}{n} G^T(z(t-\tau(t)))(\alpha+\beta)G(z(t-\tau(t))) + \sum_{i=1}^{n}(\omega_i+\sigma_i)z^T(t)\overline{C}_i(t,\varepsilon)z(t) -$$

$$- \frac{1}{n} F^T(z(t))(\omega+\sigma)F(z(t)) + \varepsilon z^T(t)Sz(t) + 2\varepsilon \sum_{i=1}^{n} \int_0^{z_i(t)} \gamma_i \Phi_i(s)\,ds, \tag{6}$$

where

$$\overline{B}_i(t,\varepsilon) = \operatorname{diag}\left(K_1^2 \frac{b_{i1}^2(\psi_{i1}^{-1}(t))}{1-\dot{\tau}_{i1}(\psi_{i1}^{-1}(t))} e^{\varepsilon\tau_{i1}(\psi_{i1}^{-1}(t))}, \ldots, K_n^2 \frac{b_{in}^2(\psi_{in}^{-1}(t))}{1-\dot{\tau}_{in}(\psi_{in}^{-1}(t))} e^{\varepsilon\tau_{in}(\psi_{in}^{-1}(t))} \right),$$

$$\overline{C}_i(t,\varepsilon) = \operatorname{diag}\left(\int_{-\infty}^{0} k_{i1}(-s)c_{i1}^2(t-s)e^{-\varepsilon s}\,ds\,L_1^2, \ldots, \int_{-\infty}^{0} k_{in}(-s)c_{in}^2(t-s)e^{-\varepsilon s}\,ds\,L_n^2 \right)$$

for all $i = 1, \ldots, n$.

Let $\eta(t) = \operatorname{diag}(\eta_1(t), \ldots, \eta_n(t))$, where $\eta_i(t) = \eta_i(z_i(t))$ such that

$$\Phi(z(t)) = \eta(t)z(t) \qquad (\forall t \in \mathbb{R}^+). \tag{7}$$

Using the inequality

$$2x^T y - y^T D y \le x^T D^{-1} x, \quad \text{where} \quad x, y \in \mathbb{R}^n, \quad D > 0, \tag{8}$$

we have

$$2z^T(t)SG(z(t-\tau(t))) - G^T(z(t-\tau(t)))\frac{\alpha}{n}G(z(t-\tau(t))) \le z^T(t)S\left(\frac{\alpha}{n}\right)^{-1}Sz(t),$$

$$2z^T(t)SF(z(t)) - F^T(z(t))\left(\frac{\omega}{n}\right)F(z(t)) \le z^T(t)S\left(\frac{\omega}{n}\right)^{-1}Sz(t),$$

$$2\Phi^T(z(t))\gamma G(z(t-\tau(t))) - G^T(z(t-\tau(t)))\frac{\beta}{n}G(z(t-\tau(t))) \le$$

$$\le \Phi^T(z(t))\gamma\left(\frac{\beta}{n}\right)^{-1}\gamma\Phi(z(t)),$$

$$2\Phi^T(z(t))\gamma F(z(t)) - F^T(z(t))\left(\frac{\sigma}{n}\right)F(z(t)) \le \Phi^T(z(t))\gamma\left(\frac{\sigma}{n}\right)^{-1}\gamma\Phi(z(t)).$$

This implies

$$\frac{dV(t,z_t)}{dt} \le -e^{\varepsilon t}z^T(t)\Big[SD(t) + D(t)S - (SA(t) - D(t)\gamma)\eta -$$

$$- \eta(A^T(t)S - \gamma D(t)) - \eta(A(t)\gamma + \gamma A^T(t))\eta - nS(\omega^{-1}+\alpha^{-1})S -$$

$$- n\eta\gamma(\sigma^{-1}+\beta^{-1})\gamma\eta - \sum_{i=1}^{n}(\alpha_i+\beta_i)\overline{B}_i(t,\varepsilon) - \sum_{i=1}^{n}(\omega_i+\sigma_i)\overline{C}_i(t,\varepsilon) -$$

$$- \varepsilon S - 2\varepsilon E\frac{1}{z^2(t)}\sum_{i=1}^{n}\int_0^{z_i(t)}\gamma_i\Phi_i(s)\,ds\Big]z(t) = z^T(t)D_1(t,\eta,\varepsilon)z(t), \tag{9}$$

where

$$D_1(t,\eta,\varepsilon) = SD(t) + D(t)S - \big(SA(t) - D(t)\gamma\big)\eta -$$
$$- \eta\big(A^T(t)S - \gamma D(t)\big) - \eta\big(A(t)\gamma + \gamma A^T(t)\big)\eta -$$
$$- nS(\omega^{-1} + \alpha^{-1})S - n\eta\gamma(\sigma^{-1} + \beta^{-1})\gamma\eta -$$
$$- \sum_{i=1}^{n}(\alpha_i + \beta_i)\overline{B}_i(t,\varepsilon) - \sum_{i=1}^{n}(\omega_i + \sigma_i)\overline{C}_i(t,\varepsilon) -$$
$$- \varepsilon S - 2\varepsilon E\,\frac{1}{z^2(t)}\sum_{i=1}^{n}\int_0^{z_i(t)}\gamma_i\Phi_i(s)\,ds.$$

Obviously, $\lim_{\varepsilon\to 0}\overline{B}_i(t,\varepsilon) = \overline{B}_i(t)$ uniformly for all $t\in\mathbb{R}^+$, $\lim_{\varepsilon\to 0}\overline{C}_i(t,\varepsilon) = \overline{C}_i(t)$ uniformly for all $t\in\mathbb{R}^+$ and $i = 1,\ldots,n$. From a assumption (H_2) we obtain

$$\frac{1}{z^2(t)}\sum_{i=1}^{n}\int_0^{z_i(t)}\gamma_i\Phi_i(s)\,ds \le \frac{1}{2}\max_{1\le i\le n}(\gamma_i H_i) \qquad (\forall t\in\mathbb{R}^+).$$

Hence, we have $\lim_{\varepsilon\to 0} D_1(t,\eta,\varepsilon) = D_1(t,\eta)$ uniformly for all $t\in\mathbb{R}^+$ and $0\le\eta\le H$. Thus, by assumption (H_3), there exists a constant $\varepsilon > 0$ such that $\lambda_{\min}\big(D_1(t,\eta,\varepsilon)\big) \ge \ge \frac{a}{2}$ for all $t\in\mathbb{R}^+$ and $0\le\eta\le H$. Therefore, by (9), we have

$$\frac{dV(t,z_t)}{dt} \le -\frac{a}{2}e^{\varepsilon t}z^T(t)z(t) \qquad (\forall t\in\mathbb{R}^+). \tag{10}$$

Secondly, we assume that (H_4) holds. By (8) we obtain

$$2\Phi^T(z(t))\gamma G(z(t-\tau(t))) + \Phi^T(z(t))\left(\frac{\gamma A(t) + A^T(t)\gamma}{2}\right)\Phi(z(t)) =$$
$$= 2\Phi^T(z(t))\gamma G(z(t-\tau(t))) - \Phi^T(z(t))\left(-\frac{\gamma A(t) + A^T(t)\gamma}{2}\right)\Phi(z(t)) \le$$
$$\le G(z(t-\tau(t)))\gamma\left(-\frac{\gamma A(t) + A^T(t)\gamma}{2}\right)^{-1}\gamma G(z(t-\tau(t))),$$
$$2\Phi^T(z(t))\gamma F(z(t)) + \Phi^T(z(t))\left(\frac{\gamma A(t) + A^T(t)\gamma}{2}\right)\Phi(z(t)) =$$
$$= 2\Phi^T(z(t))\gamma F(z(t)) - \Phi^T(z(t))\left(-\frac{\gamma A(t) + A^T(t)\gamma}{2}\right)\Phi(z(t)) \le$$
$$\le F(z(t))\gamma\left(-\frac{\gamma A(t) + A^T(t)\gamma}{2}\right)^{-1}\gamma F(z(t)),$$
$$2z^T(t)SG(z(t-\tau(t))) - G^T(z(t-\tau(t)))\,\frac{\beta}{n}\,G(z(t-\tau(t))) \le z^T(t)S\left(\frac{\beta}{n}\right)^{-1}Sz(t),$$

$$2z^T(t)SF(z(t)) - F^T(z(t))\left(\frac{\sigma}{n}\right)F(z(t)) \;\le\; z^T(t)S\left(\frac{\sigma}{n}\right)^{-1}Sz(t).$$

So from (4), (6) with $\alpha = -\alpha^*$, $\omega = -\omega^*$, we have

$$\frac{dV(t, z_t)}{dt} \;\le\; e^{\varepsilon t}\Bigg\{-2z^T(t)SD(t)z(t) + 2z^T(t)SA(t)\Phi(z(t)) +$$

$$+\; G(z(t-\tau(t)))\gamma\left(-\frac{\gamma A(t) + A^T(t)\gamma}{2}\right)^{-1}\gamma G(z(t-\tau(t))) +$$

$$+\; F(z(t))\gamma\left(-\frac{\gamma A(t) + A^T(t)\gamma}{2}\right)^{-1}\gamma F(z(t)) +$$

$$+\; nz^T(t)S(\beta^{-1} + \sigma^{-1})Sz(t) - 2\Phi^T(z(t))\gamma D(t)z(t) +$$

$$+\; \sum_{i=1}^{n}(\alpha_i + \beta_i)z^T(t)\overline{B}_i(t, \varepsilon)z(t) \;+\; \sum_{i=1}^{n}(\omega_i + \sigma_i)z^T(t)\overline{C}_i(t, \varepsilon)z(t) \;-$$

$$-\; G^T(z(t-\tau(t)))\,\frac{\alpha}{n}\,G(z(t-\tau(t))) - F^T(z(t))\left(\frac{\omega}{n}\right)F(z(t)) +$$

$$+\; \varepsilon z^T(t)Sz(t) \;+\; 2\varepsilon\sum_{i=1}^{n}\int_0^{z_i(t)}\gamma_i\Phi_i(s)\,ds\Bigg\} \;\le$$

$$\le\; -z^T(t)e^{\varepsilon t}\Bigg[SD(t) + D(t)S - (SA(t) - \gamma D(t))\eta \;-$$

$$-\; \eta(A^T(t)S - D(t)\gamma) - nS(\beta^{-1} + \sigma^{-1})S \;-$$

$$-\; \sum_{i=1}^{n}(\alpha_i + \beta_i)\overline{B}_i(t, \varepsilon) \;-\; \sum_{i=1}^{n}(\omega_i + \sigma_i)\overline{C}_i(t, \varepsilon) \;-$$

$$-\; \varepsilon S \;-\; 2\varepsilon E\,\frac{1}{z^2(t)}\sum_{i=1}^{n}\int_0^{z_i(t)}\gamma_i\Phi_i(s)\,ds\Bigg]z(t). \qquad (11)$$

Let

$$D_2(t, \eta, \varepsilon) \;=\; SD(t) \;+\; D(t)S \;-\; (SA(t) - \gamma D(t))\eta \;-$$

$$-\; \eta(A^T(t)S - D(t)\gamma) - nS(\beta^{-1} + \sigma^{-1})S \;-$$

$$-\; \sum_{i=1}^{n}(\alpha_i + \beta_i)\overline{B}_i(t, \varepsilon) \;-\; \sum_{i=1}^{n}(\omega_i + \sigma_i)\overline{C}_i(t, \varepsilon) \;-$$

$$-\; \varepsilon S \;-\; 2\varepsilon E\,\frac{1}{z^2(t)}\sum_{i=1}^{n}\int_0^{z_i(t)}\gamma_i\Phi_i(s)\,ds.$$

By a similar argument as used for $D_1(t, \eta, \varepsilon)$, we also have $\lim\limits_{\varepsilon\to 0} D_2(t, \eta, \varepsilon) = D_2(t, \eta)$ uniformly for all $t \in \mathbb{R}^+$ and $0 \le \eta \le H$. Thus, by assumption (H_4) there exists a constant $\varepsilon > 0$ such that

$$\lambda_{\min}(D_2(t,\eta,\varepsilon)) \geq \frac{a}{2}.$$

Therefore, by (11) we finally have

$$\frac{dV(t,z_t)}{dt} \leq -e^{\varepsilon t}\frac{a}{2}z^T(t)z(t) \qquad (\forall t\in\mathbb{R}^+). \tag{12}$$

From (10), (12) we further obtain

$$V(t) \leq V(0) \qquad (\forall t\geq 0). \tag{13}$$

Directly, from (4) and a assumption (H_2) we have

$$V(t) \geq z^T(t)Sz(t)e^{\varepsilon t} \geq \lambda_{\min}(S)e^{\varepsilon t}|z(t)|^2 \qquad (\forall t\geq 0), \tag{14}$$

$$V(0) = z^T(0)Sz(0) + 2\sum_{i=1}^n\int_0^{z_i(0)}\gamma_i\Phi_i(s)\,ds +$$

$$+\sum_{i=1}^n(\alpha_i+\beta_i)\sum_{j=1}^n\int_{-\tau_{ij}(0)}^0\frac{b_{ij}^2(\psi_{ij}^{-1}(s))}{1-\dot{\tau}_{ij}(\psi_{ij}^{-1}(s))}\left[g_j(x_j(s))-g_j(y_j(s))\right]^2e^{\varepsilon(s+\tau_{ij}(\psi_{ij}^{-1}(s)))}\,ds +$$

$$+\sum_{i=1}^n(\omega_i+\sigma_i)\sum_{j=1}^n\int_{-\infty}^0k_{ij}(-s)\int_s^0c_{ij}^2(u-s)\left[h_j(x_j(u))-h_j(y_j(u))\right]^2e^{\varepsilon(u-s)}\,du\,ds \leq$$

$$\leq \lambda_{\max}(S)\|\varphi-\psi\|^2+\max_{1\leq i\leq n}(\gamma_iH_i)\|\varphi-\psi\|^2+\sum_{i=1}^n(\alpha_i+\beta_i)P_i\|\varphi-\psi\|^2 +$$

$$+\sum_{i=1}^n(\omega_i+\sigma_i)O_i\|\varphi-\psi\|^2 = \Bigg(\lambda_{\max}(S)+\max_{1\leq i\leq n}(\gamma_iH_i)+\sum_{i=1}^n(\alpha_i+\beta_i)P_i +$$

$$+\sum_{i=1}^n(\omega_i+\sigma_i)O_i\Bigg)\|\varphi-\psi\|^2,$$

where

$$P_i = \max_{1\leq j\leq n}\sup_{[-\tau,0]}\left[\frac{b_{ij}^2(\psi_{ij}^{-1}(s))}{1-\dot{\tau}_{ij}(\psi_{ij}^{-1}(s))}e^{\varepsilon(s+\tau_{ij}(\psi_{ij}^{-1}(s)))}\right]K_i^2\tau,$$

$$O_i = c^2L_i^2\frac{1}{\varepsilon}\max_{1\leq j\leq n}(p_{ij}(\varepsilon)-1).$$

Hence, we have $P_0>1$ such that $|z(t)|^2 \leq P_0^2\|\varphi-\psi\|^2e^{-\varepsilon t}$ for all $t\in\mathbb{R}^+$, that is

$$|x(t)-y(t)| \leq P_0\|\varphi-\psi\|e^{-\varepsilon t/2} \qquad \text{for all} \quad t\in\mathbb{R}^+.$$

This completes the proof of Theorem 1.

4. Corollaries. In this section, as some special cases of Theorem 1, we derive some corollaries which seems to be advantageous for the stability test.

***Corollary* 1.** *Assume that the hypotheses* (H_1), (H_2) *are satisfied. If there is a constant* $a>0$ *such that*

$$\lambda_{\min}\left[2D(t) - A(t)\eta - \eta A^T(t) - \sum_{i=1}^{n}(\overline{B}_i(t) + \overline{C}_i(t)) - 2nE\right] \geq a$$

for all $t \in \mathbb{R}^+$ *and* $0 \leq \eta \leq H$, *then the system* (1) *is globally exponentially stable.*

Proof. Choosing $S = \alpha = \omega = \delta E$ $(\delta > 0)$ and $\gamma = 0$, we obtain

$$D_1(t, \eta) = \delta\left[2D(t) - A(t)\eta - \eta A^T(t) - \sum_{i=1}^{n}(\overline{B}_i(t) + \overline{C}_i(t)) - 2nE\right] -$$

$$- \sum_{i=1}^{n}(\beta_i \overline{B}_i(t) + \sigma_i \overline{C}_i(t)).$$

According to Theorem 1, this corollary holds.

Corollary 2. *Assume that the hypotheses* (H_1), (H_2) *are satisfied. If* $A(t) + A^T(t) < 0$ *and there is a constant* $a > 0$ *such that*

$$\lambda_{\min}\left[2D(t) - (A(t) - D(t))\eta - \eta(A^T(t) - D(t)) - 2nE - \right.$$

$$\left. - \sum_{i=1}^{n}((1 - \alpha_i^*)\overline{B}_i(t) + (1 - \omega_i^*)\overline{C}_i(t))\right] \geq a$$

for all $t \in \mathbb{R}^+$ *and* $0 \leq \eta \leq H$, *then the system* (1) *is globally exponentially stable, here*

$$\alpha^* = \omega^* = n \inf_{t \in \mathbb{R}^+}\left[\lambda_{\min}\left(\frac{A(t) + A^T(t)}{2}\right)^{-1}\right] E.$$

Proof. By choosing $S = \beta = \gamma = \sigma = E$, we obtain

$$D_2(t, \eta) = 2D(t) - (A(t) - D(t))\eta - \eta(A^T(t) - D(t)) - 2nE -$$

$$- \sum_{i=1}^{n}((1 - \alpha_i^*)\overline{B}_i(t) + (1 - \omega_i^*)\overline{C}_i(t)).$$

Using Theorem 1, the Corollary 2 holds.

When $D(t) = D$, $A(t) = A$, $B(t) = B$, $C(t) = C$, $\tau_{ij}(t) = \tau$, $f = g = h$, $I(t) = I$ for all $t \in \mathbb{R}^+$, system (1) degenerates into

$$\frac{dx_i(t)}{dt} = -d_i x_i(t) + \sum_{j=1}^{n} a_{ij} g_j(x_j(t)) + \sum_{j=1}^{n} b_{ij} g_j(x_j(t - \tau)) +$$

$$+ \sum_{j=1}^{n} c_{ij} \int_{-\infty}^{t} k_{ij}(t - s) g_j(x_j(s)) ds + I_i, \quad i = \overline{1, n}. \tag{15}$$

We consider the following assumptions:

(H_1') $-(A+B+C) \in P_0$; functions $k_{ij}: [0, \infty) \to [0, \infty)$, $i, j = 1, \dots, n$, are piecewise continuous on $[0, \infty)$ and satisfy $\int_0^\infty e^{\varepsilon s} k_{ij}(s)\, ds = p_{ij}(\varepsilon)$, where $p_{ij}(\varepsilon)$ are continuous functions in $[0, \delta)$, $\delta > 0$, $p_{ij}(0) = 1$.

(H_2') There are positive constants H_i, $i = 1, \dots, n$, such that $0 \le \le \frac{g_i(u) - g_i(u^*)}{u - u^*} \le H_i$ for all $u, u^* \in \mathbb{R}$ and $i = 1, \dots, n$.

(H_3') There are a positive definite symmetric matrix S, diagonal matrices

$$\alpha = \operatorname{diag}(\alpha_1, \dots, \alpha_n) > 0, \quad \beta = \operatorname{diag}(\beta_1, \dots, \beta_n) > 0,$$

$$\omega = \operatorname{diag}(\omega_1, \dots, \omega_n) > 0, \quad \sigma = \operatorname{diag}(\sigma_1, \dots, \sigma_n) > 0,$$

$$\gamma = \operatorname{diag}(\gamma_1, \dots, \gamma_n) \ge 0$$

such that $\lambda_{\min}(D_1(\eta)) > 0$ for all $0 \le \eta \le H$,

$$D_1(\eta) = SD + DS - nS(\alpha^{-1} + \omega^{-1})S - n\eta\gamma(\beta^{-1} + \sigma^{-1})\gamma\eta - \\ - (SA - D\gamma)\eta - \eta(A^T S - \gamma D) - \eta(\gamma A + A^T\gamma)\eta - \\ - \sum_{i=1}^n [(\alpha_i + \beta_i)\overline{B}_i + (\omega_i + \sigma_i)\overline{C}_i]$$

where $\eta = \operatorname{diag}(\eta_1, \dots, \eta_n)$, $H = \operatorname{diag}(H_1, \dots, H_n)$, $\overline{B}_i = \operatorname{diag}(b_{i1}^2 H_1^2, \dots, b_{in}^2 H_n^2)$, $\overline{C}_i = \operatorname{diag}(c_{i1}^2 L_1^2, \dots, c_{in}^2 L_n^2)$, $i = 1, \dots, n$.

(H_4') There are a positive definite symmetric matrix S, diagonal matrices

$$\beta = \operatorname{diag}(\beta_1, \dots, \beta_n) > 0, \quad \sigma = \operatorname{diag}(\sigma_1, \dots, \sigma_n) > 0,$$

$$\gamma = \operatorname{diag}(\gamma_1, \dots, \gamma_n) > 0$$

such that $\gamma A + A^T \gamma < 0$, $\lambda_{\min}(D_2(\eta)) > 0$ for all $0 \le \eta \le H$, where

$$D_2(t, \eta) = SD + DS - nS(\beta^{-1} + \sigma^{-1})S - (SA - D\gamma)\eta - \\ - \eta(A^T S - \gamma D) - \sum_{i=1}^n [(\beta_i - \alpha_i^*)\overline{B}_i + (\sigma_i - \omega_i^*)\overline{C}_i],$$

$$\alpha^* = \omega^* = n\lambda_{\min}\left(\gamma \left(\frac{\gamma A + A^T \gamma}{2}\right)^{-1} \gamma\right) E.$$

We have some following results.

***Corollary* 3.** *Assume that the hypotheses* (H_1'), (H_2') *and* (H_3') *or* (H_4') *are satisfied, then the system* (15) *is absolutely exponentially stable with respect to the class* PLI.

Proof. By [13], the system (15) have a unique equilibrium point for any function $g \in PLI$ and any input vector I if and only if $-(A+B+C) \in P_0$. Hence, the Corollary 3 follows from Theorem 1.

Further, as consequence of Corollaries 1 and 2 we have the following corollaries.

***Corollary* 4.** *Assume that the hypotheses* (H_1'), (H_2') *are satisfied. If*

$$\lambda_{\min}\left[2D - A\eta - \eta A^T - \sum_{i=1}^{n}(\overline{B}_i + \overline{C}_i) - 2nE\right] > 0$$

for all $0 \leq \eta \leq H$, *then the system* (15) *is absolutely exponentially stable with respect to the class* PLI.

***Corollary* 5.** *Assume that the hypotheses* (H_1'), (H_2') *are satisfied. If* $A + A^T < 0$ *and*

$$\lambda_{\min}\left[2D - (A-D)\eta - \eta(A^T - D) - 2nE - \sum_{i=1}^{n}\left((1-\alpha_i^*)\overline{B}_i + (1-\omega_i^*)\overline{C}_i\right)\right] > 0$$

for all $0 \leq \eta \leq H$, *then the system* (15) *is absolutely exponentially stable with respect to the class* PLI.

5. Conclusions. In this paper, the general neural networks with variable and unbounded time delays have been studied. We introduce two new important assumptions (H_3), (H_4) to ensure the global exponential stability of the systems. The results obtained in this paper are new and completely different from that given in [3, 6 – 8]. Comparing with [2], the results in this article improve and extend those results of [2] in many aspects. Here, the Lyapunov functional is a scale function. It shows that we can use scale Lyapunov functionals to study CNNs with variable and unbounded delays (see [7] for a criticism this method).

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