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# SPATIAL STRATEGIES OF FIRMS UNDER STACKELBERG COMPETITION

Abstract. The paper explores a spatial duopoly of firms under Stackelberg competition, when one of the firms is a leader in term of product volume. The equilibrium spatial strategies of firms are found. In the process of the analysis of equilibrium stability, it is proved that the transport tariff is a bifurcation parameter for firms. It was found that the change in the central agglomeration strategy to the differentiation strategy occurs at the point of transcritical bifurcation. The conditions for full coverage of the markets for both strategies are defined. It is obtained that the information asymmetry leads to asymmetry of equilibrium locations of firms. It is established that under differentiation strategy, the follower can due to the space factor overcome information asymmetry in the nearest markets and get a profit more than the leader.

Key words: linear city, agglomeration, differentiation, Stackelberg competition, transcritical bifurcation.

#### Introduction

In search of a solution to the Bertrand paradox, Hotelling proposed to take into account the factor of space under the price competition of firms. In Hotelling's linear city model (Hotelling, 1929), two firms compete on a segment with a unit demand at each point. Firms optimize their prices and location on the segment. Transportation delivery costs of goods are borne by consumers. Hotelling found that in an equilibrium state, firms would be minimally spatially differentiated, since they would be located in the center. This conclusion of the model analysis subsequently became a famous "principle of minimal differentiation".

In further research, the Hotelling model has developed in the following areas:

- an increase in the number of firms (Brenner, 2005, Patri and Sacco, 2017);
- increase the dimension of space (Irmen and Thisse, 1998, Mazalov and Sakaguchi, 2003);
- the complexity of the type of transport costs function (D'Aspremont, Gabszewicz and Thisse, 1979, Economides, 1986);
- generalization of the consumer's distribution density (Neven, 1986, Gupta,
   Pal and Sarkar, 1997, Tabuchi and Thisse, 1995);
- consideration of the Cournot competition (Hamilton, Klein, Sheshinski and Slutsky, 1994, Scrimitore, 2011, Hamilton, Thisse and Weskamp, 1989) and Stackelberg competition (Anderson, 1987).

In the Anderson (1987), a linear city model is investigated under Stackelberg competition, when firms optimize their locations and prices. It is found that the outcome is asymmetric in terms of Stackelberg equilibrium locations, prices and profits. In this paper, we investigate a linear city model in the framework of Stackelberg competition, when firms optimize their locations and supply volumes.

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## 1. The linear city model

Two firms sell homogeneous goods on the unit segment, at each point of which is the consumer market x,  $x \in [0,1]$ . The distance of the firms from zero point is equal  $x_1$  and  $x_2$  accordingly, and  $x_1 \le x_2$ . Each firm faces linear transportation costs of t to move one good unit per one unit of distance. Consumer arbitrage is assumed to be prohibitively costly.

The linear demand curve in the market x:

$$p(x)=1-q_1(x)-q_2(x),$$

where p(x) – the price in the market x,  $q_1(x)$ ,  $q_2(x)$  – the quantities supplied of firms in the market x, a minimum price, at which there is no demand (market potential), is equal to 1.

Let us assume that firms supply products to all markets, i.e.  $q_1(x) > 0$ ,  $q_2(x) > 0$ .

The profits of firms in the market x:

$$F_1(x) = q_1(x) \cdot (1 - q_1(x) - q_2(x) - t \cdot |x - x_1|) \rightarrow \max_{x_1, q_1(x)},$$

$$F_2(x) = q_2(x) \cdot (1 - q_1(x) - q_2(x) - t \cdot |x - x_2|) \rightarrow \max_{x_1, q_2(x)}$$

The competitive game consists of two stages. In the first stage, the firms simultaneously select their locations. In the second stage, at the given location decisions, the firms simultaneously choose their supplied quantities. The equilibrium of the model is solved by backward induction.

# 2. The Nash equilibrium

According to the backward induction method we begin with the second stage. Let us assume that firms optimize supply volumes under the conditions of the Stackelberg information asymmetry. The firm 2 (leader) knows the strategy of the firm 1 regarding the supply volume and has the right of first move. The firm 1 (follower) does not possess such information and makes decisions after leader.

Solving the first-order condition yields the reaction curve of the firm-follower:

$$q_1(x) = \frac{1 - q_2(x) - t \cdot |x - x_1|}{2}.$$
 (1)

The firm-leader knows the follower's reaction curve (1) and incorporates it into his profit function:

$$F_{2}(x) = \frac{q_{2}(x) \cdot (1 - q_{2}(x) - 2 \cdot t \cdot |x - x_{2}| + t \cdot |x - x_{1}|)}{2} \rightarrow \max_{x_{2}, q_{2}(x)}.$$

The equilibrium supply volumes of firms to the market x:

$$q_1^*(x) = \frac{1 - 3 \cdot t \cdot |x - x_1| + 2 \cdot t \cdot |x - x_2|}{4},$$

$$q_2^*(x) = \frac{1 - 2 \cdot t \cdot |x - x_2| + t \cdot |x - x_1|}{2}.$$

A coverage conditions for all markets:

$$q_1^*(x) > 0 \iff t < \frac{1}{3 \cdot |x - x_1| - 2 \cdot |x - x_2|},$$
 (2)

$$q_2^*(x) > 0 \iff t < \frac{1}{2 \cdot |x - x_2| - |x - x_1|}.$$
 (3)

Substituting into (2) the values:  $x_1 = 0$ ,  $x_2 = 1$ , x = 1 (maximum value of the denominator), we find that at t < 1/3 all markets are serviced regardless of the location of firms. Substituting into (2) the values:  $x_1 = 1/2$ ,  $x_2 = 1/2$ , x = 0 (minimum value of the denominator), we find that at any location of firms servicing all markets is possible only at t < 2. Let us note that in the equilibrium state, the location of firms depends on the transport tariff. Therefore, the analysis of conditions (2)-(3) will be continued after finding equilibrium.

The equilibrium profits of firms in the market x:

$$F_{1}^{*}(x) = \frac{\left(1 - 3 \cdot t \cdot |x - x_{1}| + 2 \cdot t \cdot |x - x_{2}|\right)^{2}}{16},$$

$$F_{2}^{*}(x) = \frac{\left(1 - 2 \cdot t \cdot |x - x_{2}| + t \cdot |x - x_{1}|\right)^{2}}{8}.$$
(4)

It follows from (4) that the ratio between the profits of the leader and follower in the market x depends on their location relative to the market x. In the case of agglomeration or symmetric location relative to the market x, the leader's profit exceeds the follower's profit exactly 2 times, how in the classical model with one market. A closer distance to the market x strengthens the market power of the leader and vice versa:

$$sign(F_2^*(x) - 2 \cdot F_1^*(x)) = sign(|x - x_1| - |x - x_2|).$$

In the first stage each firm selects a profit-maximizing location at a given location of the competitor. In the subsequent analysis, we assume that the equilibrium location of the firms obeys the condition:

$$x_1 \le 1/2, \ x_2 \ge 1/2.$$
 (5)

So, let us start with firm 1. The total profit of firm 1 in all markets:

$$F_{1} = \int_{0}^{1} F_{1}^{*}(x) dx = \int_{0}^{x_{1}} F_{1}^{*}(x) dx + \int_{x_{1}}^{x_{2}} F_{1}^{*}(x) dx + \int_{x_{2}}^{1} F_{1}^{*}(x) dx,$$

$$16 \cdot F_{1} = \int_{0}^{x_{1}} (1 - 3 \cdot t \cdot (x_{1} - x) + 2 \cdot t \cdot (x_{2} - x))^{2} dx + \int_{x_{1}}^{x_{2}} (1 - 3 \cdot t \cdot (x - x_{1}) + 2 \cdot t \cdot (x_{2} - x))^{2} dx + \int_{x_{1}}^{1} (1 - 3 \cdot t \cdot (x - x_{1}) + 2 \cdot t \cdot (x - x_{2}))^{2} dx + \int_{x_{1}}^{1} (1 - 3 \cdot t \cdot (x - x_{1}) + 2 \cdot t \cdot (x - x_{2}))^{2} dx.$$
 (6)

After integrating and identical transformations (6), we obtain:

$$\begin{split} 16 \cdot F_1 &= \frac{2 \cdot \left(1 - 2 \cdot t \cdot x_1 + 2 \cdot t \cdot x_2\right)^3}{5 \cdot t} - \frac{\left(1 - 3 \cdot t \cdot x_1 + 2 \cdot t \cdot x_2\right)^3}{3 \cdot t} + \\ &+ \frac{4 \cdot \left(1 + 3 \cdot t \cdot x_1 - 3 \cdot t \cdot x_2\right)^3}{15 \cdot t} - \frac{\left(1 - t + 3 \cdot t \cdot x_1 - 2 \cdot t \cdot x_2\right)^3}{3 \cdot t} \,. \end{split}$$

The optimal location is defined by the necessary condition:

$$\frac{16}{3 \cdot t} \cdot \frac{\partial F_1}{\partial x_1} = 4 \cdot t \cdot x_1^2 - 2 \cdot x_1 \cdot (2 + 4 \cdot t \cdot x_2 - 3 \cdot t) + 4 \cdot t \cdot x_2 \cdot (x_2 - 1) + 2 - t = 0. \quad (7)$$

The sufficient condition for the existence of profit maximum for the firm 1:

$$\frac{8}{3 \cdot t} \cdot \frac{\partial^2 F_1}{\partial x_1^2} = 4 \cdot t \cdot x_1 - 2 - 4 \cdot t \cdot x_2 + 3 \cdot t < 0 \quad \Leftrightarrow \quad x_1 < x_2 - \frac{3 \cdot t - 2}{4 \cdot t}. \tag{8}$$

The necessary condition for the existence of the equilibrium location for firm 1 is the nonnegativity of the discriminant of the square equation (7):

$$D_1 = 4 \cdot (t^2 \cdot (13 - 8 \cdot x_2) - t \cdot (20 - 16 \cdot x_2) + 4) \ge 0.$$
 (9)

It is easy to make sure that  $D_1 \ge 0$  at  $x_2 \ge 1/2$ . Therefore, due to condition (5), in the equilibrium state the discriminant (9) is always nonnegative.

Let us note that information asymmetry about location of firms can lead to monopolization of the city. With the right of first move, firm 2 (leader) can will located in the market  $x_2 < 1/2$ , create a barrier to entry for firm 1 and monopolize all markets. Therefore, we assume that firm 2 is the leader only in the second stage, and in the first stage, when choosing a location, firms compete under Cournot model.

The roots of the square equation (7) are:

$$(x_1^*)_1 = x_2 - \frac{3 \cdot t - 2}{4 \cdot t} - \frac{\sqrt{D_1}}{8 \cdot t}, \quad (x_1^*)_2 = x_2 - \frac{3 \cdot t - 2}{4 \cdot t} + \frac{\sqrt{D_1}}{8 \cdot t}.$$

The root  $(x_1^*)_2$  does not satisfy the sufficient condition (8) and therefore is not further analyzed. The root  $(x_1^*)_1$  for  $x_2 > 1/2$  always satisfies the sufficient condition (8), for  $x_2 = 1/2$  the condition (8) holds for:

$$\frac{\sqrt{D_1}}{8 \cdot t} > 0 \quad \Leftrightarrow \quad \frac{\sqrt{(3 \cdot t - 2)^2}}{4 \cdot t} > 0 \quad \Leftrightarrow \quad t \neq 2/3.$$

The total profit of firm 2 in all markets:

$$F_2 = \int_0^1 F_2^*(x) dx = \int_0^{x_1} F_2^*(x) dx + \int_{x_2}^{x_2} F_2^*(x) dx + \int_{x_2}^1 F_2^*(x) dx.$$

$$8 \cdot F_{2} = \int_{0}^{x_{1}} (1 - 2 \cdot t \cdot (x_{2} - x) + t \cdot (x_{1} - x))^{2} dx +$$

$$+ \int_{x_{1}}^{x_{2}} (1 - 2 \cdot t \cdot (x_{2} - x) + t \cdot (x - x_{1}))^{2} dx +$$

$$+ \int_{x_{1}}^{1} (1 - 2 \cdot t \cdot (x - x_{2}) + t \cdot (x - x_{1}))^{2} dx . (10)$$

After integrating and identical transformations (10), we obtain:

$$\begin{split} 8 \cdot F_2 &= \frac{2 \cdot \left(1 - 2 \cdot t \cdot x_2 + 2 \cdot t \cdot x_1\right)^3}{9 \cdot t} - \frac{\left(1 - 2 \cdot t \cdot x_2 + t \cdot x_1\right)^3}{3 \cdot t} + \\ &+ \frac{4 \cdot \left(1 + t \cdot x_2 - t \cdot x_1\right)^3}{9 \cdot t} - \frac{\left(1 - t + 2 \cdot t \cdot x_2 - t \cdot x_1\right)^3}{3 \cdot t} \,. \end{split}$$

The optimal location is defined by the necessary condition:

$$\frac{2}{t} \cdot \frac{\partial F_2}{\partial x_2} = 2 \cdot (x_2 - x_1) - t \cdot (x_2 - x_1)^2 - (2 - t) \cdot (2 \cdot x_2 - x_1 - 1/2) = 0.$$
 (11)

The sufficient condition for the existence of profit maximum for the firm 2:

$$\frac{1}{t} \cdot \frac{\partial^2 F_2}{\partial x_2^2} = t \cdot (1 + x_1 - x_2) - 1 < 0 \quad \Leftrightarrow \quad x_2 > x_1 + \frac{t - 1}{t}. \tag{12}$$

The necessary condition for the existence of the equilibrium location for firm 2 is the nonnegativity of the discriminant of the square equation (11):

$$D_2 = 4 \cdot ((t-1)^2 + t \cdot (2-t) \cdot (1/2 - x_1)) \ge 0.$$
 (13)

It is easy to make sure that  $D_2 \ge 0$  at  $x_1 \le 1/2$ . Therefore, due to condition (5), in the equilibrium state, the discriminant (13) is always nonnegative.

The roots of the square equation (11) are:

$$(x_2^*)_1 = x_1 + \frac{t-1}{t} - \frac{\sqrt{D_2}}{2 \cdot t}, \quad (x_2^*)_2 = x_1 + \frac{t-1}{t} + \frac{\sqrt{D_2}}{2 \cdot t}.$$

The root  $(x_2^*)_1$  does not satisfy the sufficient condition (12) and therefore is not further analyzed. The root  $(x_2^*)_2$  for  $x_1 < 1/2$  always satisfies the sufficient condition (12), for  $x_1 = 1/2$  the condition (12) holds for:

$$\frac{\sqrt{D_2}}{2 \cdot t} > 0 \quad \Leftrightarrow \quad \frac{\sqrt{(t-1)^2}}{t} > 0 \quad \Leftrightarrow \quad t \neq 1.$$

Thus, we received the reaction curves of firms:

$$x_1 = x_2 + \frac{2 - 3 \cdot t - \sqrt{t^2 \cdot (13 - 8 \cdot x_2) - t \cdot (20 - 16 \cdot x_2) + 4}}{4 \cdot t},$$
(14)

$$x_2 = x_1 + \frac{t - 1 + \sqrt{(t - 1)^2 + t \cdot (2 - t) \cdot (1/2 - x_1)}}{t}.$$
 (15)

To solve the system of equations (14)-(15) we introduce a new variable:

$$w = \sqrt{t^2 \cdot \left(13 - 8 \cdot x_2\right) - t \cdot \left(20 - 16 \cdot x_2\right) + 4} \; , \; \; w \ge 0 \; .$$

Then we obtain from (14):

$$x_1 = x_2 + \frac{2 - 3 \cdot t - w}{4 \cdot t}, \quad x_2 = \frac{w^2 - 13 \cdot t^2 + 20 \cdot t - 4}{8 \cdot t \cdot (2 - t)}.$$
 (16)

Substituting (16) into (15), and solving with respect to w, we obtain:

$$w_1 = 2 - 3 \cdot t$$
,  $w_2 = \frac{7 \cdot t - 2}{3}$ . (17)

Substituting (17) into (16), we find solutions of the system (14)-(15):

$$x_1^{agg} = x_2^{agg} = 1/2, (18)$$

$$x_1^{dis} = \frac{8 - 7 \cdot t}{18 \cdot t}, \quad x_2^{dis} = \frac{17 \cdot t - 4}{18 \cdot t}.$$
 (19)

So, we obtained two equilibrium strategies for the location of firms: central agglomeration and dispersion.

It follows from condition  $w \ge 0$  that the solution (18) is defined for  $t \le 2/3$ , the solution (19) is defined for  $t \ge 2/7$ . For t = 1/2, the solutions (18) and (19) coincide. From the location condition,  $x_1 \le x_2$ , it follows that firms can apply the dispersion strategy only when  $t \ge 1/2$ .

Let us finish off an analysis of the market coverage conditions (2)-(3). Under central agglomeration strategy, firms will be able to serve all markets on condition:

$$t < \frac{1}{3 \cdot \left| x - x_1^{agg} \right| - 2 \cdot \left| x - x_2^{agg} \right|} \quad \Leftrightarrow \quad t < 2.$$

Under dispersion strategy, firms will be able to serve all markets on condition:

$$t < \frac{1}{3 \cdot \left| x - x_1^{dis} \right| - 2 \cdot \left| x - x_2^{dis} \right|} \iff t < \frac{1}{3 \cdot \left( 1 - x_1^{dis} \right) - 2 \cdot \left( 1 - x_1^{dis} \right)} \iff t < 50/73.$$

Under central agglomeration strategy the firms minimize a total distance of traffic, therefore full market coverage may be possible with a higher transport tariff.

In previous studies (Gupta, Pal and Sarkar, 1997, Hamilton, Klein, Sheshinski and Slutsky, 1994, Hamilton, Thisse and Weskamp, 1989) it is proved that firms in the Cournot equilibrium are always located symmetrically with respect to the center. We have obtained that the information asymmetry about supply volumes will lead to the asymmetry of equilibrium location of firms. It follows from (19)

that in the equilibrium state the firm-leader is 2 times closer to the center than the firm follower:

$$1/2 - x_1^{dis} = 2 \cdot (x_2^{dis} - 1/2).$$

## 3. The analysis of the stability of equilibrium

Let us analyze a stability of the solutions (18)-(19). For this we consider a twodimensional map:

$$x_{1}(n+1) = x_{2}(n) + \frac{2 - 3 \cdot t - \sqrt{t^{2} \cdot (13 - 8 \cdot x_{2}(n)) - t \cdot (20 - 16 \cdot x_{2}(n)) + 4}}{4 \cdot t},$$

$$x_{2}(n+1) = x_{1}(n) + \frac{t - 1 + \sqrt{(t-1)^{2} + t \cdot (2 - t) \cdot (1/2 - x_{1}(n))}}{t},$$
(20)

where *n* is a time moment,  $n = 0, 1, 2, ..., x_1(0) = 0, x_2(0) = 1$ .

As is known, the nature of the stability of fixed points is determined by their multipliers. The multipliers are eigenvalues of the Jacobian matrix in a fixed point, and their number is equal to the dimension of map.

The Jacobian matrix of the map (20) in the fixed point (18):

$$J = \begin{pmatrix} 0 & -\frac{2 \cdot t}{2 - 3 \cdot t} \\ -\frac{t}{2 \cdot (1 - t)} & 0 \end{pmatrix}. \tag{21}$$

From (21) we obtain two real multipliers:

$$\mu_{1,2} = \pm \sqrt{\frac{t^2}{(1-t)\cdot(2-3\cdot t)}}, \quad t < 2/3.$$
(22)

For  $|\mu_{1,2}| < 1$  the fixed point is stable, for  $|\mu_{1,2}| > 1$  the fixed point is unstable, for  $|\mu_{1,2}| = 1$  the bifurcation occurs.

From (22) we find that the fixed point (18) is stable at t < 1/2 and is unstable at t > 1/2. The loss of stability occurs at the bifurcation point: t = 1/2.

The Jacobian matrix of the map (20) in a fixed point (19):

$$J = \begin{pmatrix} 0 & \frac{10 \cdot t - 8}{7 \cdot t - 2} \\ \frac{5 \cdot t - 4}{2 \cdot (t + 1)} & 0 \end{pmatrix}. \tag{23}$$

From (23) we obtain two real multipliers:

$$\mu_{1,2} = \pm \sqrt{\frac{(5 \cdot t - 4)^2}{(t+1) \cdot (7 \cdot t - 2)}}, \quad t > 2/7.$$
(24)

From (24) we find that the fixed point (19) is unstable at t < 1/2 and is stable at t > 1/2. The acquisition of stability occurs at the bifurcation point: t = 1/2.

Thus, at the value of the transport tariff t = 1/2, occurs a transcritical bifurcation, in which fixed points change a nature of stability (Fig. 1).

The equilibrium profit dynamics of the firm-leader, depending on the transport tariff, is presented in Fig. 2.

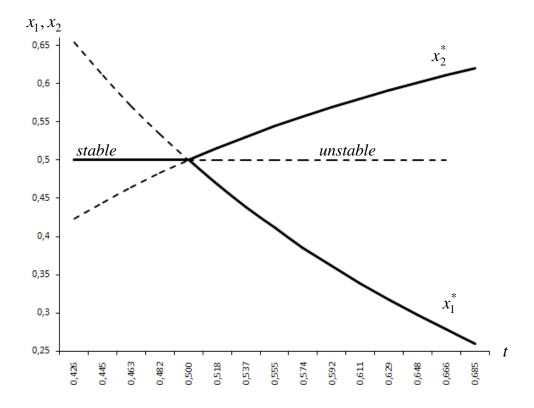


Fig. 1 – Equilibrium spatial strategies of firms depending on transportation tariffs

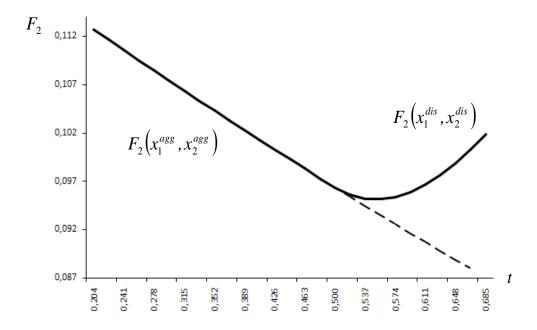


Fig. 2 – Dynamics of the equilibrium profit of the firm-leader, depending on the transport tariff

The Fig. 2 illustrates the effects that affect spatial strategies of firms. Before bifurcation point the effect of minimizing transport costs is dominate (Scrimitore, 2011). Firms choose the central agglomeration strategy to minimize a total distance of transportation. The growth of the transport tariff leads to a decrease in the total profit. Due to information asymmetry, the leader's profit in all markets is twice as high as the profit of the follower (Fig. 3).

In the bifurcation point begins to dominate the effect of reducing competition. Firms choose a differentiation strategy to monopolize adjacent markets. The growth of the transport tariff leads to an increase in total profits. The growth of total profit with growth of the transport tariff is due to the fact that when differentiation strategy, the firms supply more to adjoining markets and less to distant markets. Due to information asymmetry, the leader almost monopolizes the markets to the right of himself and at the same time is present in the markets to the left of the follower. This is clearly seen in Fig. 4. In the equilibrium state, for t=2/3, the leader in the follower "territory",  $x \in \left[0, x_1^{dis}\right]$ , receives 19,2% of the total profit both firms, and the follower in the leader "territory",  $x \in \left[x_2^{dis}, 1\right]$ , receives only 1,9% of the total profit both firms.

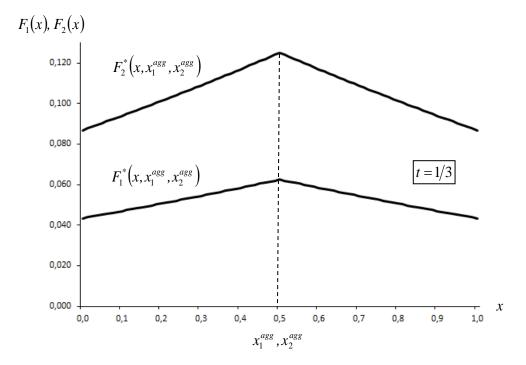


Fig. 3 – Equilibrium profits of firms under central agglomeration

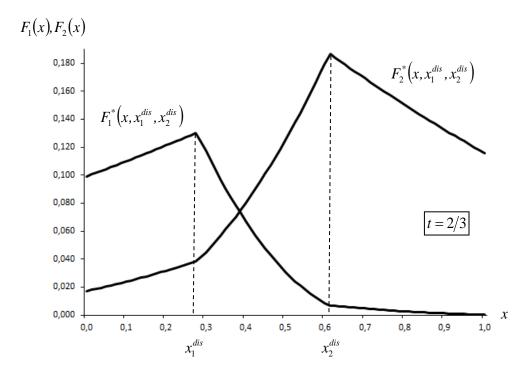


Fig. 4 – Equilibrium profits of firms under differentiation

## **Conclusions**

The paper explores the spatial duopoly of firms under Stackelberg competition, when one of the firms is the leader in term of supply volume. The equilibrium spatial strategies of firms are found. In the process of the analysis of equilibrium stability, it is proved that the transport tariff is a bifurcation parameter for firms. It was found that the change in the central agglomeration strategy to the differentiation strategy occurs at the point of transcritical bifurcation. The conditions for full coverage of the markets for both strategies are defined. It is obtained that the information asymmetry leads to asymmetry of equilibrium locations of firms. It is established that under differentiation strategy, the follower can due to the space factor overcome information asymmetry in the nearest markets and get a profit more than the leader. It is proved that spatial differentiation enhances a market power of firms and allows monopolization of neighboring markets.

The purpose of further research is to analyze the competitive interaction of firms in the Hotelling's linear city model under the conditions of other asymmetry types.

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