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STRESSED STATE IN A FINITE CYLINDER WITH A CIRCULAR CRACK AT NON- STATIONARY TORSION

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This paper considers a solution to an axially symmetric dynamic problem of determining the stress-state in the vicinity of a circular crack in a finite cylinder. The cylinder lower base is rigidly fixed, and the upper one is loaded with time-dependent tangential stresses. In contrast to the traditional analytical methods based on the use of the integral Laplace transform, the proposed one consists in the difference approximation of only the time derivative. To do this, specially selected unequally spaced nodes and a special representation of the solution in these nodes are used. Such an approach allows the initial problem to be reduced to a sequence of boundary problems for the homogeneous Helmholtz equation. Each such problem is solved by applying the finite Fourier and Hankel integral transforms with their subsequent inversion. As a result, an integral representation was obtained for the angular displacement through an unknown displacement jump in the crack plane. With regard to the derivative of this jump from the boundary condition on the crack, an integral equation was obtained which, as a result of the integral Weber-Sonin operator application and a series of transformations, was reduced to the Fredholm integral equation of the second kind regarding the unknown function associated with the jump. An approximate solution of this equation was carried out by the method of collocations, with the integrals being approximated by quadratic Gaussian-Legendre formulas. The numerical solution found made it possible to obtain an approximate formula for calculating the stress intensity factor (SIF). Using this formula, we studied the effect of the nature of the load and the geometric parameters of the cylinder on the time dependence of this factor. The analysis of the results showed that for all the types of loading considered, the maximum value of SIF can be observed during the transient process. When a sudden, constant load is applied, this maximum is 2-2.5 times higher than the static value. In the case of a sudden harmonic load, SIF maximum also significantly exceeds the values it acquires with steady-state oscillations, in the absence of resonance. Increasing the cylinder height and reducing the crack area result in an increase in the duration of the transient process and a decrease in the value of SIF maximum. The same effect can be observed when the crack plane approaches the stationary end of the cylinder.

Keywords: stress intensity coefficient (SIF), axially symmetric dynamic problem, finite differences, finite cylinder, circular crack, torque moment.

Introduction

A large number of machine elements and structures are cylindrical in shape. Their having such defects as cracks significantly reduces their operational characteristics and can lead to destruction, especially under the conditions of dynamic loading. Therefore, the study of stress distribution in cylindrical bodies with cracks under dynamic loading is an important task.

The analysis of modern scientific literature shows that the stress state of finite and infinite cylindrical bodies with cracks under static loading has been studied sufficiently enough. Examples of solving such problems are given in [1–6]. In solving dynamic tasks, basically, unbounded bodies with cracks were considered, most often circular. A detailed analysis of the results is presented in papers [7, 8]. As far as harmonic oscillations are concerned, there are a number of works, for example [9, 10], where circular cracks are considered in plates and infinite cylinders, and [11, 12] where circular and ring cracks are considered in a finite cylinder.

It is also proposed to use mixed numerical-experimental methods to determine the stress intensity factors in cylindrical bodies with outer ring cracks [13, 14]. But these methods, like all the experimental ones, are characterized by the disadvantages associated with the need to carry out experiments for each particular sample. This complicates the study of the impact of cylinder geometric dimensions on CIF values.

The complexity of theoretical studies of dynamic problems is due to the necessity of using the Laplace integral transform in time with subsequent numerical inversion. However, this task is not only mathematically complicated, but incorrect. Recently, there have appeared works in which the modified method of finite difference in time is applied. With the help of this method, this paper solves the problem of determining SIF in the vicinity of the plane circular crack in a finite cylinder under torsional loading. So far, such a problem has been considered only in a stationary formulation [15], for a harmonic moment [11] or for a cylinder with a cover plate [16].

Problem Formulation

An isotropic finite elastic cylinder with height a and radius r_0 is considered (Fig. 1). The cylinder is related to the cylindrical system of coordinates, whose center coincides with the center of the lower base, and the Oz axis with the cylinder axis. The lower base is considered to be stationary, and to the upper one at the initial time $t = 0$ is under the tangent load $G \cdot \bar{p}(r, t)$. At a height of $z=c$, $0 < c < a$, in parallel with the cylinder ends, there is a circular crack of radius $b < r_0$, whose center is on the axis. Both the cylinder side surface and crack surface are considered to be free of stresses. Under these conditions, the cylinder is in a state of axisymmetric torsional deformation and only the angular displacement $\bar{w}(r, z, t)$ will be different from 0.

Next, in order to formulate the initial boundary-value problem, it is expedient to pass on to dimensionless quantities using the formulas

$$\bar{w}(r_0\eta, a\zeta, r_0\tau/c_2) = r_0 \cdot w(\eta, \zeta, \tau), \quad \gamma = a/r_0, \quad l = c/a, \quad \beta = b/r_0, \\ r = r_0\eta, \quad z = a\zeta, \quad 0 \leq \eta, \zeta \leq 1, \quad t = r_0\tau/c_2, \quad \tau \in (0, +\infty), \quad c_2^2 = G \cdot \rho^{-1},$$

where ρ , G is the density and shear moduli for the cylinder material.

Then, dimensionless displacement will satisfy the equation

$$D_{\eta\zeta} w = \frac{\partial^2 w}{\partial \tau^2}, \quad D_{\eta\zeta} = \frac{\partial^2}{\partial \eta^2} + \frac{1}{\eta} \frac{\partial}{\partial \eta} - \frac{1}{\eta^2} + \frac{1}{\gamma^2} \frac{\partial^2}{\partial \zeta^2}. \quad (1)$$

Equation (1) is considered as having zero initial conditions.

We formulate boundary conditions in relative dimensionless quantities.

On the cylinder ends, they have the form

$$w|_{\zeta=0} = 0, \quad \left. \frac{\partial w}{\partial \zeta} \right|_{\zeta=1} = \gamma p(\eta, \tau), \quad 0 \leq \eta \leq 1, \quad \tau \in [0, +\infty), \quad (2)$$

where $p(\eta, \tau) = \bar{p}(r/r_0, c_2 t/r_0)$.

On the lateral surface of the cylinder, there must be fulfilled the equality

$$\tau_{\phi r}(1, \zeta, \tau) = 0, \quad 0 \leq \zeta \leq 1, \quad \tau \in [0, +\infty). \quad (3)$$

For the conditions on the crack we have

$$\tau_{\phi z}(\eta, l, \tau) = 0, \quad 0 \leq \eta \leq \beta, \quad \tau \in [0, +\infty), \quad \chi(\eta, \tau) = \tilde{\chi}(r/r_0, c_2 \tau/r_0), \quad (4)$$

where $\chi(\eta, \tau) \equiv 0$, $\eta \geq \beta$, and $\tilde{\chi}(r, t)$ is an unknown jump of displacements in the plane of the crack.

To solve the formulated initial-boundary problem (1) – (4), we apply a method based on the difference approximation of time derivatives, detailed in [17]. For this purpose, we create a time grid

$$\tau_k = \sum_{v=1}^k h_v, \quad h_v = \tau_k - \tau_{k-1}, \quad (\tau_0 = 0), \quad k = 1, 2, 3, \dots \quad h_i \neq h_j.$$

We introduce the designation $w(\eta, \zeta, \tau_k) = w_k(\eta, \zeta)$ and use the left difference time derivatives. Then, from the initial conditions, we find $w_0(\eta, \zeta) = 0$ and from equation (1) we find the following differential equations:

$$D_{\eta\zeta} w_1 - \frac{w_1}{h_1^2} = 0, \quad D_{\eta\zeta} w_k - \frac{w_k}{h_k^2} = \frac{w_{k-2}}{h_k h_{k-1}} - \frac{w_{k-1}}{h_k} \left(\frac{1}{h_k} + \frac{1}{h_{k-1}} \right), \quad k = 2, 3, \dots \quad (5)$$

The analytical solution to equations (5) is considerably complicated by the fact that on the right side there are displacement values in the two previous moments of time. To avoid these difficulties, according to [17], we write the angular displacement and stress in the form of a linear combination of new functions

$$w_k = \sum_{v=1}^k C_{kv} U_v, \quad \tau_{\phi rk} = \sum_{v=1}^k C_{kv} \tau_{\phi rv}, \quad \tau_{\phi zk} = \sum_{v=1}^k C_{kv} \tau_{\phi zv}, \quad p_k = \sum_{v=1}^k C_{kv} P_v, \quad (6)$$

where U_v is a new unknown function.

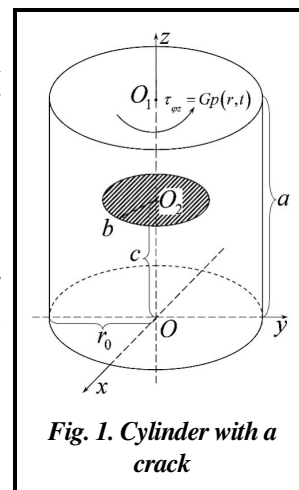


Fig. 1. Cylinder with a crack

In [17], it is shown that if we choose the coefficients in formulas (6) according to the formulas

$$C_{kk} = 1, \quad k = 1, 2, 3, \dots, \quad C_{k,k-1} = \frac{h_{k-1}}{h_{k-1} - h_k}, \quad k = 2, 3, \dots,$$

$$C_{k,v} = \frac{h_v^2}{h_k^2 - h_v^2} \cdot \left(\frac{h_k}{h_{k-1}} C_{k-2,v} - \left(1 + \frac{h_k}{h_{k-1}} \right) C_{k-1,v} \right), \quad k = 3, 4, \dots; \quad v = 1, 2, \dots, k-2,$$

then the functions $U \pm$ satisfy the homogeneous Helmholtz equations

$$D_{\eta\zeta} U_v - \kappa^2 U_v = 0, \quad v = 1, 2, 3, \dots, \quad \kappa_v = h_v^{-1} \quad (7)$$

The boundary conditions on the cylinder surfaces with respect to these functions can be written as follows:

$$U_v|_{\zeta=0} = 0, \quad \frac{\partial U_v}{\partial \zeta} \Big|_{\zeta=1} = \gamma P_v, \quad \tau_{\phi,rv} \Big|_{\eta=1} = 0. \quad (8)$$

The conditions for the crack will take the form

$$\tau_{\phi,zv} \Big|_{\zeta=l} = 0, \quad 0 \leq \eta \leq \beta, \quad \langle U_v \rangle \Big|_{\zeta=l} = \chi_v(\eta), \quad \chi_v(\eta) \equiv 0, \quad \eta \geq \beta, \quad \chi_k = \sum_{v=1}^k C_{kv} \chi_v. \quad (9)$$

Reducing the Problem to an Integral Equation and its Solution

We represent the solution to the resultant boundary-value problem (7), (8), (9) as the sum

$$U_v(\eta, \zeta) = U_v^0(\eta, \zeta) + U_v^1(\eta, \zeta).$$

The first term is the solution to the problem in the absence of crack. It satisfies the conditions on the cylinder ends and lateral surface and is given by the formula

$$U_v^0(\eta, \zeta) = \int_0^{\infty} \frac{\gamma \operatorname{sh}(\alpha_v \zeta)}{\alpha_v \operatorname{ch}(\alpha_v)} \overline{P}_v(\lambda) \lambda J_1(\eta \lambda) dt,$$

where $\alpha_v^2 = \gamma^2(t^2 + \kappa_v^2)$, $\overline{P}_v(\lambda) = \int_0^{\infty} \eta P_v(\eta) J_1(\eta \lambda) d\eta$.

The second term is the solution to equation (7). It satisfies the zero conditions on the cylinder ends and lateral surface

$$U_v^1 \Big|_{\zeta=0} = 0, \quad \frac{\partial U_v^1}{\partial \zeta} \Big|_{\zeta=1} = 0, \quad \tau_{\phi,rv} \Big|_{\eta=1} = 0,$$

where as on the surface of the crack it is discontinuous with a jump (9) and satisfies the conditions

$$\tau_{\phi,zv}^1(\eta, l) = -\tau_{\phi,zv}^0(\eta, l), \quad 0 \leq \eta \leq \beta, \quad \langle U_v^1 \rangle \Big|_{\zeta=l} = \chi_v(\eta), \quad \chi_v(\eta) \equiv 0, \quad \eta \geq \beta. \quad (10)$$

The solution to this boundary value problem is constructed by the integral transform method, analogous to papers [11, 16], and it has the form

$$U_v^1(\eta, \zeta) = \int_0^1 \xi \chi_v(\xi) [S(\xi, \eta, \zeta) + D(\xi, \eta, \zeta)] d\xi,$$

where

$$S(\xi, \eta, \zeta) = \frac{1}{2} \int_0^{\infty} \lambda J_1(\eta \lambda) J_1(\xi \lambda) [F(\lambda, \zeta - l) + F(\lambda, \zeta + l)] d\lambda,$$

$$D(\xi, \eta, \zeta) = 2 \sum_{j=1}^{\infty} \frac{\lambda_j}{\gamma^2} \cos \lambda_j l \cdot \sin \lambda_j \zeta \cdot \frac{K_2(q_{jv})}{I_2(q_{jv})} I_1(q_{jv} \xi) I_1(q_{jv} \eta),$$

$$F(\lambda, \zeta \pm l) = \operatorname{sgn}(\zeta \pm l) \frac{\operatorname{sh}(\alpha_v(1 - |\zeta \pm l|))}{\operatorname{sh} \alpha_v},$$

$$\alpha_v = \gamma \sqrt{\lambda^2 + \kappa_v^2}, \quad q_{jv} = \frac{\lambda_j^2}{\gamma^2} + \kappa_v^2, \quad \lambda_j = \frac{\pi}{2}(2j-1).$$

This solution contains an unknown function $\chi_v(\xi)$. If we use condition (10) on the crack, we obtain an equation with respect to the function $\chi_v(\xi)$, which, after integration by parts, will take the form:

$$\int_0^\beta \xi \psi_v(\xi) [F_1(\xi, \eta) + D_1(\xi, \eta)] d\xi = \int_0^\infty \frac{\gamma \operatorname{ch}(\alpha_v l)}{\operatorname{ch}(\alpha_v)} \overline{P}_v(\lambda) \lambda J_1(\eta \lambda) d\lambda, \quad 0 \leq \eta \leq \beta, \quad (11)$$

where

$$\psi_v(\xi) = \frac{1}{\xi} \frac{d}{d\xi} (\xi \chi_v(\xi)),$$

$$D_1(\xi, \eta) = 2 \sum_{j=1}^{\infty} \frac{\lambda_j^2}{\gamma^2} \cos^2 \lambda_j l \cdot \frac{K_2(q_{jv})}{q_{jv} I_2(q_{jv})} I_0(q_{jv} \xi) I_1(q_{jv} \eta),$$

$$F_1(\xi, \eta) = - \int_0^\infty \frac{\alpha_v}{\operatorname{sh} \alpha_v} \operatorname{ch}(\alpha_v(1-l)) \cdot \operatorname{ch}(\alpha_v l) \cdot J_1(\eta \lambda) J_0(\xi \lambda) d\lambda.$$

To solve equation (11) we reduce it to the Fredholm equation of the second kind according to the known method [11, 16]. To do this, we introduce a new unknown function $\phi_v(\tau)$:

$$\psi_v(\xi) = - \frac{2}{\pi} \frac{1}{\xi} \frac{d}{d\xi} \int_\xi^\beta \frac{\tau \cdot \phi_v(\tau)}{\sqrt{\tau^2 - \xi^2}} d\tau$$

and to both parts of equation (11) we apply the operator:

$$D_2[f] = \frac{d}{dx} \int_0^x \frac{y dy}{\sqrt{x^2 - y^2}} \int_0^y f(\eta) d\eta.$$

Due to these transformations, the introduction of designations

$$\sigma_{jv} = \sqrt{(2j-1)^2 + \left(\frac{2\gamma\kappa_v}{\pi}\right)^2}, \quad \theta_{jv} = \frac{\pi\sigma_{jv}}{4\gamma}, \quad \lambda = u\kappa_v, \quad p = \sqrt{u^2 + 1},$$

$$\tau = \beta y, \quad \phi_v(\tau) = \beta g_v(y), \quad x = \beta s,$$

and the even extension of the function $g_v(y)$ on the interval $[-1; 1]$, equation (11) is reduced to the Fredholm integral equation of the second kind

$$g_v(s) - \frac{2\beta}{\pi\gamma} \int_{-1}^1 g_v(y) [B(y, s) + Q(y) - Q(y-s)] dy - g_v(0) = Z_v(s), \quad (12)$$

where

$$Z_v(s) = -4\kappa_v\beta \int_0^1 \eta P_v(\eta) \int_0^\infty \frac{\operatorname{ch}(\gamma\kappa_v p l)}{\operatorname{ch}(\gamma\kappa_v p)} J_1(\eta\kappa_v u) \sin^2\left(\frac{\kappa_v u \beta s}{2}\right) du d\eta,$$

and $B(y, s)$ and $Q(Y)$ are represented as uniformly convergent series and proper integrals.

An approximate solution to equation (12), as in [11, 16], is sought in the form of an interpolation polynomial. To solve equation (12), we approximate its integrals according to the quadrature Gauss-Legendre formula [18] and obtain a system of linear algebraic equations with respect to the values of the unknown function in the interpolation nodes

$$g_{vj} - \frac{2\beta}{\pi\gamma} \sum_{m=1}^n g_{vm} A_m [B(y_m, y_j) + Q(y_m) - Q(y_m - y_j)] - \sum_{m=1}^n b_m^0 g_{vm} = Z(y_j), \quad (13)$$

where

$$B(y_m, y_j) = G(y_m, y_j) + R(y_m, y_j),$$

$$A_m = \frac{2}{(1 - y_m^2) [P_n'(y_m)]^2}, \quad b_m^0 = -\frac{P_n(0)}{y_m P_n'(y_m)}.$$

After solving the system, the unknown function is approximated by the interpolation polynomial

$$g_v(y) \approx g_v^n(y) = \sum_{m=1}^n g_{vm} \frac{P_n(y)}{(y - y_m) P_n'(y_m)}, \quad g_{vm} = g_v(y_m), \quad m = 1, 2, 3, \dots, n,$$

where $P_n(y)$ is the n -th Legendre polynomial, and y_m is the polynomial root.

The resultant solution allows us to determine the stress state at any point in the cylinder.

For the criteria of destruction, an important role is played by SIF, which is determined by the formula

$$\tilde{K}(t_k) = \lim_{r \rightarrow b+0} \sqrt{r-b} \cdot \tilde{\tau}_{\phi z}^1(r, c, t_k).$$

The dimensionless value of SIF after the solution of system (18) can be obtained from the formula

$$K(t_k) = \frac{\tilde{K}(t_k)}{G\sqrt{b}}, \quad K(\tau_k) = \sum_{v=1}^k C_{kv} K_v, \quad K_v = -\frac{1}{\sqrt{2\beta\pi\gamma}} g_v^n(1). \quad (14)$$

Results of Numerical Studies

Using formulas (14), there was performed a numerical study of the dependence of SIF on the dimensionless time $\tau = c_2 t / r_0$ for different load cases. The time grid nodes were condensed near the point $\tau = 0$.

The function determining the load on the cylinder end in condition (2) was presented as the product

$$p(\eta, \tau) = \eta \cdot f(\tau).$$

After discretization by formulas (6) we received

$$P_v = \eta \cdot f_v,$$

where f_v can be found from the recurrence relation $f(\tau_k) = \sum_{v=1}^k C_{kv} f_v$.

The results of calculations are shown in Fig. 2 in the form of graphs of time dependencies of relative SIFs. During these calculations it was considered that the relative height of the cylinder is $\gamma = a/r_0 = 2$, the relative crack radius is $\beta = b/r_0 = 0.5$ and the crack is located in the middle plane of the cylinder $l = c/r_0 = 0.5$. The charts in Fig. 2 have been constructed for the case of the action of a suddenly applied torsional load $f(\tau) = H(\tau)$ (curve 1), the case of specifying the torsional load by a suddenly applied moment of the unit length $f(\tau) = H(\tau) - H(\tau - 1)$ (curve 2), as well as for the case of the action of a suddenly applied harmonic torque load $f(\tau) = H(\tau) \cdot \cos(3\tau)$.

From the graphs in Fig. 2 it can be seen that in all considered types of loading, during the transient process, the maximum SIF values are observed. When a sudden constant load is applied, this maximum is 2–2.5 times higher than the static value of SIF. In the case of sudden harmonic loading, the maximal value of SIF also significantly exceeds the value it acquires during steady-state oscillations, in the absence of resonance. Hence, it is most likely that the destruction of the cylinder will occur during the transient period.

A numerical study of the influence of the cylinder geometric characteristics on the time dependence of SIF was also conducted. Calculations were made for the case of a suddenly applied torsional load (Fig. 3–5).

Curves 1–3 are constructed for the values of the relative cylinder height $\gamma = a/r_0 = 1; 2; 4$. As can be seen from this figure, an increase in the relative length of the cylinder leads to a decrease in the value of SIF and a decrease in the time of the transient process.

In Fig. 4 different values of the relative crack location height $l = c/r_0 = 0.25; 0.5; 0.75$ correspond to curves with numbers 1–3. An analysis of this figure shows that an increase in the values of SIFs is observed during the approach of the crack to the loaded end of the cylinder.

In Fig. 5 values of the relative crack radius $\beta=b/r_0=$: 0.25; 0.5; 0.75 correspond to curves 1–3. These curves demonstrate the fact that in the case of an increase in the relative crack radius, an increase in SIF values can be observed.

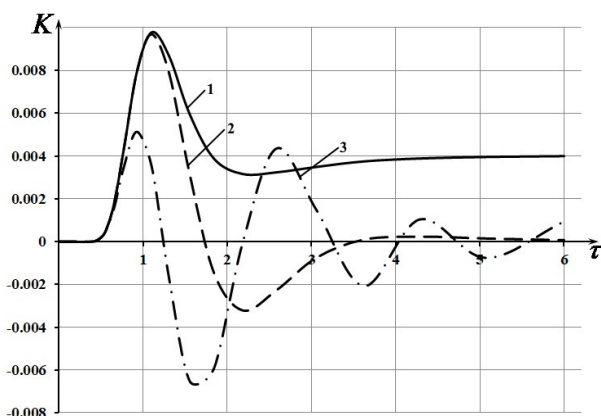


Fig. 2. Time dependence of relative SIFs for different types of load

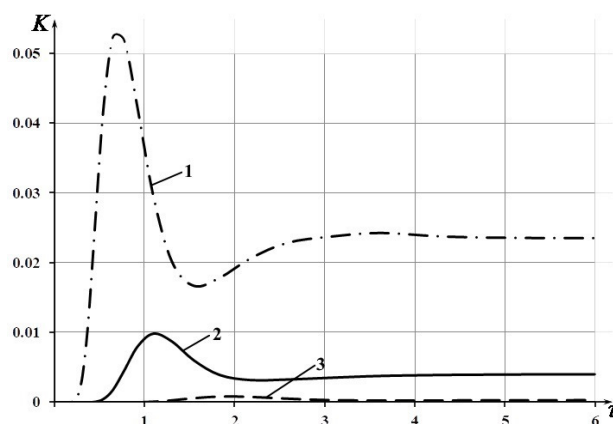


Fig. 3. Influence of cylinder height on SIF value

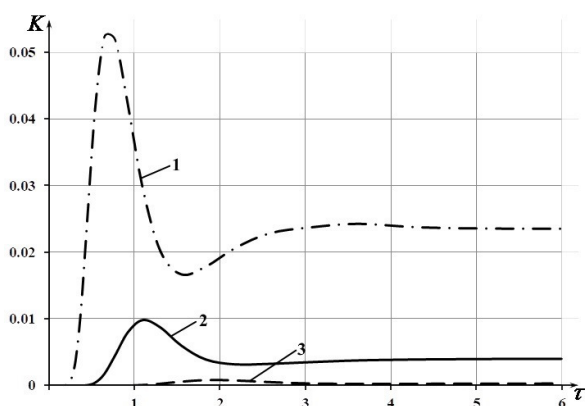


Fig. 4. Influence of the crack location height on SIF value

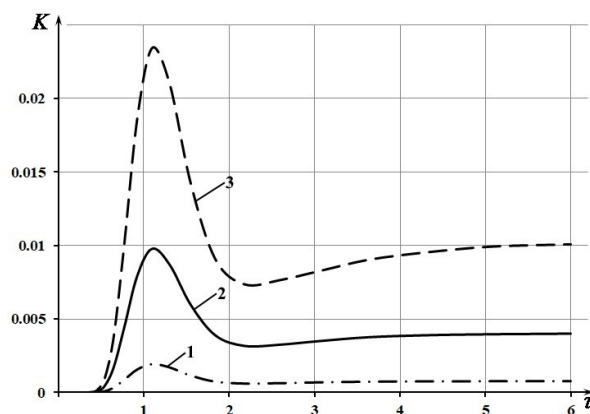


Fig. 5. Influence of the relative crack radius on the value of the SIF

Conclusions

The article proposes a method for solving the problem of determining the stress-strain state of an elastic finite cylindrical body with an internal circular crack that is under torsional loading. This technique is based on the differential approximation of the time derivative and use of a time grid with specially selected nodes. Numerical results demonstrate the effectiveness of such an approach when investigating the transient processes that occur immediately after load application. It is to be noted that the presence of several cracks is not critical for the application of the proposed method, but, of course, solving such a problem is technically more complicated, since one will have to solve the system of integral equations. The appearance of boundary conditions on the cylinder surfaces does not limit the capability of the method, since these conditions only determine the type of integral transformations that are used.

It should also be noted that, in the framework of the above problem statement, no crack can be indefinitely moved nearer to the cylinder ends, since in the case of the crack approaching them the convergence of integrals and series that determine the solution and kernels of integral equations deteriorates significantly, and upon reaching the very ends, the integrals in general become singular. Consequently, for these borderline cases, it is necessary to solve individual problems.

There also arise some problems in applying this technique for large amounts of time, due to the step-by-step accumulation of errors.

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У статті розв'язана вісесиметрична динамічна задача з визначення напруженого стану в околі кругової тріщини в скінченному циліндрі. Нижня основа циліндра жорстко закріплена, а верхня навантажена тангенціальними напруженнями, які залежать від часу. На відміну від традиційних аналітичних методів, що ґрунтуються на використанні інтегрального перетворення Лапласа, запропонований метод полягає в різниціевій апроксимації тільки похідної за часом. Для цього використовуються спеціальним чином підібрані нерівновіддалені вузли та спеціальне подання розв'язку в цих вузлах. Такий підхід дозволяє звести вихідну задачу до послідовності крайових задач для однорідного рівняння Гельмгольца. Кожна така задача розв'язується шляхом застосування скінченних

інтегральних перетворень Фур'є і Ганкеля з подальшим їх оберненням. В результаті було отримано інтегральне подання для кутового переміщення через невідомий стрибок цього переміщення в площині тріщини. Відносно похідної цього стрибка з граничної умови на тріщині отримано інтегральне рівняння, яке в результаті застосування інтегрального оператора Вебера-Соніна і ряду перетворень зведено до інтегрального рівняння Фредгольма другого роду відносно невідомої функції, пов'язаної зі стрибком. Наближене розв'язання цього рівняння здійснено методом колокацій, причому інтегралі наближали квадратними формулами Гаусса-Лежандра. Знайдений числовий розв'язок дав можливість отримати наближену формулу для розрахунку коефіцієнта інтенсивності напружень (КІН). Користуючись цією формулою, провели дослідження впливу характеру навантаження і геометричних параметрів циліндра на почасову залежність цього коефіцієнта. Аналіз результатів показав, що у всіх розглянутих видах навантаження максимум значень КІН спостерігається під час перехідного процесу. Під час прикладення раптового постійного навантаження цей максимум у 2–2,5 рази перевищує статичне значення. У разі раптового гармонічного навантаження максимум КІН теж значно перевищує значення, яких він набуває за усталених коливань, за відсутності резонансу. Збільшення висоти циліндра і зменшення площі тріщини призводять до збільшення тривалості перехідного процесу і зменшення величини максимуму КІН. Той самий ефект спостерігається, коли площина тріщини наближається до нерухомого кінця циліндра.

Ключові слова: коефіцієнт інтенсивності напружень (КІН), вісесиметрична динамічна задача, скінченні різниці за часом, скінченний циліндр, кругова тріщина, крутний момент.

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