моделей скінченних елементів і зробити процес розрахунку більш правильним і компактним. Для вирішення поставленого завдання була складена система матричних рівнянь. Вона грунтується на використанні залежностей енергетичного балансу під час механічної контактної взаємодії елементів ротора, а також теплового балансу у разі впливу нестаціонарного теплового потоку. Під час створення чисельного алгоритму розв'язання поставленої задачі використовувалося пряме розкладання Холецького. Для додання розв'язку більшої компактності застосовувалася схема Шермана. Всі розрахунки полів переміщень і температур проведені для двох широко поширених типів з'єднань, які використовуються для створення таких роторів, а саме: з'єднань з зазором та натягом.

***Ключові слова**: тривимірні скінченні елементи, ротори газових турбін, поля переміщень і температур, задача контактної термопружності, зазор, натяг.*

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STEPWISE OPTIMIZATION OF I-SECTION FLEXIBLE ELEMENTS UNDER A FUZZY APPROACH TO TAKING INTO ACCOUNT CORROSION AND PROTECTIVE PROPERTIES OF ANTICORROSIVE COATING

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This paper is a continuation of research in the field of optimal design of structures under a combined approach to the measurement of corrosion and anticorrosion protective properties of coatings. As noted earlier, such coatings are barrier layers that impede the penetration of aggressive media to the surface of a structure and delay the onset of the process of intense corrosion. In this case, it is important to take into account not only the corrosive effect on the structure, but also be able to estimate the period of time for which the anticorrosive coating loses its protective properties. Since structural elements with damaged protective coatings are able to continue to be subjected to current loads over a considerable period of time, their accelerated corrosion wear is to be taken into account in the damaged areas of coatings. Consequently, the work of the structures protected by coatings consists of two periods: with a protective coating (during which the coating loses protective properties and collapses) and with a damaged protective coating (when there is a severe corrosion wear of unprotected structural areas). The model proposed in the previous study (and implemented on the example of optimization of the flexible elements of a rectangular beam) allows taking into account the smooth transition of the work of structures with protective coatings and the time when the protective properties of anticorrosive coatings practically do not work. This paper considers a solution to a more complicated (due to its multi-extremity) problem of optimization (finding the optimal form) of I-section (double-T) flexible structural elements under a fuzzy approach to taking into account corrosion and anticorrosive coating protective properties.

***Keywords:** corrosion, anticorrosive coatings, optimization.*

Introduction

The work of structures in corrosive environments leads to their corrosive wear. In this case, it should not be forgotten that when constructing mathematical models of corrosive wear of structures, it is also necessary to take into account the work of protective coatings and determine the duration of the incubation period, which is the durability of the applied protective coatings. Constructive elements with damaged protective coatings are able to continue to be subjected to acting loads for a considerable period of time, and their accelerated corrosion wear should be taken into account in zones with damaged areas of coatings. Consequently, the work of the structures protected by coatings consists of two periods: with a protective coating (during which this coating loses its protective properties and collapses) and with a damaged protective coating (when there is a severe corrosive wear of unprotected structure sections).

To date, a number of models have been built, that take into account the reduction in the protective properties of polymer coatings, and models of deforming structures with protective polymer coatings, for example [1–3]. This work is a continuation of research in the field of optimal design of structures under a combined approach to taking into account corrosion and the corrosion resistance of coatings, carried out in [4]. The proposed (and implemented on the example of optimization of rectangular flexible elements in [4] model allows taking into account the smooth transition of the work of structures both with and without a protective coating.

This paper considers a solution to a more complicated problem (due to its multi-extremity) than that in [4]. It is the problem of optimization (finding the optimal form) of I-section flexible structural elements under a fuzzy approach to taking into account corrosion and the protective properties of anticorrosive coating.

Problem statement

We choose, as the basic equation of corrosion, the model proposed by V.M. Dolinsky [5], which takes into account the effect of stresses on the corrosive wear of structures (Fig. 1)

$$\frac{dS}{dt}=\begin{cases}0, & \text{when } t<t_{ink}\\ -2\left(\alpha+\beta\left|\sigma_m\right|\right), & \text{when } t\geq t_{ink}\end{cases}, \tag{1}$$

where α and β are constant coefficients; S_0 and S are the initial and current thicknesses of an I-beam flange (Fig. 1); σ_m, t_{ink} are the maximum stresses and time during which the structure completely loses its anticorrosive properties in the current section, respectively.

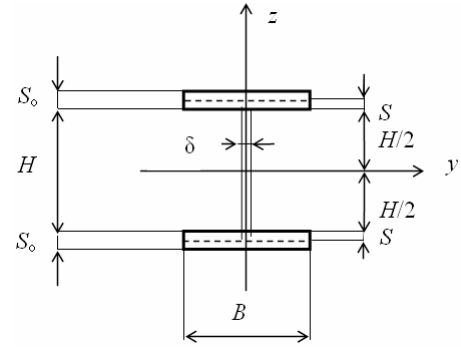

Fig.1. Cross-section of a flexible I-beam element

It is assumed that the upper and lower faces of the section are susceptible to corrosion and the following fuzzy model of corrosion wear is proposed, taking into account the decrease in coating protective properties [4]

$$\frac{dS}{dt}=\begin{cases}-2\left(\alpha+\beta\left|\sigma_m\right|\right)(1-D), & \text{when } 0<D\leq 1\\ -2\left(\alpha+\beta\left|\sigma_m\right|\right), & \text{when } D=0,\end{cases} \tag{2}$$

where D is the parameter characterizing the protective properties of the coating under consideration (at the initial moment of time it is taken to be unity, and at the moment of loosing protective properties $D=D_k$) it is determined from the equation [4]

$$dD/dt=-A(1+m\sigma), \tag{3}$$

where A is the coefficient that takes into account the effect of the type of protective coating and the nature of the corrosive environment; m is the coefficient that takes into account the effect of stress state level on the kinetics of the decrease in the protective properties of the coating; σ is equivalent stresses.

2. Solving equations of corrosion and determining the time for which a structure completely loses its corrosion protection coating

We now turn to the solution to the equations (2). From the equation (3) we have

$$dt=-\frac{dD}{A(1+m\sigma)}. \tag{4}$$

Substituting (4) into the upper part of the equation (2) and dividing the variables, we obtain

$$\frac{A(1+m|\sigma|)dS}{2(\alpha+\beta|\sigma|)} = (1-D)dD \tag{5}$$

Assuming that the bending of an I-section structure occurs in the $x\,z$ plane and that, in the case of bending, mainly the I-section flanges operate, we find its geometric characteristics.

The moment of inertia of an I-section

$$I_y \approx 2BS(H/2+S/2)^2 = 2BS\left(H^2/4+HS/2+S^2/4\right) \approx 2BS\left(H^2/4+HS/2\right) = HBS(H/2+S)$$

Then the moment of resistance of an I-section and the maximum stresses in it are determined respectively as

$$W_y = \frac{I_y}{H/2+S} = \frac{HBS(H/2+S)}{H/2+S} = HBS \quad \text{and} \quad \sigma = |\sigma_{\max}| = \frac{M}{W_y} = \frac{M}{HBS}. \tag{6}$$

Substituting (6) into (5) and proceeding to integration, we have

$$\int_1^0 (1-D)dD = \frac{A}{2}\int_{S_o}^{S} \frac{(1+mM/BSH)}{(\alpha+\beta M/BSH)}dS. \tag{7}$$

After integrating (7), we obtain the following solution to the upper part of the equation (2):

$$S_o - S + (a-b)\ln\frac{S_o+b}{S+b} = \frac{\alpha}{A}, \tag{8}$$

where $a = mM/BH$; $b = \beta M/BH\alpha$.

Before proceeding to the solution of the lower part of the equation (2), we find the time T_* for which the an I-section structure completely loses its anticorrosion coating (the upper and lower faces of the I-section flanges are meant). The derivation of the expression is carried out in a similar manner, as in [4].

Taking the upper limit of the integral on the right-hand side of the equation (7) for D, after integrating, we obtain an equation analogous to (8)

$$D^2 - 2D + 1 + \frac{A}{\alpha}\left[S - S_o + (a-b)\ln\frac{S+b}{S_o+b}\right] = 0.$$

Hence

$$D = 1 + \sqrt{-\frac{A}{\alpha}\left[S - S_o + (a-b)\ln\frac{S+b}{S_o+b}\right]}. \tag{9}$$

Differentiating the left and right sides of the equation (9), we have

$$dD = \frac{1}{2}\frac{A/\alpha\left[-1+(b-a)(S+b)\right]}{\sqrt{\frac{A}{\alpha}\left[(S_o-S)+(a-b)\ln\frac{S_o+b}{S+b}\right]}}dS. \tag{10}$$

Substituting (10) into (4), after integrating, we obtain the following integral expression for T_*:

$$T_* = \frac{1}{2\alpha}\int_{S_o}^{S} \frac{1+(a-b)/(S+b)}{(1+a/S)\sqrt{\frac{A}{\alpha}\left[S_o - S + (a-b)\ln\frac{S_o+b}{S+b}\right]}}dS.$$

The approximate value T_* can be found from (4) (at $D_k = 0$) by the formula

$$T_* \approx \frac{1}{A(1+M/BHS_{cp})},$$

where $S_{cp}=(S_0+S)/2$.

To solve the lower part of the equation (2), we divide the variables in it

$$\int_{S}^{S_k} \frac{dS}{1+\beta M / BHS\alpha} = -2\alpha \int_{0}^{T_k} dt, \tag{11}$$

where S_k is the critical thickness of an I-beam, determined from the principle of equal stress of a structure at the final moment of its operation T by the formula $S_k = M/[\sigma]BH$; $[\sigma]$ is the maximum permissible structural stresses; T_k is the structure operating time after a complete loss of corrosion protection, determined (as in [4]) by the formula $T_k = T - T_*$.

After integrating (11), we finally have

$$-\frac{M}{[\sigma]BH} + S + b\ln\frac{M/[\sigma]BH + b}{S + b} = 2\alpha(T - T_*). \tag{12}$$

After solving the equations (2), as well as determining the time T_* during which the upper and lower faces of I-section flanges completely lose the anticorrosive coating, it is possible to directly proceed to the optimization process.

3. Step-by-step solution to an optimization problem

Accepting (as in [4]) as the goal function the initial weight (or volume) of the construction, it should be noted that the process of its (goal function) minimization is more complicated than in the case of optimizing a rectangular cross-section. Preliminary calculations, using the algorithm of the random search method [6], showed that if all the dimensions of an I-beam are included in the vector of variable parameters, i.e. for each fixed value of x $\overline{X} = \{x_1, x_2, x_3, x_4, x_5\}^{\mathrm{T}} = \{S_0, S, H, B, \delta\}^{\mathrm{T}}$ is taken as the vector of variable parameters, then the optimization problem is multi-extremal (has a lot of local minima) and is difficult to solve. In this case, the optimization problem was divided into 2 stages.

3.1. The first stage of optimization

Let's consider the first optimization stage. Here, the vector of variable parameters, for each fixed value of x, includes the initial flange thickness, the flange thickness at the time T_*, the B flange width, and the I-beam wall thickness δ, i.e.,

$$\overline{X} = \{x_1, x_2, x_3, x_4\}^{\mathrm{T}} = \{S_0, S, B, \delta\}^{\mathrm{T}}$$

The value of the I-beam depth H along the structure length is assumed to be fixed.

As a numerical implementation (as in [4]), we consider the optimization of a cantilever beam with a force F at the end. The initial data of the problem: F=10 κN; length of the beam L=1 m; m=0.005 MPa^{-1}; α=1 mm/year; A=0.732 year^{-1}; β=1×10^{-3} mm/(MPa×year); $[\sigma]$=210 MPa; T=5 years.

The following constructive limitations were accepted: 1) $B/S_k \le 24$; 2) $H/\delta \le 60$; 3) $\delta \ge 3$ mm; 4) $B \ge 3$ mm; 5) $S_0 \le 30$ mm. During the optimization, three variants of different values of the I-beam depth H: a) H_1=80 mm; b) H_2=100 mm; c) H_3=120 mm.

The optimal dimensions of the initial thickness of I-beam flanges $S_0(x)$, their appearance at the time T_* – $S(x)$ and at the final moment of the structure operating time $S_k(x)$ are shown in figure 2. The optimal dimensions of flange widths and the wall thickness of an I-section cantilever beam in all its points are shown in figure 3. Dependences of change in the cross-sectional area $A_0(x)$ long the entire length of the beam (for variants a, b and c) are shown in figure 4.

As can be seen from figure 2, in all three cases, the optimum thickness S_0 reaches its maximum (30 mm) practically along the entire length of a cantilever beam, that is, restriction 5 is active in the process of searching for optimal solutions. The exception is the cantilever beam area, close to the support ($x \le 100$ mm), starting from which there is a sharp decrease in the initial thickness, its final value at x=0 being $S_0 = \alpha(2T - 1/A) = 8.63$ mm. This analytic expression can be easily obtained from the system of equations (8) and (12) taken at x=0

$$\begin{cases} S_o - S = \alpha / A \\ S = 2\alpha(T - T_*), \end{cases} \tag{13}$$

where $T_* = 1/A$.

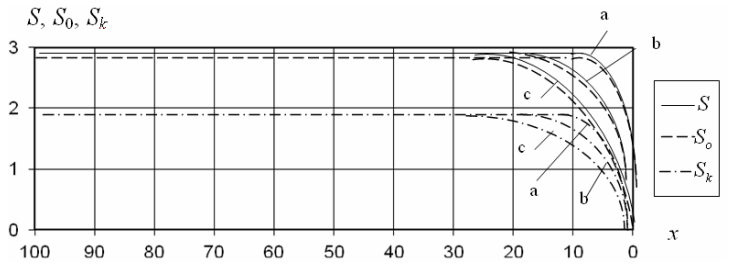

Fig. 2. Optimal dimensions of initial flange widths and their view at the transitional and final moments of time

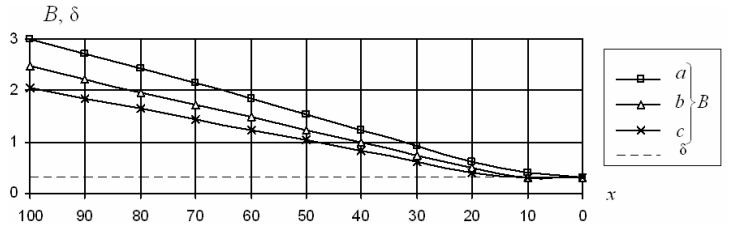

Fig. 3 Optimal shapes of flange widths and wall thickness of an I-beam

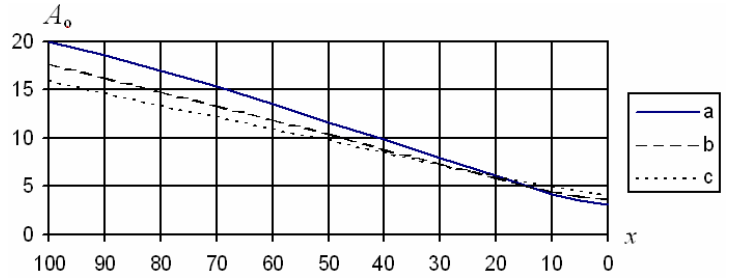

Fig.4. Dependences of change of a beam cross-section area

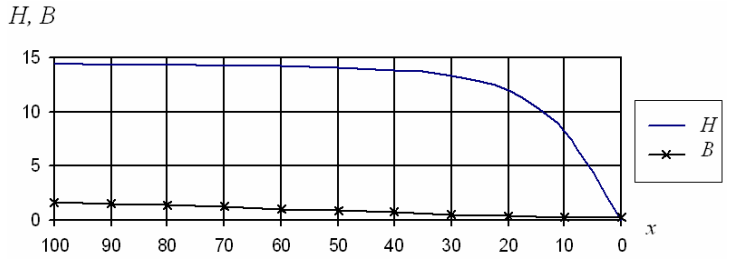

Fig. 5. Optimal shapes of I-beam flange width and wall depth

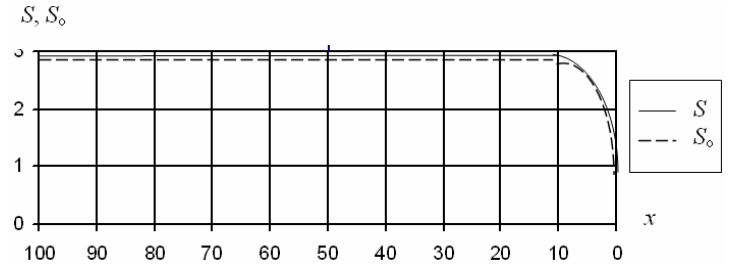

Fig. 6. Optimal shape of the initial width of I-beam flanges and their view at the moment T_*

The tendency to a sharp decrease after an equal section (similarly to $S_0(x)$) is retained for the curves $S(x)$ and $S_k(x)$, the value S being little different from the value of S along the entire length of a cantilever beam (practically by the thickness of the protective anticorrosive layer, as was also noted for a rectangular beam). From (13) it can be seen that for x=0 the difference S_0–S=α/A=1.37 mm.

The optimal value of the I-section wall reaches its minimum along the entire length of the cantilever beam, that is, δ=3 mm, and in all calculation variants (see Figure 3).

As regards the curves $B(x)$, it is evident that the optimum flange width in the corresponding points is inversely proportional to the corresponding I-beam wall height H: a) H_1=80 mm; b) H_2=100 mm; c) H_3=120 mm.

So, at H=80 mm, we have the flange width B (practically along the entire length of the cantilever) greater than in variant b, where H=100 mm. The same pattern is obvious when comparing variants b and c.

This tendency is typical for all the points of a cantilever beam, again except for the area at x≤100 mm, where the curves $B(x)$ asymptotically approach their minimum value (B=30 mm), which explains the sharp decrease of the curves $S_0(x)$, $S(x)$ and $S_k(x)$ in this area, and the resulting optimization (minimization) of the cross-section A_0.

Comparing the dependence curves of a cantilever cross-section along its length $A_0(x)$ for all the variants (Fig. 4), it can be concluded that for x>200 mm the cross-sectional area is inversely proportional to the given wall depth H, at x=200 mm they are practically equal (A_0≈60 мм) – the curves intersect, and at x<200 mm there is a direct dependence of A_0 on H.

As a result of optimization of the first stage, in all the variants there is a smooth transition from the I-section (in the area 100 mm≤x≤1000 mm) to the rectangular one (at x≤100 mm), figure 3.

3.2. The second stage of optimization

Taking into account the results obtained above, we proceed to the second stage. Since both the initial thickness of the flange and the wall thickness δ do not change in the area of 100 mm≤x≤1000 mm during the optimization process, they are taken here as fixed values, namely, S_0=30 mm and δ=3 mm. In this case, the vector of variable parameters in this area is taken as $\overline{X} = \{H, B\}^T$. At x≤100 mm, the widths of the flanges and the I-beam wall remain unchanged. In this area, they can be assumed to be fixed, namely, B=δ=3 mm. Here, the vector of variable parameters is taken as $\overline{X} = \{S_0, S, H\}^T$.

The results of the second stage of optimization, obtained as above, using the algorithm of the random search method [6], are shown in figures 5, 6. The optimal cantilever beam shape is shown in figure 7.

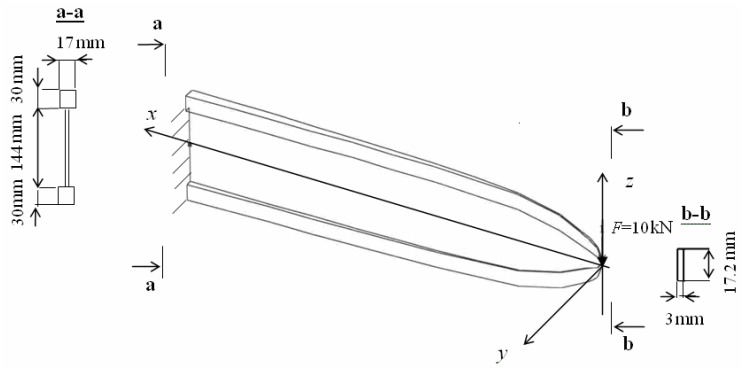

Fig.7. Optimal view of a beam

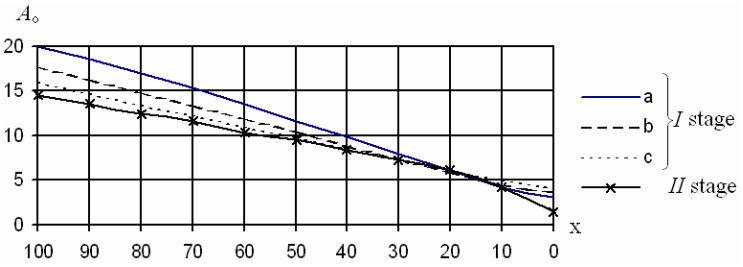

Fig. 8. Analysis of the two-stage optimization

Conclusions

The model of the combined approach to the calculation of corrosion and protective properties of anticorrosion coatings proposed in [4] was realized when determining the optimal dimensions (Fig. 5, 6) and the shape (Fig. 7) of the I-section flexible elements in the example of a cantilever beam. As can be seen from figure 7, as a result of the stage-by-stage optimization, it was established that the I-section of a beam (along its entire length, that is, at 0≤x≤1000 mm) smoothly transforms into a rectangular one, which is the case for x=0.

A comparative analysis of the two optimization stages is shown in figure 8.

From figure 8 it can be seen that at all the points x of a cantilever beam the optimal cross-sectional areas obtained in the second stage are less (or at least equal to) the corresponding cross-sectional areas obtained in the first stage, that is, $A_o^{II}(x) \le A_o^{I}(x)$. This is the proof that only as a result of a stage-by-stage optimization is the construction of the minimum weight. In conclusion, it should be noted that the proposed model (2), implemented in the optimization of structural elements of rectangular [4] and I-sections, operating under corrosion conditions, can be used both in analytical solutions and with the help of numerical methods.

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Поетапна оптимізація згинних елементів двотаврового перерізу при нечіткому підході до врахування корозії та захисних властивостей антикорозійного покриття

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Ця робота є продовженням дослідження в області оптимального проектування конструкцій при комбінованому підході до обліку корозії і антикорозійних захисних властивостей покриттів. Як зазначалося раніше, такі покриття являють собою бар'єрні шари, що утрудняють проникнення агресивного середовища до поверхні конструкції і відсувають початок процесу інтенсивної корозії. У цьому випадку важливо враховувати не тільки корозійний вплив на конструкцію, але також вміти оцінити час, за який антикорозійне покриття втрачає свої захисні властивості. Оскільки конструктивні елементи зі зруйнованим захисним покриттям може продовжувати сприймати діючі навантаження протягом значного проміжку часу, потрібно враховувати їхній прискорений корозійний знос в зонах зі зруйнованими ділянками покриттів. Отже, робота захищених покриттями конструкцій складається з двох періодів: із захисним покриттям (протягом якого це покриття втрачає захисні властивості і руйнується) та зі зруйнованим захисним покриттям (коли має місце інтенсивний корозійний знос незахищених ділянок конструкцій). Запропонована в попередньому дослідженні модель (і реалізована на прикладі оптимізації згинних елементів прямокутного перерізу) дозволяє враховувати плавний перехід роботи конструкцій із захисним покриттям і часом роботи конструкції, коли захисні властивості антикорозійного покриття практично не діють. У цій роботі розглядається розв'язок складнішої (через свою багатоекстремальність) задачі оптимізації (знаходження оптимальної форми) згинних елементів конструкцій 1-го (двотаврового) перерізу при нечіткому підході до обліку корозії і захисних властивостей антикорозійного покриття.

***Ключові слова**: корозія, антикорозійні покриття, оптимізація.*

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