Andrei Vladimirovich Roiter

To the 75th anniversary

A. V. Roiter was born on November 30, 1937 in Dnepropetrovsk in the family of the well-known Ukrainian scientist V. A. Roiter. In 1955 he entered Kyiv Taras Shevchenko University, and in 1958 he moved to Leningrad State University, where he received his Diploma (M.S.) in Mathematics in 1960. In the same year he started his graduate study. The advisor of his Master and Ph.D. Thesis was a prominent mathematician D. K. Faddeev. In 1961 A. V. Roiter was hired as a researcher at the Institute of Mathematics of the Academy of Sciences of Ukraine, where he worked until his death on July 26, 2006. He received his Ph.D. in 1963 and Doctor of Sciences degree (habilitation) in 1969. Since 1991 he was the Head of the Department of Algebra.

His first papers were devoted to the theory of integral representations. They were inspired by D. K. Faddeev, who was then an active propagandist of this branch of algebra. The theory was very young then and contained, perhaps, only one positive result — the description of integral representations of cyclic groups of prime order by Diederichsen [7]. The latter paper contained one more result: Diederichsen claimed that cyclic groups of order p^2 (p prime) have infinitely many non-isomorphic indecomposable representations. Certainly, this result negatively affected further investigations, so it is easy to understand why the first paper of A. V. Roiter [25] excited many researchers. He showed that Diederichsen was *wrong*: the group of order 4 has only finitely many (namely, 9) non-isomorphic indecomposable representations. Soon other mathematicians, such as Berman, Gudivok, Heller, Reiner and Jones, obtained more general results and showed at last that a finite group G has only finitely many non-isomorphic indecomposable integral representations if and only if, for each prime p, its Sylow p-subgroup is cyclic of order at most p^2 [5, 6, 15, 16, 18].

At this time the main interest of A. V. Roiter was in integral representations of *rings*, more precisely, of *orders in finite dimensional algebras*, where he was, together with D. K. Faddeev, a pioneer of the theory and the author of several basic papers. The best known result of A. V. Roiter in the theory of integral representations is certainly his paper [29] on *genera* of representations, i.e. representations having isomorphic localizations at all primes. The Jordan–Zassenhaus Theorem [9] implies that every genus only contains finitely many isomorphism classes. A. V. Roiter proved that for every order Λ in a semisimple $\mathbb{Q}$-algebra the numbers of isomorphism classes in the genera of representations of Λ are bounded. This paper had a great influence on the whole subject and inspired a series of papers and results on the structure of genera.

Other papers of A. V. Roiter are devoted to the *local theory*, i.e. to the study of p-adic representations. Here the best known are his results on the so-called *Bass orders* [28, 11] and the criterion for a commutative order to have finitely many non-isomorphic indecomposable representations [10].[1] An important tool here was his theory of *divisibility of modules* [26, 27]. Undoubtedly, the latter was one of the sources of his first paper on representations of finite dimensional algebras [30], which was immediately evaluated as outstanding.

Remind its history. It is related with two conjectures of Brauer and Thrall (see [19]). Let A be a finite dimensional algebra over a field $\mathbf{k}$. A is said to be *of finite type* if it only has finitely many non-isomorphic indecomposable (finite dimensional) modules. Otherwise it is said to be *of infinite type*. It is said to be of *bounded type* if the dimensions of indecomposable A-modules are bounded and of *unbounded type* otherwise. Finally, it is said to be of *strongly unbounded type* if there are infinitely many dimension d such that there are infinitely many non-isomorphic indecomposable A-modules of dimension d. Certainly, if the field $\mathbf{k}$ is finite, *finite* and *bounded* types coincide and strongly unbounded is impossible. Brauer and Thrall conjectured that, even if $\mathbf{k}$ is infinite, *infinite type* implies *unbounded* (the 1st Brauer–Thrall Conjecture) and, moreover, *strongly unbounded* (the 2nd Brauer–Thrall Conjecture). Some (very special) results in this direction were obtained by Yoshii [34], Curtis and Jans [8]. In [30] A. V. Roiter completely proved the 1st Brauer–Thrall Conjecture using quite new approach.

[1] Independently and in another formulation, this criterion was also obtained by Jacobinski [17].

That paper became the origin of further investigations of representations of finite dimensional algebras, which were the main work of A. V. Roiter for the rest of his life. In particular, it led him to the *theory of matrix problems*. The latter originated in the calculations used before by several authors when studying representations of algebras and orders. But the first paper where this theory got its own individuality was his paper with Nazarova [23] on representations of partially ordered sets. In particular, in this paper they proved that the 2nd Brauer–Thrall Conjecture holds for these representations. It became the starting point for developing a new technique, which is now widely used in the representation theory and related areas. In the preprint [24] Nazarova and Roiter applied this technique to attack the 2nd Brauer–Thrall Conjecture. This paper made obvious that the theory of matrix problems should be developed and studied *per se* as a particular branch of algebra. A. V. Roiter himself made important contribution in this theory, especially in its foundations, where he in particular proposed a general definition of matrix problems as *representations of differential graded categories* [33, 20] and, equivalently, as *representations of BOCSs* [31].

The way to the 2nd Brauer–Thrall Conjecture was rather long. The crucial step in this direction was the result obtained by A. V. Roiter together with Bautista, Gabriel, and Salmerón [11, 2]. They have proved that a finite dimensional algebra of finite type over an algebraically closed field has a *multiplicative basis* i.e. such a basis that the product of any two of its elements either is zero or belongs to the same basis. Using this result, Bautista [1], Bongartz [4] and Fischbacher [12] (independently) finally proved the 2nd Brauer–Thrall Conjecture for finite dimensional algebras over an arbitrary algebraically closed field.

The outstanding book of Gabriel and Roiter [14] was (and is) of great influence on the modern development of the theory of representations of finite dimensional algebras as well as on the theory of matrix problems.

The last ideas of A. V. Roiter concerned representations in Hilbert spaces. In the papers [22, 21] he introduced the notion of *locally scalar* representations of quivers (or, the same, graphs) in Hilbert spaces. Together with Krugliak they constructed for such representations *Coxeter functors* analogous to those of Bernstein–Gelfand–Ponomarev [3] and applied them to the study of locally scalar representations. In particular, they proved that a graph has only finitely many indecomposable locally scalar representations (up to unitary isomorphism) if and only if it is a Dynkin graph (the result quite analogous to that of Gabriel [13] for the "usual" representations of quivers). Unfortunately, his untimely death interrupted his fruitful activity.

Being one of the most prominent mathematicians, A. V. Roiter was also an active teacher and organizer. In 1961 he organized in Kyiv a seminar on the theory of representations. A lot of students of Taras Shevchenko University participated in this seminar, so it became the foundation of the Kyiv school of the representation theory, which is now well-known and highly esteemed. Among the participants of this seminar, V. M. Bondarenko, Yu. A. Drozd, V. V. Kirichenko, S. A. Krugliak, S. A. Ovsienko, L. A. Nazarova, V. V. Sergeichuk and A. G. Zavadskij became Doctors of Sciences (habilitated). A. V. Roiter himself was the supervisor of 13 Ph.D. Theses. In 2007 A. V. Roiter was awarded (unfortunately, posthumously) with the State Prize of Ukraine as one of the main authors of a cycle of papers and monographs on the theory of representations.

His death took away a great mathematician and we will always miss him.

Yu. Drozd, V. Kirichenko, S. Krugliak,
M. Kleiner, V. Bondarenko, S. Ovsienko

References

[1] R. Bautista. On algebras of strongly unbounded representation type. Comment. Math. Helv. 60 (1985), 392-399.

[2] R. Bautista, P. Gabriel, A. Roiter and L. Salmerón. Representation-finite algebras and multiplicative bases. Invent. Math. 81 (1985), 217–285.

[3] I. N. Bernstein, I. M. Gelfand and V. A. Ponomerev. Coxeter functors and a theorem of Gabriel. Uspehi Mat. Nauk, 28, no. 2 (1973), 19–33.

[4] K. Bongartz. Indecomposables are standard. Comment. Math. Helv. 60 (1985), 400-410.

[5] S. D. Berman and P. M. Gudivok. On integral representations of finite groups. Dokl. Acad. Nauk SSSR 145 (1962), 1199–1201.

[6] S. D. Berman and P. M. Gudivok. On integral representations of finite groups. Doklady i Soobschenija Uzhgorod University 5 (1962), 74–76.

[7] F. E. Diederichsen. Über die Ausreduktion ganzzahliger Gruppendarstellungen bei arithmetischer Äquivalenz. Hamb. Abh. 14 (1938), 357–412.

[8] C. W. Curtis and J. P. Jans. On algebras with a finite number of indecomposable modules. Trans. Amer. Math. Soc. 114 (1965), 122–132.

[9] C. W. Curtis and I. Reiner. Methods of representation theory, with applications to finite groups and orders. Vol. I. John Wiley & Sons, 1981.

[10] Y. A. Drozd and A. V. Roiter. Commutative rings with a finite number of indecomposable integral representations. Izv. Akad. Nauk SSSR. Ser. Mat. 31 (1967), 783–798.

[11] Y. A. Drozd, V. V. Kirichenko and A. V. Roiter. Hereditary and Bass orders. Izv. Akad. Nauk SSSR Ser. Mat. 31 (1967), 1415–1436.

[12] U. Fischbacher. Une nouvelle preuve d'un théorème de Nazarova et Roiter. C. R. Acad. Sci., Paris, Sér. I. 300 (1985), 259–262.

[13] P. Gabriel. Unzerlegbare Darstellungen I. Manuscripta Math. 6 (1972), 71–103.

[14] P. Gabriel and A. Roiter. Algebra VIII: Representations of finite-dimensional algebras. Encyclopaedia of Mathematical Sciences. 73. Springer-Verlag, 1992.

[15] A. Heller and I. Reiner. Representations of cyclic groups in rings of integers, I. Ann. Math. 76 (1962), 73–92.

[16] A. Heller and I. Reiner. Representations of cyclic groups in rings of integers, II. Ann. Math. 77 (1963), 318–328.

[17] H. Jacobinski. Sur les ordres commutatifs avec un nombre fini de réseaux indecomposables. Acta Math. 118 (1967), 1-31.

[18] A. Jones. Groups with a finite number of indecomposable integral representations. Mich. Math. J. 10 (1963), 257–261.

[19] J. P. Jans. On the indecomposable representations of algebras. Ann. Math. 66 (1957), 418–429.

[20] M. M. Kleiner and A. V. Roiter. Representations of differential graded categories. Matrix problems. Inst. Math. Akad. Sci. Ukraie. Kiev, 1977, pp. 5–70.

[21] S. A. Kruglyak, L. A. Nazarova and A. V. Roiter. Orthoscalar representations of quivers in the category of Hilbert spaces. Zap. Nauchn. Semin. POMI 338 (2006), 180-201.

[22] S. A. Kruglyak and A. V. Roiter. Locally scalar graph representations in the category of Hilbert spaces. Funkts. Anal. Prilozh. 39, No. 2 (2005), 13-30.

[23] L. A. Nazarova and A. V. Roiter. Representations of partially ordered sets. Zap. Nauchn. Semin. LOMI. 28 (1972), 5–31.

[24] L. A. Nazarova and A. V. Roiter. Categorical matrix problems and the problem of Brauer–Thrall. Kiev, 1973. (German translation: Mitt. Math. Sem. Giessen Heft 115, 1975)

[25] A. V. Roiter. On the representations of the cyclic group of fourth order by integral matrices. Vestnik Leningrad. Univ. 15 (1960), no. 19, 65–74.

[26] A. V. Roiter. Categories with division and integral representations. Dokl. Akad. Nauk SSSR 153 (1963), 46–48.

[27] A. V. Roiter. Divisibility in the category of representations over a complete local Dedekind ring. Ukrain. Mat. J. 17 (1965), no. 4, 124–129.

[28] A. V. Roiter. An analogue of the theorem of Bass for modules of representations of non-commutative orders. Dokl. Akad. Nauk SSSR 168 (1966), 1261–1264.

[29] A. V. Roiter. Integer-valued representations belonging to one genus. Izv. Akad. Nauk SSSR Ser. Mat. 30 (1966), 1315–1324.

[30] A. V. Roiter. Unboundedness of the dimensions of the indecomposable representations of an algebra which has infinitely many indecomposable representations. Izv. Akad. Nauk SSSR Ser. Mat. 32 (1968), 1275–1282.

[31] A. V. Roiter. Matrix problems and representations of BOCSs. Representations and quadratic forms. Inst. Math. Akad. Sci. Ukraine. Kiev, 1979, pp. 3–38.

[32] A. V. Roiter. A generalization of a theorem of Bongartz. Preprint no. 17, Inst. Math. Akad. Sci. Ukraine. Kiev, 1981.

[33] A. V. Roiter and M. M. Kleiner. Representations of differential graded categories. Representations of algebras (Proc. Internat. Conf., Carleton Univ., Ottawa, Ont., 1974.) Lecture Notes in Math., Vol. 488, Springer, Berlin, 1975, pp. 316–339.

[34] T. Yoshii. Note on algebras of bounded representation type. Proc. Japan Acad. 32 (1956), 441–445.