ISSN 1027-5495. Functional Materials, 25, No.2 (2018), p. 406-411
doi:https://doi.org/10.15407/fm25.02.406

A new stress-based multiaxial high-cycle fatigue damage criterion

Xin Li[1]*, Jianwei Yang*[2]*, Dechen Yao*[2]

[1] School of Mechanical-electronic and Automobile Engineering, Beijing University of Civil Engineering and Architecture, Beijing 100044, China
[2] Beijing Key Laboratory of Performance Guarantee on Urban Rail Transit Vehicles, School of Machine-electricity and Automobile Engineering, Beijing University of Civil Engineering and Architecture

Received February 2, 2018

A new stress-based high-cycle fatigue damage criterion for multiaxial load cases, $D = \alpha D_{\sigma}^{\beta} D_{\tau}^{\gamma}$ is presented. This criterion is based on a critical plane approach that the damage parameter is a function of normal stress amplitude and shear stress amplitude on critical plane of maximum shear stress range. The coefficient α, β and γ in the damage function are material parameters. Tensions with torsion test data are required to ascertain these coefficients. This criterion matches the test results well and shows accurate predictions of fatigue failure life compared to some present methods.
Keywords: Multiaxial high-cycle fatigue; critical plane; stress-based fatigue analysis.

Предложен новый критерий определения повреждения материалов за счет напряжений, возникающих при многоциклической нагрузке $D = \alpha D_{\sigma}^{\beta} D_{\tau}^{\gamma}$. Этот критерий основан на приближении, что параметр повреждения является функцией нормальной амплитуды напряжения и амплитуды сдвигового напряжения. Коэффициенты α, β и γ в функции повреждения являются экспериментальными параметрами. Для определения этих коэффициентов используется напряженность, возникающая при испытаниях на кручение. Этот критерий хорошо соответствует результатам испытаний и показывает более точные прогнозы усталостных отказов по сравнению с некоторыми существующими современными методами.

Новий метод визначення втоми матеріалів, заснований на багатоціклічному навантаженні. *Xin Li, Jianwei Yangb, Dechen Yaoc*

Запропоновано новий критерій визначення пошкодження матеріалів за рахунок напружень, що виникають при багатоциклічному навантаженні $D = \alpha D_{\sigma}^{\beta} D_{\tau}^{\gamma}$. Цей критерій заснований на наближенні, що параметр пошкодження є функцією нормальної амплітуди напруги і амплітуди сдвигової напруги. Коефіцієнти α, β і γ у функції ушкодження є експериментальними параметрами. Для визначення цих коефіцієнтів використовується напруженість, що виникає при випробуваннях на крутіння. Цей критерій добре відповідає результатам випробувань і показує більш точні прогнози втомних відмов у порівнянні з деякими існуючими сучасними методами.

1. Introduction

Almost every mechanical component is subjected to multiaxial loadings. How to predict the life of these mechanical parts is a key factor for designers. Early fatigue researchers use equivalent stress, such as Von Mises criterion or Tresca criterion, to put forward their fatigue life prediction criterions, which are essentially extension of static yield criteria [1]. The aim of these methods is reducing the multiaxial stress components to an equivalent uniaxial stress, expecting to transfor the multiaxial fatigue to a simple uniaxial fatigue problem. However, a difficulty of this type of procedure is definition of a mean stress, whose meaning is unclear within multiaxial stress conditions. In present, it is believed that the critical plane method is an efficient way of fatigue life prediction, for it has a clearer physical meaning than other fatigue life prediction methods. Many critical plane methods have been proposed to assist in the mechanical design [2]. A typical critical plane criterion is proposed by Findley [3]. He uses a linear combination of normal stress and shear stress in the critical plane as a mean stress. A similar concept depends on the test of proportional combination of cyclic bending and torsion was proposed by Stulen and Cummings [4]. McDiarmid [5] pointed out that the important factors of high cycle fatigue (HCF) are the maximum shear stress range and the normal stress in the critical plane of the maximum shear stress amplitude. Rashed et al. [6] propose a stress-strain criterion in consideration of the nonproportional multiaxial loading condition. Chen et al [7]. discuss the applicability of several mutliaxial fatigue damage criterions under variable amplitude loading.

Another class of fatigue criterion is the energy based parameters, which also use the critical plane concept. Smith et al. [8] proposed Smith-Watson-Topper (SWT) criterion which is well-known and widely accepted. Chu et al. [9] proposed a criterion that extends the SWT criterion to include a shear term. Liu [10] used a virtual energy method for shear and normal facture. Glinka et al. [11] proposed a criterion based on the summation of the product of normal stress and strain ranges, and that of shear stress and strain ranges on the critical shear plane.

In this paper, a stress-based fatigue damage criterion for multiaxial fatigue is proposed. Experiment data of three metals [12-14] were used to estimate the criterion, with the lives ranging from 8.5×10^3 to 1.3×10^6 cycles, and two classical critical plane criterions are used to compare with this criterion. The results show that the new criterion correlates with the experimental results very well and effects for multiaxial high-cycle fatigue cases.

2. The establish of the stress-based high cycle multiaxial fatigue damage criterion

A load-based criterion for damage to multiaxis fatigue cycles is established. The new fatigue damage criterion is based on four concepts as follows:

Concept 1: Multiaxial fatigue damage occurred on the plane of maximum shear stress range, which is defined as the critical plane. Critical plane method is developed on the basis of phenomenological observations of fatigue crack development. The critical plane approach receives the most attention because of its good correlation with test data and its explicit physical meaning. Metal fatigue experiments show that the cracks generally propagate along the plane of maximum shear stress range[15]. In this paper, the critical plane is defined as the plane which suffers the maximum shear stress range.

Concept 2: The maximum shear stress amplitude and the normal stress amplitude on the critical plane are the two main factors contribute to the fatigue damage. The critical plane method generally uses combination of shear stress and normal stress or stress amplitudes on the critical plane as damage parameters for HCF [16,17]. The shear stress induces dislocation movement along slip line, while the normal stress opens the crack, reduces friction between the crack surfaces and accelerates the propagation of the crack [18]. In the new fatigue damage criterion, the damage parameters are shear stress amplitude and normal stress amplitude.

Concept 3: In uniaxial conditions (tension or torsion), the stress and fatigue circle follow the Basquin function. Under macroscopically elastic uniaxial loading conditions, fatigue life can be described in terms of a S-N (or Wöhler) curve which associates the stress amplitude with the life (described in the number of cycles to failure) of a experimental specimen or a engineering component. Generally, the relationship between the stress amplitude and the fatigue life can be described using an exponential equation. The mostly used equation is known as Basquin function as follows:

$$S = S_f^{'}\left(2N_f\right)^n \quad (1)$$

Where S is the uniaxial stress amplitude, N_f is the number cycles to fatigue, $S_f^{'}$ and n are material parameters.

For tension or torsion fatigue test, Basquin function can be described as

$$\begin{aligned} \sigma_a &= \sigma_f^{'}\left(2N_f\right)^b \\ \tau_a &= \tau_f^{'}\left(2N_f\right)^c \end{aligned} \quad (2)$$

Where σ_a is the normal stress amplitude, $\sigma_f^{'}$ is fatigue strength coefficient, b is fatigue strength exponent, τ_a is shear stress amplitude, $\tau_f^{'}$ is shear fatigue strength coefficient, c is shear fatigue strength exponent.

Table 1 Tensile performance of metals

Metal type	SM45C steel	6082-T6 AL	7075-T651 AL
Young modulus (GPa)	208.6	69.4	71.7
Yield stress (MPa)	418	301	501
Tensile strength (MPa)	731	343	561

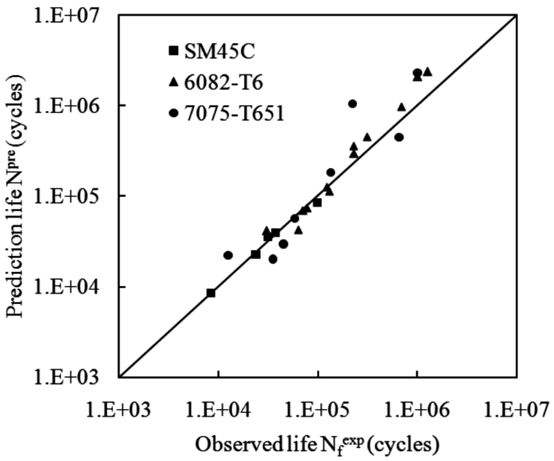

Fig.1 The comparison of observed lives and the new criterion prediction lives.

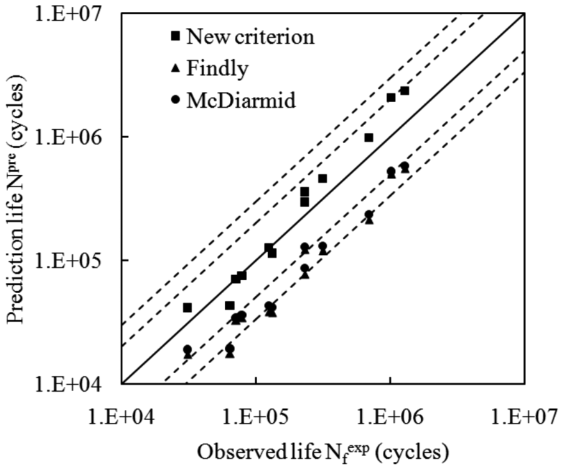

Fig.2 The prediction lives comparison of different criterions for SM45C steel.

Concept 4: The total fatigue damage is a function of the damage caused by normal stress amplitude and shear stress amplitude on the critical plane. Based on concept 2, we can assume that the total damage in one cycle is a function of the damages caused by normal stress and shear stress on the critical plane. That is to say

$$D = f\left(D_\sigma, D_\tau\right) \qquad (3)$$

Where D is the total damage of one cycle, $D=1/N_f$, D_σ is the damage caused by normal stress amplitude on the critical plane, D_τ is the damage caused by shear stress amplitude on the critical plane.

The fatigue damage caused by normal stress and shear stress can be converted from Equation (2)

$$D_\sigma = \left(\frac{\sigma_{n,a}}{\sigma_f^{'}}\right)^{-\frac{1}{b}},\ D_\tau = \left(\frac{\tau_a}{\tau_f^{'}}\right)^{-\frac{1}{c}} \qquad (4)$$

Where $\sigma_{n,a}$ is the amplitude of the normal stress on the critical plane, τ_a is the amplitude of the shear stress on the critical plane.

With the analysis of the test data of different metals, we define the form of the damage function as follows

$$D = \alpha D_\sigma^\beta D_\tau^\gamma \qquad (5)$$

Where α, β and γ are coefficients of material.

Substituted Equation (4) in Equation (5), the damage function of the new fatigue damage criterion is

$$D\left(\sigma_{n,a}, \tau_a\right) = \alpha\left(\frac{\sigma_{n,a}}{\sigma_f^{'}}\right)^{-\frac{\beta}{b}}\left(\frac{\tau_a}{\tau_f^{'}}\right)^{-\frac{\gamma}{c}} \qquad (6)$$

The fatigue life is reciprocal of the damage.

3. Criterion assessment

Material test data. Combined axial load or bending and torsion experimental data taken from the published literature [12-14] were used in order to assessment the new fatigue damage criterion. The test materials conclude SM45C structural steel, 6082-T6 and 7075-T651 aluminum alloys. The tensile performances of these metals are listed in Table 1, the bending and torsion experimental data are listed in Table 2. The test load can be described as follows

$$\begin{aligned} \sigma_x(t) &= \sigma_{x,m} + \sigma_{x,a}\sin(\omega t) \\ \tau_{xy}(t) &= \tau_{xy,m} + \tau_{xy,a}\sin(\omega t + \varphi) \end{aligned} \qquad (7)$$

The material coefficients used in the fatigue damage criterion are listed in Table 3.

The contrastive criterions. Two critical plane criterions proposed by Findly [3] and McDiarmid [5] are compared with the new criterion.

Findly proposed a linear combination of normal stress and shear stress in the critical plane for a given number of cycles to failure

$$\tau_a + k\sigma_n = \tau_f^{'}\left(2N_f\right)^c \qquad (8)$$

Table 2 The bending and torsion loading parameters and observed lives N_f^{exp} . Prediction lives N^{FD} from Findly, N^{MD} from McDiarmid and N^{NEW} from the new criterion

No.	$\sigma_{x,a}$ (MPa)	$\sigma_{x,m}$ (MPa)	$\tau_{xy,a}$ (MPa)	$\tau_{xy,m}$ (MPa)	φ (°)	N_f^{exp} (cycles)	N^{FD} (cycles)	N^{MD} (cycles)	N^{NEW} (cycles)
SM45C steel bearing-torsion									
1	390	0	151	0	0	8500	3894	12203	8465
2	349	0	148	0	0	24000	18723	56609	22622
3	325	0	153	0	0	32000	34589	100033	35283
4	372	0	93	0	0	38000	75341	268707	39016
5	309	0	134	0	0	100000	172938	518088	84512
6082-T6 aluminum alloy bearing-torsion									
6	70	-3	118	0	0	71255	32493	33992	69769
7	71	-1	117	1	1	78730	34065	35670	74325
8	59	-1	100	1	-7	230750	121693	127282	356997
9	53	-1	83	1	-2	1018775	497186	521716	2071817
10	52	-2	82	0	2	1289550	551221	578247	2350208
11	147	-2	106	1	-4	31000	17275	18860	41134
12	151	-4	104	0	-3	64090	17510	19165	42524
13	163	-2	81	0	-5	124460	38276	42627	125552
14	147	1	90	-1	-8	132215	37471	41279	113714
15	146	-3	76	-1	-6	232370	77265	85848	294717
16	118	-3	82	1	-5	315795	119040	130226	454012
17	119	1	72	-1	0	694062	211765	233434	973212
7075-T651 aluminum alloy bearing-torsion									
18	127.2	0	170.5	0	0	59194	20360	62695	55842
19	166.1	0	110.4	0	0	136646	143785	460252	182242
20	201.3	0	130	0	0	45500	35333	115236	29302
21	153.5	0	100.3	0	0	662627	287777	931454	440128
22	137.4	0	79.7	0	0	1018000	1093868	3819905	2280773
23	205.8	0	137.5	0	0	35804	25825	82384	20037
24	147.5	0	86.9	0	0	225000	589466	2038538	1037802
25	203.8	0	136.3	0	0	12708	27784	88576	22032

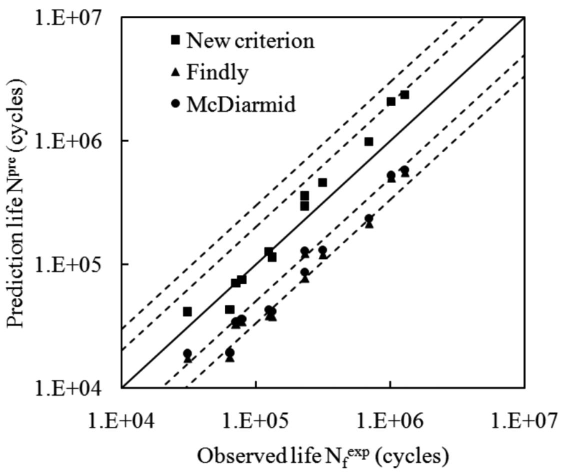

Fig.3 The prediction lives comparison of different criterions for 6082-T6 aluminum alloy

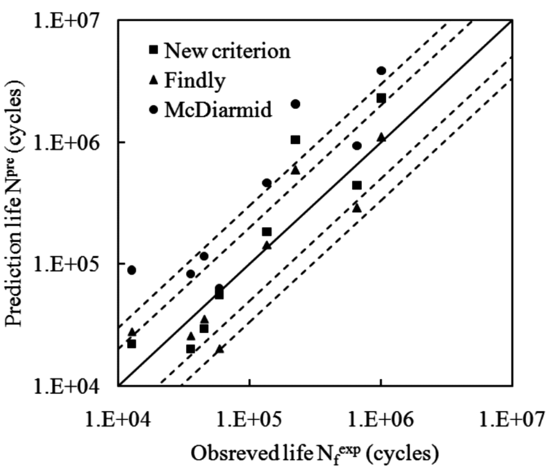

Fig.4 The prediction lives comparison of different criterions for 7075-T651 aluminum alloy.

Where k is a material coefficient. The critical plane is defined as the plane experience the maximum value of fatigue damage. This criterion was effective for proportional combination of bending and torsion condition.

McDiarmid noticed that the important parameters of loading in HCF are the maximum

Table 3 Material coefficients used in the fatigue damage criterion

	σ_f' (MPa)	b	τ_f' (MPa)	c
SM45C steel	1024.3	–0.0946	441.44	–0.0511
6082-T6 Al	1053.1	–0.1426	470.27	–0.1258
7075-T651 Al	1072.6	–0.1246	760.33	–0.1264

Table 4 Parameters of these Findely criterion, McDiarmid criterion and the New criterion

Material	Findly criterion	McDiarmid criterion		New criterion		
	k	τ_{-1} (MPa)	σ_u (MPa)	α	β	γ
SM45C steel	0.219	197.2	731	3.789	0.326	0.409
6082-T6 Al	0.12	68.2	343	9.295	0.0138	1.225
7075-T651 Al	0.377	109.9	561	74.346	0.427	0.905

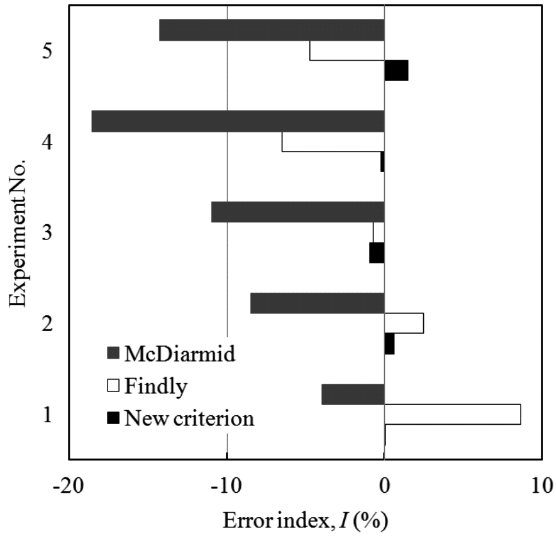

Fig.5 Error indexes of different criterions for SM45C steel.

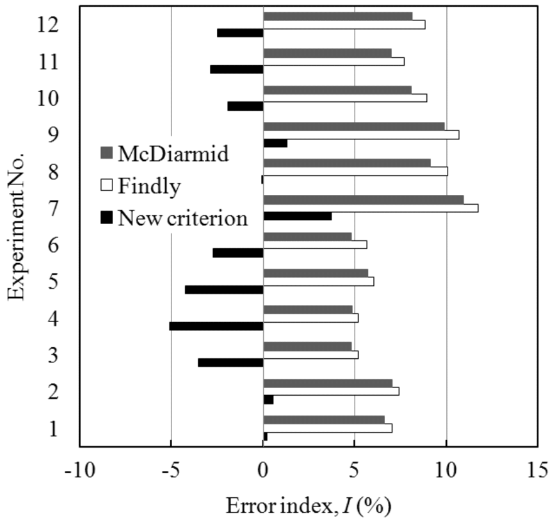

Fig.6 Error indexes of different criterions for 6082-T6 aluminum alloy.

shear stress range and the normal stress on the critical plane. The critical plane is defined as the plane of maximum shear stress range. The criterion is

$$\tau_a + \left(\tau_{-1}/2\sigma_u\right)\sigma_{n,\max} = \tau_f^{'}\left(2N_f\right)^c \quad (9)$$

Where τ_{-1} is fatigue limit of torsion, σ_u is ultimate strength, $\sigma_{n,\max}$ is the maximum normal stress on the critical plane.

The particular parameters of these criterions are listed in Table 4. Some parameters (k in Findly criterion, α, β and γ in the new criterion) are obtained from fitting of the bending-torsion test data.

4. Results and discussion

The comparison of the observed lives and new criterion prediction lives of different materials are showed in Fig.1. Fig.2 to Fig.4 demonstrate the new criterion prediction lives in comparison with Findly criterion and McDiarmid criterion prediction lives for the three materials respectively. In all figures, the central diagonal line presents the perfect agreement between the observed lives and the prediction lives. For Fig.2 to Fig.4, the dashed lines represent factor 2 and factor 3 bandwidths.

Fig.1 proves that the new criterion matches the test result well. Fig.2 to Fig.4 show that for different materials, the new criterion's prediction result is the best among the three criterions. Generally, most prediction life points of Findly criterion are in factor 3 band, while some prediction life points of McDiarmid criterion are out of factor 3 band. For 6082-T6 aluminum alloy, these two criterions perform almost the same.

For a quantitative judgment of the prediction quality of the new criterion and the comparison of different criterions, an error index is introduced as follows

$$I = \frac{\ln N_f^{\exp} - \ln N^{pre}}{\ln N_f^{\exp}} \times 100\% \quad (10)$$

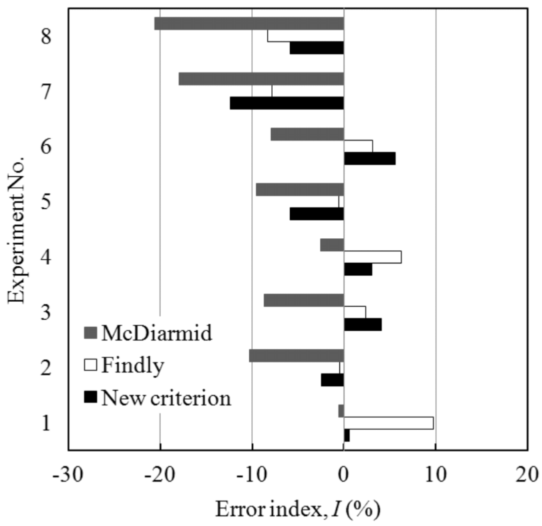

Fig.7 Error indexes of different criterions for 7075-T651 aluminum alloy.

The comparisons of different criterions for the three materials are shown in Fig.5 to Fig.7 respectively. For SM45C steel, the error (absolute value) of the new criterion is less than 1.5%, while the maximum errors (absolute value) of Findly and McDiarmid are up to 8.6% and 18.5% respectively; for 6082-T6 aluminum alloy, the error (absolute value) of the new criterion is less than 5.2%, while the maximum errors (absolute value) of Findly and McDiarmid are up to 11.7% and 10.9% respectively; for 7075-T651 aluminum alloy, the error (absolute value) of the new criterion is less than 12.4%, while the maximum errors (absolute value) of Findly and McDiarmid are up to 9.7% and 20.5% respectively. Generally the prediction error of the new criterion is less than Findly criterion and McDiarmid criterion. But for 7075-T651 aluminum alloy, the maximum error of Findly criterion is less than the new criterion. The prediction result of McDiarmid criterion is the worst among the three criterions.

3. Conclusion

A new concept is introduced to formulate the new fatigue damage criterion. This new criterion fits the test data of different materials perfectly; most of the prediction lives points are in factor 2 band; the error analysis shows that the error of the new criterion generally within the range of ±5%. It is more accurate than Findly criterion and McDiarmid criterion. This research provides a new thinking for high-cycle fatigue life prediction.

Acknowledgement

Project funded by China Postdoctoral Science Foundation (2016M591060) and Beijing Postdoctoral Research Foundation (2016ZZ-79) are gratefully acknowledged.

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