2.5D SIMULATION OF PLASMA WAKEFIELD EXCITATION BY A NONRESONANT TRAIN OF RELATIVISTIC ELECTRON BUNCHES

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By using 2d3v code LCODE the investigations in detail of the reconstruction of a long nonresonant train of relativistic electron bunches into resonant one at plasma wakefield excitation are presented. Because of bunch repetition and plasma frequencies detuning the wakefield beatings are occurred. The bunches in the maximum of beating experience defocusing radial force and go out the interaction with wakefield. It leads to the shortening of the beating period and to the asymmetry between the energy loss of decelerated bunches of the beating front and energy gain of accelerated bunches of the beating rear. In result along with wakefield amplitude beating the monotonical growth takes place with the rate equivalent to the resonant train with smaller current.

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1. INTRODUCTION

Resonant plasma wakefield excitation by long train of relativistic electron bunches is difficult because realization of homogeneous and stationary plasma in experiments is difficult. Results of 2.5D numerical simulation by the code LCODE [1] of plasma wakefield excitation by a nonresonant train of relativistic electron bunches (repetition frequency ωm is not coincided with plasma frequency ωp) with parameters close to experiment [2] have been presented in [3]. Train of short relativistic electron bunches of energy 2 MeV, charge 0.32 nC, rms length σz=1.7 cm, rms radius σr=0.5 cm, rms angular spread σθ=0.05 mrad, repetition period 360 ps excites a wakefield. The plasma of density, smaller than resonant one 1011 cm-3, is simulated, so the frequency of the excited wave is smaller than the frequency of bunches repetition ωp<ωm. The mechanism of the wakefield excitation in this case is the self-consistent self-cleaning of electron bunches. As a result of interaction with wakefield the train becomes more resonant.

In this paper we consider the mechanism of this excitation more detaily, taking into account bunch defocusing in radial field and changing of the coupling of bunches with wakefield.

2. RESULTS OF SIMULATION

It is difficult to support plasma strongly resonant (ωm is coincided with ωp) in experiments. Therefore, the wakefield excitation by nonresonant train of electron bunches in plasma is of great interest. We simulate dynamics, using cylindrical coordinate system (r, z). At first we consider 500 electron bunches. Fig.1 shows the amplitude of the on-axis longitudial electric field as a function of the coordinate along the plasma and the number of bunches for ωp=0.97ωm while ωm=1.0025ωm. At small coordinate along the plasma, we observe beatings of the field, as it is excited by a periodic force of a different frequency. Number of bunches in beating equals N=1/(1-ωp/ωm)=39. Number of beatings along train equals N(NL)=500(1-ωp/ωm)=13.

Some distance downstream the wakefield grows, because the train becomes resonant due to self-cleaning at radial defocusing of nonresonant bunch electrons [3]. The effect of reconstruction of resonant train due to radial defocusing of nonresonant bunch electrons is useful, because during evolution the nonlinear wave frequency ωNL change takes place. Really, from Fig.1 one can estimate this nonlinear frequency change (ωNL/ωp-1)=0.1% using increase of beating number at amplitude growth.

We simulate and compare the case, close to optimal ωp=1.0025ωm (initial compensation of nonlinear ωp decrease), and the nonresonant case ωp=0.97ωm. For optimized difference ωp=ωp the wakefield amplitude reaches value 4.5 MV/m, corresponding to 15% of the wave breaking limit. From Fig.2 one can see that near...
exit of the system after 500 bunches the wakefield in the case \( \omega_p = 0.97 \omega_m \) is smaller only in 1.5 times in comparison with the case, close to optimal \( \omega_p \approx 1.0025 \omega_m \). 

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One can see self-cleaning due to defocusing in Fig.4. Near exit of the plasma the bunch electrons, which get in large radial defocusing fields and consequently in small longitudinal electrical field \( E_z \), are defocused.

For the train of 318 bunches (Fig.4), we observe that approximately 100 bunches lose their energy linearly. Further bunches lose their energy slower. Let us consider energy exchange of bunches with wakefield for small difference \( \omega_m \) and \( \omega_p \). Energy losses of \( N \)-th bunch on wakefield excitation on \( \lambda \) equals

\[ \epsilon_N = 2 \pi e n_{bc} E_0 \omega_p / \omega_p, \]

\[ E_0 = E_{0N} + (\beta - 1) \delta E_N, \beta \approx 1/2. \]

Then wave energy \( W \) changes on

\[ W_N - W_{N-1} = \eta \epsilon_N, \]

\[ \eta \approx 1.7 \times 10^{-3}, \ell_b / \lambda = 1/6, \]

\( \eta \) is the part of volume, occupied by bunches in comparison with volume, occupied by wakefield

\[ \delta E_N = E_{N-1} - \delta E_{N}, \delta E_N = E_1 = e n_{bc} \eta / (2 \pi \omega_p). \]

Energy losses of train of bunches \( \Sigma \epsilon_i \) equal

\[ \Sigma \epsilon_i = \pi \eta (e n_{bc} N^2 \pi / \omega_p)^2. \]

Energy losses of train of \( N \) bunches are proportional to \( N^2 \). On 1st asymptotic the wakefield amplitude grows linearly with time

\[ \ddot{E}_N = 2 \pi e n_{bc} \eta = \text{const.} \]

When bunches completely lose the energy

\[ \Sigma \epsilon_i = (N-K) m c^2 (\gamma_b - 1) n_b + \pi e n (e n_{bc} K^2 \pi / \omega_p)^2. \]

When each bunch loses a significant part of the energy, the wakefield amplitude grows with time as \( \sqrt{t} \).

From Fig.5 one can see that there is electron group which is decelerated and there is no another similar electron group which is accelerated.

3. SELF-CLEANING MECHANISM

Let us consider self-cleaning mechanism of the train of 32 bunches in the case \( \omega_p = 0.92 \omega_m \). From Fig.6 one can see that the radial distribution of bunch density is asymmetrical along beating. In front of beating the bunch electrons are located closer to the axis and in the rear of beating they are located far from the axis.

From Figs.6-8 one can see that the middle radii of bunches are larger (Fig.6) and the mean field \( E_0 = \int E_z n_b d\Gamma / \int n_b d\Gamma \) is smaller (Fig.7) in rear of beating (phases of electron acceleration) in comparison with front of beating (phases of electron deceleration).

Because the wakefield is excited by nonresonant train the wakefield represents beatings. Near injection boundary the symmetry between phases of decelerated electrons in wakefield in front of beating and phases of accelerated electrons in wakefield in rear of beating is realized. As a result of bunch radial defocusing in the middle of beating the wavelength becomes larger far from the injection boundary. This leads to phase symmetry braking, i.e. accelerated bunches in rear of beating get in the smaller (larger) longitudinal (radial) fields comparatively to decelerated bunches in front of beating. Therefore middle radii of bunches are larger in rear of beating, than in front of beating. It decreases coupling of bunches with wakefield in rear of beating in
comparison with front of beating. The train continues to excite the wakefield.

CONCLUSIONS

As a result of bunch radial defocusing in the middle of beating, excited by nonresonant train of electron bunches, the wavelength becomes larger. This leads to bunch phase shift, in rear of beating accelerated bunches get in the smaller (larger) longitudinal (radial) fields comparatively to decelerated bunches in front of beating. The train continues to excite the wakefield.

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