Ukrainian Mathematical Journal, Vol. 49, No. 5, 1997

ON ASYMPTOTIC DECOMPOSITIONS OF o-SOLUTIONS IN THE THEORY OF QUASILINEAR SYSTEMS OF DIFFERENCE EQUATIONS

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UDC 517.949

We consider a quasilinear system of difference equations with certain conditions. We prove that there exists a formal partial o-solution of this system in the form of functional series of special type. We also prove a theorem on the asymptotic behavior of this solution.

Consider the system of difference equations

$$\Delta y_k(t) = q_k(t) + \sum_{i=1}^{n} p_{ki}(t) y_i(t) + \sum_{k_i+\ldots+k_n=2}^{\infty} p_{kk_1\ldots k_n}(t) y_1^{k_1}(t)\ldots y_n^{k_n}(t), \tag{1}$$

$$t \in N, \quad t \geq t_0, \quad k = 1, \ldots, n,$$

with the conditions

$$\text{(i)} \quad \left| p_{kk_1\ldots k_n}(t) \right| \leq AR^{-(k_1+\ldots+k_n)}, \tag{2}$$

$$k = 1, \ldots, n, \quad k_1 + \ldots + k_n \geq 2, \quad A, R \in \mathbb{R}_+;$$

$$\text{(ii)} \quad q_k(t) = o(1), \quad k = 1, \ldots, n, \quad t \to +\infty;$$

$$\text{(iii)} \quad \exists P_0 = \lim_{t\to+\infty} P(t), \quad P_0 \in \mathbb{C}^{n\times n}, \quad P(t) = (p_{ki}(t))_n^n.$$

Assume that the characteristic numbers λ_k, $k = 1, \ldots, n$, of the matrix P_0 possess the property $|1 + \lambda_k| \neq 0$, $k = 1, \ldots, n$.

Inequality (2) guarantees the absolute and uniform convergence of the series in system (1) in any domain of the form

$$\Gamma\left\{ t \in N, \ t \geq t_0, \ \sum_{i=1}^{n} |y_i(t)| \leq R_0 < R \right\}, \quad R_0 \in \mathbb{R}_+.$$

Furthermore, we assume that the functions $q_k(t)$, $p_{ki}(t)$, and $p_{kk_1\ldots k_n}$, $k, i = 1, \ldots, n$, $k_1 + \ldots + k_n \geq 2$, admit, in a certain sense (see Definitions 6 and 7), formal expansions into series of the form

$$\sum_{k_1+\ldots+k_p=0}^{\infty} c_{k_1k_2\ldots k_p} f_1^{k_1}(t) f_2^{k_2}(t) \ldots f_p^{k_p}(t), \qquad c_{k_1k_2\ldots k_p} \in \mathbb{C}, \tag{3}$$

Odessa University, Odessa. Translated from Ukrainskii Matematicheskii Zhurnal, Vol. 49, No. 5, pp. 672–677, May, 1997. Original article submitted May 15, 1995.

0041–5995/97/4905–0747 $18.00

where $f_k(t)$, $k = 1, \dots, p$, is a fixed set of functions such that

$$\Delta^i f_k(t) = o(1), \quad \Delta^0 f_k(t) \overset{\text{def}}{=} f_k(t), \quad k = 1, \dots, p, \ i = 0, 1, 2, \dots .$$

In what follows, we denote the set of functions $f_k(t)$, $k = 1, \dots, p$, by (f).

We rewrite system (1) as

$$\Delta Y(t) = Q(t) + P(t)Y(t) + \Psi(t, Y(t)). \tag{4}$$

Consider the series

$$\sum_{s=0}^{\infty} c_k \sigma_k(t), \quad \sigma_k(t) = \prod_{i=0}^{s-1} \left(\Delta^i f_1(t)\right)^{k_i} \dots \prod_{i=0}^{s-1} \left(\Delta^i f_p(t)\right)^{l_p}, \tag{5}$$

where $k = k_0 \dots k_{s-1} \dots l_0 \dots l_{s-1}$ and, for any fixed value s, the exponents $k_0, \dots, k_{s-1}, \dots, l_0, \dots, l_{s-1}$ can be integer nonnegative numbers satisfying the condition

$$k_0 + 2k_1 + \dots + sk_{s-1} + \dots + l_0 + 2l_1 + \dots + sl_{s-1} = s.$$

The coefficients c_k are columns of the same dimension n. The numbers s are called the orders of the corresponding terms in (5).

Definition 1. *A vector function $\varphi(t)$, $t \in N$, $t \geq t_0$, which is a finite sum of the type*

$$\varphi(t) = \sum_{s=s_0} c_k^* \sigma_k(t), \quad c_k^* \in \mathbb{C}^{n \times 1}, \quad k = k_0 \dots k_{s-1} \dots l_0 \dots l_{s-1},$$

where the terms have the same order s_0, is called a function of order s_0, which is denoted as follows: $\Pi(\varphi(t)) = s_0$.

Property 1. If $\Pi(\varphi(t)) = s_0$ and $c \in \mathbb{C}$, then $\Pi(c\varphi(t)) = s_0$.

Property 2. If $\Pi(\varphi_1(t)) = s_0$ and $\Pi(\varphi_2(t)) = s_0$, then

$$\Pi(\varphi_1(t) + \varphi_2(t)) = s_0.$$

Property 3. If $\Pi(\varphi_1(t)) = s_1$ and $\Pi(\varphi_2(t)) = s_2$, then

$$\Pi(\varphi_1(t)\varphi_2(t)) = s_1 + s_2.$$

Definition 2. *The following series are called, respectively, the sum, difference, and product of two formal series $\sum_{s=0}^{\infty} W_s$ and $\sum_{s=0}^{\infty} V_s$ of type (5):*

$$\sum_{s=0}^{\infty} (W_s + V_s), \quad \sum_{s=0}^{\infty} (W_s - V_s), \quad \sum_{s=0}^{\infty} (W_s V_s + W_1 V_{s-1} + \dots + W_s V_0).$$