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RESEARCH ARTICLE

Hamming distance between the strings generated by adjacency matrix of a graph and their sum

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Abstract. Let $A(G)$ be the adjacency matrix of a graph G. Denote by $s(v)$ the row of the adjacency matrix corresponding to the vertex v of G. It is a string in the set $\mathbb{Z}_2^n$ of all n-tuples over the field of order two. The Hamming distance between the strings $s(u)$ and $s(v)$ is the number of positions in which $s(u)$ and $s(v)$ differ. In this paper the Hamming distance between the strings generated by the adjacency matrix is obtained. Also $H_A(G)$, the sum of the Hamming distances between all pairs of strings generated by the adjacency matrix is obtained for some graphs.

1. Introduction

Let $\mathbb{Z}_2 = \{0, 1\}$ and $(\mathbb{Z}_2, +)$ be the additive group, where $+$ denotes addition modulo 2. For any positive integer n,

$$\begin{aligned} \mathbb{Z}_2^n &= \mathbb{Z}_2 \times \mathbb{Z}_2 \times \cdots \times \mathbb{Z}_2 \ (n \ \text{ factors}) \\ &= \{(x_1, x_2, \ldots, x_n) | x_1, x_2, \ldots, x_n \in \mathbb{Z}_2\}. \end{aligned}$$

Thus every element of $\mathbb{Z}_2^n$ is an n-tuple $(x_1, x_2, \ldots, x_n)$ written as $x = x_1x_2 \ldots x_n$ where every x_i is either 0 or 1 and is called a *string* or *word*. The number of 1's in $x = x_1x_2 \ldots x_n$ is called the *weight* of x and is denoted by $wt(x)$.

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Let $x = x_1x_2 \dots x_n$ and $y = y_1y_2 \dots y_n$ be the elements of $\mathbb{Z}_2^n$. Then the sum $x + y$ is computed by adding the corresponding components of x and y under addition modulo 2. That is, $x_i + y_i = 0$ if $x_i = y_i$ and $x_i + y_i = 1$ if $x_i \neq y_i$, $i = 1, 2, \dots, n$.

The *Hamming distance* $H_d(x, y)$ between the strings $x = x_1x_2 \dots x_n$ and $y = y_1y_2 \dots y_n$ is the number of i's such that $x_i \neq y_i$, $1 \leqslant i \leqslant n$. Thus $H_d(x, y) =$ Number of positions in which x and y differ $= wt(x + y)$.

Example 1. Let $x = 01001$ and $y = 11010$. Therefore $x + y = 10011$. Hence $H_d(x, y) = wt(x + y) = 3$.

Lemma 1 ([8])**.** *For all $x, y, z \in \mathbb{Z}_2^n$, the following conditions are satisfied:*

(i) $H_d(x, y) = H_d(y, x)$;

(ii) $H_d(x, y) \geqslant 0$;

(iii) $H_d(x, y) = 0$ *if and only if* $x = y$;

(iv) $H_d(x, z) \leqslant H_d(x, y) + H_d(y, z)$.

A graph G with vertex set $V(G)$ is called a *Hamming graph* [4, 7] if each vertex $v \in V(G)$ can be labeled by a string $s(v)$ of a fixed length such that $H_d(s(u), s(v)) = d_G(u, v)$ for all $u, v \in V(G)$, where $d_G(u, v)$ is the length of shortest path joining u and v in G.

Hamming graphs are known as an interesting graph family in connection with the error-correcting codes and association schemes. For more details see [1, 2, 4 - 7, 9 - 11, 13, 14].

Motivated by the work on Hamming graphs, in this paper we study the Hamming distance between the strings generated by the adjacency matrix of a graph. Also we obtain the sum of the Hamming distances between all pairs of strings for certain graphs.

2. Preliminaries

Let G be an undirected graph without loops and multiple edges with n vertices and m edges. Let $V(G) = \{v_1, v_2, \dots, v_n\}$ be the vertex set of G. The vertices adjacent to the vertex v are called the *neighbours* of v. The *degree* of a vertex v, denoted by $deg_G(v)$ is the number of neighbours of v. A graph is said to be *r-regular* if the degree of each vertex is equal to r. The vertices which are adjacent to both u and v simultaneously are called the *common neghbours* of u and v and the vertices which are neither adjacent to u nor adjacent to v are called *non-common neighbours* of u and v. The *distance* between two vertices u and v in G is the length of shortest path joining u and v and is denoted by $d_G(u, v)$.

The *adjacency matrix* of G is a square matrix $A(G) = [a_{ij}]$ of order n, in which $a_{ij} = 1$ if the vertex v_i is adjacent to v_j and $a_{ij} = 0$, otherwise. Denote by $s(v)$ the row of the adjacency matrix corresponding to the vertex v. It is a string in the set $\mathbb{Z}_2^n$ of all n-tuples over the field of order two.

Sum of Hamming distances between all pairs of strings generated by the adjacency matrix of a graph G is denoted by $H_A(G)$. That is,

$$H_A(G) = \sum_{1 \leqslant i < j \leqslant n} H_d(s(v_i), s(v_j)).$$

$H_A(G)$ is a graph invariant. For graph theoretic terminology we refer to the books [3, 12].

Example 2.

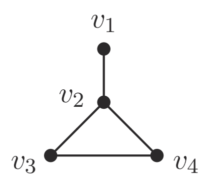

FIGURE 1. Graph G.

Adjacency matrix of a graph G of Fig. 1 is

$$A(G) = \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

and the strings are $s(v_1) = 0100$, $s(v_2) = 1011$, $s(v_3) = 0101$, $s(v_4) = 0110$.

$$H_d(s(v_1), s(v_2)) = 4, \quad H_d(s(v_1), s(v_3)) = 1, \quad H_d(s(v_1), s(v_4)) = 1,$$
$$H_d(s(v_2), s(v_3)) = 3, \quad H_d(s(v_2), s(v_4)) = 3, \quad H_d(s(v_3), s(v_4)) = 2.$$

Therefore $H_A(G) = 4 + 1 + 1 + 3 + 3 + 2 = 14$.

3. Hamming distance between strings

In this section we obtain the Hamming distance between a given pair of strings generated by the adjacency matrix of a graph.

Theorem 1. *Let G be a graph with n vertices. Let the vertices u and v of G have k common neighbours and l non-common neighbours.*

(i) *If u and v are adjacent vertices, then*

$$H_d(s(u), s(v)) = n - k - l.$$

(ii) *If u and v are nonadjacent vertices, then*

$$H_d(s(u), s(v)) = n - k - l - 2.$$

Proof. (i) Let u and v be the adjacent vertices of G. Let u and v have k common neighbours and l non-common neighbours. Therefore remaining $n-k-l-2$ vertices other than u and v are adjacent to either u or v but not to both. Therefore the strings of u and v from $A(G)$ will be in the form

$$s(u) = x_1x_2x_3 \dots x_{k+1}x_{k+2}x_{k+3} \dots x_{k+l+2}x_{k+l+3} \dots x_n$$

and

$$s(v) = y_1y_2y_3 \cdots y_{k+1}y_{k+2}y_{k+3} \cdots y_{k+l+2}y_{k+l+3} \cdots y_n$$

where $x_1 = 0$, $x_2 = 1$, $y_1 = 1$, $y_2 = 0$, $x_i = y_i = 1$ for $i = 3, 4, \dots, k+2$, $x_i = y_i = 0$ for $i = k+3, k+4, \dots, k+l+2$ and $x_i \neq y_i$ for $i = k+l+3, k+l+4, \dots, n$.

Therefore $s(u)$ and $s(v)$ differ at $n-k-l-2+2 = n-k-l$ places. Hence $H_d(s(u), s(v)) = n-k-l$.

(ii) Let u and v be the nonadjacent vertices of G. Let u and v have k common neighbours and l non-common neighbours. Therefore remaining $n-k-l-2$ vertices other than u and v are adjacent to either u or v but not to both. Therefore the strings of u and v from $A(G)$ will be in the form

$$s(u) = x_1x_2x_3 \dots x_{k+1}x_{k+2}x_{k+3} \dots x_{k+l+2}x_{k+l+3} \dots x_n$$

and

$$s(v) = y_1y_2y_3 \cdots y_{k+1}y_{k+2}y_{k+3} \cdots y_{k+l+2}y_{k+l+3} \cdots y_n$$

where $x_1 = 0$, $x_2 = 0$, $y_1 = 0$, $y_2 = 0$, $x_i = y_i = 1$ for $i = 3, 4, \dots, k+2$, $x_i = y_i = 0$ for $i = k+3, k+4, \dots, k+l+2$ and $x_i \neq y_i$ for $i = k+l+3, k+l+4, \dots, n$.

Therefore $s(u)$ and $s(v)$ differ at $n-k-l-2$ places. Hence $H_d(s(u), s(v)) = n-k-l-2$. $\square$

Lemma 2. *Let G be a graph with n vertices. Let the vertices u and v of G have k common neighbours and l non-common neighbours.*

(i) *If u and v are adjacent vertices, then*

$$deg_G(u) + deg_G(v) = n + k - l.$$

(ii) *If u and v are nonadjacent vertices, then*

$$deg_G(u) + deg_G(v) = n + k - l - 2.$$

Proof. (i) Let u and v be the adjacent vertices of G. Let u and v have k common neighbours and l non-common neighbours. Therefore remaining $n - k - l - 2$ vertices other than u and v are adjacent to either u or v but not to both. Therefore

$$deg_G(u) + deg_G(v) = k + k + (n - k - l - 2) + 2 = n + k - l.$$

(ii) Let u and v be the nonadjacent vertices of G. Let u and v have k common neighbours and l non-common neighbours. Therefore remaining $n - k - l - 2$ vertices other than u and v are adjacent to either u or v but not to both. Therefore

$$deg_G(u) + deg_G(v) = k + k + (n - k - l - 2) = n + k - l - 2. \qquad \square$$

Theorem 2. *Let G be an r-regular graph with n vertices. Let u and v be the distinct vertices of G. If u and v have k common neighbours, then $H_d(s(u), s(v)) = 2r - 2k$.*

Proof. We consider here two cases.

Case 1. Let u and v be the adjacent vertices of G. Let u and v have k common neighbours and l non-common neighbours. Then by Theorem 1 (i),

$$H_d(s(u), s(v)) = n - k - l. \tag{1}$$

But from Lemma 2 (i), $2r = n + k - l$. Which implies $l = n + k - 2r$. Substituting this in (1),

$$H_d(s(u), s(v)) = n - k - (n + k - 2r) = 2r - 2k.$$

Case 2. Let u and v be the nonadjacent vertices of G. Let u and v have k common neighbours and l non-common neighbours. Then by Theorem 1 (ii),

$$H_d(s(u), s(v)) = n - k - l - 2. \tag{2}$$

But from Lemma 2 (ii), $2r = n+k-l-2$. Which implies $l+2 = n+k-2r$. Substituting this in (2),

$$H_d(s(u), s(v)) = n - k - (n + k - 2r) = 2r - 2k. \qquad \square$$

A connected acyclic graph is called *tree*.

Theorem 3. *Let G be a tree with n vertices. Let u and v be the distinct vertices of G.*

(i) *If $d_G(u, v) \neq 2$, then $H_d(s(u), s(v)) = deg_G(u) + deg_G(v)$.*

(ii) *If $d_G(u, v) = 2$, then $H_d(s(u), s(v)) = deg_G(u) + deg_G(v) - 2$.*

Proof. (i) Let $d_G(u, v) \neq 2$. We consider here two cases.

Case 1. Let $d_G(u, v) = 1$. Then u and v have zero common neighbours and $n - (deg_G(u) + deg_G(v))$ non-common neighbours. Therefore from Theorem 1 (i),

$$\begin{aligned} H_d(s(u), s(v)) &= n - 0 - [n - (deg_G(u) + deg_G(v))] \\ &= deg_G(u) + deg_G(v). \end{aligned}$$

Case 2. Let $d_G(u, v) > 2$. Then u and v have zero common neighbours and $n - (deg_G(u) + deg_G(v) + 2)$ non-common neighbours. Therefore from Theorem 1 (ii),

$$\begin{aligned} H_d(s(u), s(v)) &= n - 0 - [n - (deg_G(u) + deg_G(v) + 2)] - 2 \\ &= deg_G(u) + deg_G(v). \end{aligned}$$

(ii) Let $d_G(u, v) = 2$. Then u and v have 1 common neighbour and $n - (deg_G(u) + deg_G(v) + 1)$ non-common neighbours. Therefore from Theorem 1 (ii),

$$\begin{aligned} H_d(s(u), s(v)) &= n - 1 - [n - (deg_G(u) + deg_G(v) + 1)] - 2 \\ &= deg_G(u) + deg_G(v) - 2. \end{aligned} \qquad \square$$

4. Sum of Hamming distances of some graphs

As usual, by K_n, P_n, C_n and $K_{a,n-a}$ we denote respectively the complete graph, the path, the cycle and the complete bipartite graph on n vertices.

Theorem 4. *For a complete graph K_n, $H_A(K_n) = n(n-1)$.*

Proof. Complete graph K_n on n vertices is a regular graph of degree $n-1$. In a complete graph every pair of adjacent vertices has $n-2$ common neighbours and zero non-common neighbours. Also there is no pair of nonadjacent vertices.

Therefore from Theorem 2, $H_d(s(u), s(v)) = 2(n-1) - 2(n-2) = 2$ for every pair of vertices u and v of K_n and there are $\binom{n}{2}$ such pairs in K_n.

Therefore

$$H_A(K_n) = \sum_{\{u,v\} \subseteq V(K_n)} H_d(s(u), s(v)) = \sum_{\binom{n}{2}} 2 = n(n-1). \qquad \square$$

Theorem 5. *For a complete bipartite graph $K_{p,q}$, $H_A(K_{p,q}) = pq(p+q)$.*

Proof. The graph $K_{p,q}$ has $n = p+q$ vertices and $m = pq$ edges. Every pair of adjacent vertices of $K_{p,q}$ has zero common neighbours and zero non-common neighbours. Therefore from Theorem 1 (i), $H_d(s(u), s(v)) = p+q$ for adjacent vertices u and v.

Let V_1 and V_2 be the partite sets of the vertices of a graph $K_{p,q}$, where $|V_1| = p$ and $|V_2| = q$. Let u and v be the nonadjacent vertices. If $u, v \in V_1$ then u and v has q common neighbours and $p-2$ non-common neighbours. Therefore from Theorem 1 (ii), $H_d(s(u), s(v)) = (p+q)-(q)-(p-2)-2 = 0$. Similarly, if $u, v \in V_2$, then $H_d(s(u), s(v)) = 0$. Therefore

$$\begin{aligned} H_A(K_{p,q}) &= \sum_{d_G(u,v)=1} H_d(s(u), s(v)) + \sum_{d_G(u,v)\neq 1} H_d(s(u), s(v)) \\ &= \sum_{m} (p+q) = m(p+q) = pq(p+q). \qquad \square \end{aligned}$$

Theorem 6. *For a cycle C_n on $n \geqslant 3$ vertices, $H_A(C_n) = 2n(n-2)$.*

Proof. For $n = 3$ and $n = 4$, the result follows from the Theorems 4 and 5 respectively. Now we prove for $n \geqslant 5$. Cycle C_n is a regular graph of degree $r = 2$. In C_n, $n \geqslant 5$, every pair of adjacent vertices has zero common neighbours. Therefore from Theorem 2, $H_d(s(u), s(v)) = 2r - 0 = 4$ for every pair of adjacent vertices u and v of C_n.

Let $d_{C_n}(u, v) = 2$. Then u and v have 1 common neighbour. Therefore from Theorem 2, $H_d(s(u), s(v)) = 2r - 2 = 2$.

Let $d_{C_n}(u, v) \geqslant 3$. Then u and v have zero common neighbours. Therefore from Theorem 2, $H_d(s(u), s(v)) = 2r - 0 = 4$.

Therefore

$$
\begin{aligned}
H_A(C_n) &= \sum_{d_{C_n}(u,v)=1} H_d(s(u),s(v)) + \sum_{d_{C_n}(u,v)=2} H_d(s(u),s(v)) \\
&\quad + \sum_{d_{C_n}(u,v)>2} H_d(s(u),s(v)) \\
&= m(4) + n(2) + \left(\binom{n}{2} - m - n\right)(4) = 2n(n-2). \qquad \square
\end{aligned}
$$

A graph G is said to be *strongly regular* with parameters (n, r, a, b) if it is non complete r-regular graph with n vertices in which every pair of adjacent vertices has a common neighbours and every pair of nonadjacent vertices has b common neighbours.

Theorem 7. *Let G be a strongly regular graph with parameters (n, r, a, b). Then $H_A(G) = n(n-1)(r-b) + nr(b-a)$.*

Proof. Let G be a strongly regular graph with parameters (n, r, a, b) and m edges.

From Theorem 2, $H_d(s(u), s(v)) = 2r - 2a$, if u and v are adjacent vertices and $H_d(s(u), s(v)) = 2r - 2b$, if u and v are nonadjacent vertices. Therefore

$$
\begin{aligned}
H_A(G) &= \sum_{d_G(u,v)=1} H_d(s(u),s(v)) + \sum_{d_G(u,v)\neq 1} H_d(s(u),s(v)) \\
&= \sum_{m}(2r-2a) + \sum_{\binom{n}{2}-m}(2r-2b) \\
&= m(2r-2a) + \left(\binom{n}{2} - m\right)(2r-2b) \\
&= \frac{nr}{2}(2r-2a) + \left(\binom{n}{2} - \frac{nr}{2}\right)(2r-2b) \\
&= n(n-1)(r-b) + nr(b-a). \qquad \square
\end{aligned}
$$

Theorem 8. *Let G be a graph with n vertices and m edges and $\overline{G}$ be the complement of G. Then*

$$
H_A(\overline{G}) = H_A(G) + n(n-1) - 4m. \tag{3}
$$

Proof. Let u and v be the vertices of G. If the adjacent vertices u and v have k_1 common neighbours and l_1 non-common neighbours in G, then from Theorem 1 (i)

$$H_d(s(u), s(v)) = n - k_1 - l_1. \tag{4}$$

Further, if the nonadjacent vertices u and v have k_2 common neighbours and l_2 non-common neighbours in G, then from Theorem 1 (ii)

$$H_d(s(u), s(v)) = n - k_2 - l_2 - 2. \tag{5}$$

Therefore

$$\begin{aligned} H_A(G) &= \sum_{d_G(u,v)=1} H_d(s(u), s(v)) + \sum_{d_G(u,v)\neq 1} H_d(s(u), s(v)) \\ &= \sum_{d_G(u,v)=1} (n - k_1 - l_1) + \sum_{d_G(u,v)\neq 1} (n - k_2 - l_2 - 2) \\ &= \sum_{d_G(u,v)=1} (n-k_1-l_1) + \sum_{d_G(u,v)\neq 1} (n-k_2-l_2) - 2\left(\binom{n}{2} - m\right). \end{aligned} \tag{6}$$

If the vertices u and v are adjacent (nonadjacent) in G, then they are nonadjacent (adjacent) in $\overline{G}$. Therefore from (4) and (5), in $\overline{G}$, $H_d(s(u), s(v)) = n - k_1 - l_1 - 2$ for nonadjacent pairs of vertices and $H_d(s(u), s(v)) = n - k_2 - l_2$ for adjacent pairs of vertices. Therefore

$$\begin{aligned} H_A(\overline{G}) &= \sum_{d_{\overline{G}}(u,v)\neq 1} H_d(s(u), s(v)) + \sum_{d_{\overline{G}}(u,v)=1} H_d(s(u), s(v)) \\ &= \sum_{d_{\overline{G}}(u,v)\neq 1} (n - k_1 - l_1 - 2) + \sum_{d_{\overline{G}}(u,v)=1} (n - k_2 - l_2) \\ &= \sum_{d_G(u,v)=1} (n - k_1 - l_1 - 2) + \sum_{d_G(u,v)\neq 1} (n - k_2 - l_2) \\ &= \sum_{d_G(u,v)=1} (n - k_1 - l_1) - 2m + \sum_{d_G(u,v)\neq 1} (n - k_2 - l_2). \end{aligned} \tag{7}$$

From (6) and (7) the result follows. □

A graph is said to be *selfcomplementary* if it is isomorphic to its complement.

Theorem 9. *Let G be a graph with n vertices and m edges. Let $\overline{G}$ be the complement of G. Then $H_A(G) = H_A(\overline{G})$ if and only if G is selfcomplementary graph.*

Proof. If G is a selfcomplementary graph, then $G \cong \overline{G}$. Therefore $H_A(G) = H_A(\overline{G})$.

Conversely, let $H_A(G) = H_A(\overline{G})$. Therefore from (3), $n(n-1)-4m = 0$. This gives that $m = (n(n-1))/4$ implying G is a selfcomplementary graph [12]. □

Theorem 10. *Let G be a tree on n vertices. Then*

$$H_A(G) = \sum_{d_G(u,v)\neq 2} [deg(u) + deg(v)] + \sum_{d_G(u,v)=2} [deg(u) + deg(v) - 2].$$

Proof. Follows from the Theorem 3. □

Theorem 11. *For a path P_n on $n \geqslant 2$ vertices, $H_A(P_n) = 2n^2 - 6n + 6$.*

Proof. Let $v_1, v_2, \dots, v_n$ be the vertices of P_n where v_i is adjacent to v_{i+1}, $i = 1, 2, \dots, n-1$. There are $n-2$ pairs of vertices which are at distance two in P_n. Out of these $n-2$ pairs, two pairs (v_1, v_3) and (v_{n-2}, v_n) have Hamming distance equal to one and the remaining pairs have Hamming distance 2.

There are $\binom{n}{2} - (n-2)$ pairs of vertices in P_n, which are at distance 1 or at distance greater than 2. Out of these pairs, the one pair (v_1, v_n) has Hamming distance 2, $2n-6$ pairs of vertices, in which exactly one vertex is end vertex, have Hamming distance 3 and the remaining $\binom{n}{2} - 3n + 7$ pairs of vertices have Hamming distance 4. Therefore from Theorem 3,

$$\begin{aligned} H_A(P_n) &= \sum_{d_{P_n}(u,v)=2} H_d(s(u), s(v)) + \sum_{d_{P_n}(u,v)\neq 2} H_d(s(u), s(v)) \\ &= \sum_{d_{P_n}(u,v)=2} (deg_{P_n}(u) + deg_{P_n}(v) - 2) \\ &\quad + \sum_{d_{P_n}(u,v)\neq 2} (deg_{P_n}(u) + deg_{P_n}(v)) \\ &= [2(3-2) + (n-4)(4-2)] \\ &\quad + \left\{[(1)(2) + (2n-6)(3)] + \left[\binom{n}{2} - (n-2) - (2n-6) - 1\right](4)\right\} \\ &= 2n^2 - 6n + 6. \end{aligned}$$

□

5. Conclusion

Theorems 1 and 2 gives the Hamming distance between the strings generated by adjacency matrix of a graph in terms of number of common neighbours and non-common neighbours. Results of Section 4 gives the sum of Hamming distances between all pairs of strings generated by the adjacency matrix for some standard graphs like complete graph, complete bipartite graph, tree, cycle, path, strongly regular graph. Further there is a scope to extend these reults to graph valued functions such as line graph, total graph, product graphs.

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