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First sound velocity in liquid ^{4}He

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Based on the many-boson system structure factor, which takes into account three- and four-particle direct correlations, there was found the first sound velocity temperature behaviour in liquid ^{4}He in the post-RPA approximation. The expression received for the sound velocity matches with the well-known results in both low and high temperature limits. The results of this paper can be used to analyze the contributions of three- and four-particle correlations into thermodynamic and structural features of liquid ^{4}He.

Key words: *boson systems, liquid ^{4}He, first sound velocity*

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1. Introduction

The study of the sound velocity in Fermi and Bose systems has been attracting attention of researchers for a long time and still remains relevant today [1–9]. There are a lot of various many-particle systems, in which these investigations take place nowadays, for example: mixtures of ^{3}He and ^{4}He [10–12], ^{4}He in aerogel [1, 2], solid ^{3}He [3], plasma [13], trapped Bose gas [9], etc.

Experimental study of the sound velocity temperature behavior in many-boson system began almost 80 years ago. Barton [14] and a year later Findlay with colleagues [15, 16] using ultrasonic method made measurements in liquid ^{4}He under saturated vapor pressure (experiments were also carried out at higher pressures). The range of the research began from temperature $T = 1.75$ K. The precision of measurements was estimated less than 0.5 percent by authors. The curve of the then obtained temperature behavior perfectly conforms with the current results [17].

Hronevold [18] interpreted these experimental data. He proved that the ultrasonic waves in helium II are adiabatic despite their high frequency and the large thermal conductivity of helium. He also made attempts to explain a much smaller gap of the sound velocity at the λ-point transition compared to the value which stems from the Ehrenfest equation.

Another issue that caused interest of researchers was the sound attenuation in the ^{4}He liquid. Pellet and Skvayyer [19] sought it experimentally and showed that the experimental data are in a good agreement (at least for the temperature region from 3.2 K to 4.2 K) with theoretical results obtained from the classical formulae in assumption that the reasons of sound attenuation are only viscosity and heat losses. Later on, the attenuation and the sound velocity in ^{4}He were calculated using the Landau-Khalatnikov kinetic equations and the phonon Boltzmann equation [20]. In paper [21], the authors compared the theoretical results of the critical behavior of the sound propagation in liquid ^{4}He (as well as in other liquids) near the gas-liquid critical point which was derived within the field-theoretic renormalization group formalism with the experimental data. Another phenomenon that is studied nowadays is the first sound reflection in liquid ^{4}He [22].

Theoretical study of the sound velocity was also carried out by the collective variables representative method, but only in the limit of low temperatures [23–27]. Particularly in papers [24, 26], correlation was found between the first sound velocity at absolute zero temperature and the Fourier coefficient of pair interparticle interaction energy in the “one sum over the wave vector” approximation. This result was obtained in two different ways. The first one went through finding longwave asymptotics of structural quantities, which take into account direct three- and four-particle correlations, and the second one was based on the well-known thermodynamic relation: $mc^2/N = \partial^2 E_0/\partial N^2$, where c is the first sound velocity, N is the number of particles, m is the particle mass, E_0 is ground state energy taken in the pair correlations approximation. The above mentioned relations in higher approximation ("two sums over the wave vector") were found in paper [28].

The objective of this paper is to find the temperature behavior of the first sound velocity in liquid ^{4}He. We proceed from the exact relation that links the sound velocity with the longwave limit of the structure factor [29], as well as from the results which we obtained earlier for the pair structure factor in the "one sum over the wave vector" approximation that takes into account three- and four-particles correlations [30]. It should be noted that the calculation of the sound velocity through the structure factor in pair correlations approximation only leads to a constant value in pre-critical region and gives a light growth in the above-critical region (this behavior does not conform with the experimental data). The results of the sound velocity calculation in the post-RPA approximation show a fairly good agreement with the experimental data. The resulting expression for the first sound velocity in the limit of low temperatures matches with the already known one [24, 26].

2. Structure factor and first sound velocity in many-boson system

It is known [29] that there is a relation between the sound velocity value in the wide temperature region and the longwave limit of the structure factor:

$$\lim_{q\to 0} S(q) = \frac{T}{mc^2(T)}\,. \tag{2.1}$$

Sound velocity temperature behaviour obtained from the expression for the structure factor in pair correlations approximation yields incorrect results in the above-critical region, as it was already mentioned. Therefore, we should use the post-RPA approximation which takes into account three- and four-particle correlations [28]:

$$S_q = \frac{S_0(q)}{1+(\lambda_q+\Pi_q)S_0(q)}\,, \tag{2.2}$$

where $S_0(q)$ is the two-particle structure factor of the ideal bose gas,

$$\begin{aligned}\Pi_q = &-\frac{1}{2N}\frac{1}{S_0^2(q)}\sum_{\mathbf{k}\neq 0}\frac{\lambda_k S_0^{(4)}(\mathbf{q},-\mathbf{q},\mathbf{k},-\mathbf{k})}{1+\lambda_k S_0(k)} + \frac{1}{2N}\frac{1}{S_0^2(q)}\sum_{\mathbf{k}\neq 0}\frac{\lambda_k\lambda_{|\mathbf{q}+\mathbf{k}|}\left[S_0^{(3)}(\mathbf{q},\mathbf{k},-\mathbf{q}-\mathbf{k})\right]^2}{[1+\lambda_k S_0(k)][1+\lambda_{|\mathbf{q}+\mathbf{k}|}S_0(|\mathbf{q}+\mathbf{k}|)]}\\ &+4C_2(\mathbf{q}) + \frac{12}{N}\sum_{\mathbf{k}\neq 0}\frac{C_3(\mathbf{q},\mathbf{k},-\mathbf{q}-\mathbf{k})S_0^{(3)}(\mathbf{q},\mathbf{k},-\mathbf{q}-\mathbf{k})}{S_0(q)[1+\lambda_k S_0(k)][1+\lambda_{|\mathbf{q}+\mathbf{k}|}S_0(|\mathbf{q}+\mathbf{k}|)]} + \frac{8}{N}\sum_{\mathbf{k}\neq 0}\frac{C_4(\mathbf{q},\mathbf{k})S_0(k)}{1+\lambda_k S_0(k)}\\ &+\frac{72}{N}\sum_{\mathbf{k}\neq 0}\frac{C_3^2(\mathbf{q},\mathbf{k},-\mathbf{q}-\mathbf{k})S_0(k)S_0(|\mathbf{q}+\mathbf{k}|)}{[1+\lambda_k S_0(k)][1+\lambda_{|\mathbf{q}+\mathbf{k}|}S_0(|\mathbf{q}+\mathbf{k}|)]}\,,\end{aligned} \tag{2.3}$$

$$\lambda_q = \alpha_q\tanh(\beta E_q) - \tanh(\beta\varepsilon_q), \qquad \alpha_q = \sqrt{1+\frac{2N}{V}\nu_q\bigg/\frac{\hbar^2 q^2}{2m}}\,, \tag{2.4}$$

$\nu_q = \int \mathrm{e}^{-\mathrm{i}\mathbf{q}\mathbf{r}}\Phi(r)\mathrm{d}\mathbf{r}$ is the Fourier coefficient of the pair interparticle interaction energy; $\beta = 1/T$, T is temperature.

Generally, we are not interested in the explicit form of the pair interparticle interaction energy because finally we express its Fourier coefficient from the experimentally measured structure factor. At the same time, the existence and finiteness of the Fourier coefficient of the pair interparticle interaction energy in the ^{4}He liquid (it is an important issue in our theory) follow from the fact of the existence of the investigated system, for which we have experimentally measured the scattering length.

The explicit look of the expressions for the quantities $C_2(\mathbf{q}_1)$, $C_3(\mathbf{q}_1,\mathbf{q}_2,\mathbf{q}_3)$, $C_4(\mathbf{q}_1,\mathbf{q}_2)$ is described in paper [30]. In the longwave limit, we obtain such expressions for these quantities as well as for λ_q:

$$C_2^0(T)=\lim_{q\to 0}C_2(\mathbf{q})=\frac{1}{32N}\sum_{\mathbf{k}\neq 0}\frac{(\alpha_k^2-1)^2}{\alpha_k^4}\left[\frac{\beta^2E_k^2}{\sinh^2(\beta E_k)}+\beta E_k\coth(\beta E_k)-2\right],\tag{2.5}$$

$$C_3^0(k,T)=\lim_{q\to 0}C_3(\mathbf{q},\mathbf{k},-\mathbf{q}-\mathbf{k})=\frac{1}{24}\frac{\alpha_k^2+1}{\alpha_k}\tanh\left(\frac{\beta}{2}E_k\right)-\frac{1}{12}\tanh\left(\frac{\beta}{2}\varepsilon_k\right)+\frac{1}{48}\frac{\beta\varepsilon_k(1-\alpha_k^2)}{\cosh^2\left(\frac{\beta}{2}E_k\right)},\tag{2.6}$$

$$\begin{aligned}C_4^0(k,T)=\lim_{q\to 0}C_4(\mathbf{q},\mathbf{k})=\frac{1}{32}\Bigg[&\frac{\beta^2E_k^2(\alpha_k^2-1)^2}{4\alpha_k^3}\frac{\tanh\left(\frac{\beta}{2}E_k\right)}{\cosh^2\left(\frac{\beta}{2}E_k\right)}+\frac{\beta E_k}{\cosh^2\left(\frac{\beta}{2}E_k\right)}\frac{(\alpha_k^2-1)^2+2(\alpha_k^4-1)}{4\alpha_k^3}\\&-\frac{(\alpha_k^2-1)^2+2(\alpha_k^4-1)}{2\alpha_k^3}\tanh\left(\frac{\beta}{2}E_k\right)-\frac{4}{\alpha_k}\tanh\left(\frac{\beta}{2}E_k\right)+4\tanh\left(\frac{\beta}{2}\varepsilon_k\right)\Bigg],\end{aligned}\tag{2.7}$$

$$\lim_{q\to 0}\lambda_q=\beta\rho\nu_0,\tag{2.8}$$

where $\varepsilon_k=\hbar^2k^2/2m$, $E_k=\varepsilon_k\alpha_k$, ρ is the equilibrium density of liquid ^{4}He. Using expressions for pair, three- and four-particle structure factors of ideal bose-gas (see Appendix) we can also find the values for $1/S_0(q)$, $S_0^{(3)}(\mathbf{q},\mathbf{k},-\mathbf{q}-\mathbf{k})/S_0(q)$ and $S_0^{(4)}(\mathbf{q},\mathbf{k})/S_0^2(q)$ in longwave limit:

$$S_2^0(T)\equiv\lim_{q\to 0}\frac{1}{S_0(k)}=\begin{cases}0, & (T\leqslant T_\mathrm{c}),\\ \dfrac{1}{1+F_1(T)}, & (T>T_\mathrm{c}),\end{cases}\tag{2.9}$$

$$S_3^0(k,T)\equiv\lim_{q\to 0}\frac{S_0^{(3)}(\mathbf{q},\mathbf{k},-\mathbf{q}-\mathbf{k})}{S_0(q)}=\begin{cases}2n_k+1, & (T<T_\mathrm{c}),\\ 1+2\dfrac{S_0(k)-1+F_2(k,T)}{1+F_1(T)}, & (T>T_\mathrm{c}),\end{cases}\tag{2.10}$$

$$S_4^0(k,T)\equiv\lim_{q\to 0}\frac{S_0^{(4)}(\mathbf{q},\mathbf{k})}{S_0^2(q)}=\begin{cases}0, & (T\leqslant T_\mathrm{c}),\\ 2[S_0(k)-1+2F_2(k,T)+F_3(k,T)], & (T>T_\mathrm{c}),\end{cases}\tag{2.11}$$

where T_c is the critical temperature, $n_k=[z_0^{-1}\exp(\beta\varepsilon_k)-1]^{-1}$ are occupation numbers, $z_0=\exp(\beta\mu)$ is activity, μ is chemical potential.

The explicit look for $F_1(T)$, $F_2(k,T)$, $F_3(k,T)$ functions is as follows:

$$F_1(T)=\frac{1}{N}\sum_{\mathbf{p}\neq 0}n_p^2=\frac{1}{2\pi^2\rho}\int_0^\infty\frac{p^2\mathrm{d}p}{\left[z_0^{-1}\exp(\beta p^2)-1\right]^2},\tag{2.12}$$

$$\begin{aligned}F_2(k,T)=\frac{1}{N}\sum_{\mathbf{p}\neq 0}n_p^2n_{|\mathbf{p}+\mathbf{k}|}=\frac{1}{4\pi^2\rho}\int_0^\infty&\frac{p^2\mathrm{d}p}{\left[z_0^{-1}\exp(\beta p^2)-1\right]^2}\\&\times\left(\frac{1}{2\beta pk}\ln\left\{\frac{z_0^{-1}\exp\left[\beta(p+k)^2\right]-1}{z_0^{-1}\exp\left[\beta(p+k)^2\right]-1}\right\}-2\right),\end{aligned}\tag{2.13}$$

$$F_3(k,T) = \frac{1}{N}\sum_{\mathbf{p}\neq 0} n_p^2 n_{|\mathbf{p}+\mathbf{k}|}^2 = \frac{1}{8\pi^2\rho\beta k}\int_0^\infty \frac{p^2\mathrm{d}p}{\left[z_0^{-1}\exp(\beta p)-1\right]^2}\left(-\ln\left\{\frac{z_0^{-1}\exp\left[\beta(p+k)^2\right]-1}{z_0^{-1}\exp\left[\beta(p+k)^2\right]-1}\right\}\right.$$
$$\left. +4\beta pk - \frac{1}{z_0^{-1}\exp\left[\beta(p+k)^2\right]-1} + \frac{1}{z_0^{-1}\exp\left[\beta(p-k)^2\right]-1}\right). \tag{2.14}$$

As a result we obtain:

$$c^2(T) = \rho\nu_0 + \frac{T}{m}\left\{S_2^0(T) + \frac{1}{2N}\sum_{\mathbf{k}\neq 0}\frac{\lambda_k S_4^0(k,T)}{1+\lambda_k S_0(k)} - \frac{1}{2N}\sum_{\mathbf{k}\neq 0}\frac{\left[\lambda_k S_3^0(k,T)\right]^2}{[1+\lambda_k S_0(k)]^2}\right.$$
$$\left. +4C_2^0(T) + \frac{12}{N}\sum_{\mathbf{k}\neq 0}\frac{C_3^0(k,T)S_3^0(k,T)}{[1+\lambda_k S_0(k)]^2} + \frac{8}{N}\sum_{\mathbf{k}\neq 0}\frac{C_4^0(k,T)S_0(k)}{1+\lambda_k S_0(k)} + \frac{72}{N}\sum_{\mathbf{k}\neq 0}\frac{\left[C_3^0(k,T)S_0(k)\right]^2}{[1+\lambda_k S_0(k)]^2}\right\}. \tag{2.15}$$

3. Low and high temperature limit

In the low temperature limit

$$\lim_{T\to 0} TC_2^0(T) = \frac{1}{32N}\sum_{\mathbf{k}\neq 0}\frac{\varepsilon_k\left(\alpha_k^2-1\right)^2}{\alpha_k^3}, \qquad \lim_{T\to 0} TC_3^0(k,T) = 0, \qquad \lim_{T\to 0} TC_4^0(k,T) = 0,$$
$$\lim_{T\to 0} TS_2^0(k,T) = 0, \qquad \lim_{T\to 0} TS_3^0(k,T) = 0, \qquad \lim_{T\to 0} TS_4^0(k,T) = 0. \tag{3.1}$$

That is why

$$c^2 \equiv \lim_{T\to 0} c^2(T) = \frac{\rho\nu_0}{m} - \frac{1}{8mN}\sum_{\mathbf{k}\neq 0}\frac{\varepsilon_k\left(\alpha_k^2-1\right)^2}{\alpha_k^3}. \tag{3.2}$$

The same result we obtain if we take the second derivative of the energy (in the pair correlation approximation) on the number of particles:

$$c^2 = \frac{N}{m}\frac{\partial^2 E}{\partial N^2}. \tag{3.3}$$

Taking the square root from the equation (3.2) and using the smallness of the second term compared with the first one, we have:

$$c \equiv \lim_{T\to 0} c(T) = \sqrt{\frac{\rho\nu_0}{m}} - \frac{1}{16N\sqrt{m\rho\nu_0}}\sum_{\mathbf{k}\neq 0}\frac{\varepsilon_k\left(\alpha_k^2-1\right)^2}{\alpha_k^3}. \tag{3.4}$$

The other way to the same result goes through formula [24, 26]:

$$c = -\frac{\hbar}{m}\lim_{q\to 0} q\tilde{a}_2(q), \tag{3.5}$$

where the value $\tilde{a}_2(q)$ has such a look:

$$\tilde{a}_2(q) = -\frac{1}{2}(\alpha_q - 1) + \frac{1}{N}\sum_{\mathbf{k}\neq 0}\left[\frac{k^2}{2q^2\alpha_q}a_4(\mathbf{q},-\mathbf{q},\mathbf{k},-\mathbf{k}) + \frac{(\mathbf{k},\mathbf{q}+\mathbf{k})}{q^2\alpha_q}a_3(\mathbf{q},\mathbf{k},-\mathbf{q}-\mathbf{k})\right]. \tag{3.6}$$

In the high temperature limit, the contributions of three- and four-particle correlations are equal to zero [28]. Taking into account that $\lim\limits_{T\to 0} S_0(q) = 1$, we obtain a well-known classical expression for the first sound velocity in a high temperature region [29]:

$$c(T) = \sqrt{\frac{T}{m} + \frac{\rho\nu_0}{m}}. \tag{3.7}$$

4. Numeric calculations

We use the expression (2.15) for the first sound velocity numeric calculation. The transition from the summation to integration is carried out quite simply:

$$\frac{1}{N}\sum_{\mathbf{k}\neq 0} = \frac{1}{2\pi^2\rho}\int\limits_0^\infty k^2 \mathrm{d}k. \tag{4.1}$$

We can find the unknown quantity ν_0 using the value of the first sound velocity at zero temperature, which we obtain by extrapolating the experimental data. Therefore, from relation (3.2) we have:

$$\nu_0 = \frac{m}{\rho}\left[c^2 + \frac{1}{8mN}\sum_{k\neq 0}\frac{\varepsilon_k\left(\alpha_k^2 - 1\right)^2}{\alpha_k^3}\right]. \tag{4.2}$$

We use the value for ν_0 (4.2) only in the expressions that reproduce the approximation of pair correlations in order not to exceed accuracy. We use $\nu_0 = mc^2/\rho$ in expressions under the sum. We do numeric calculation using the effective mass of helium atom in liquid instead of real mass, in order to eliminate infra-red divergence [31]. The use of the effective mass is a phenomenological one, but it is a necessary step, which allows us to get rid of the essential consequences of approximate calculations (breaking of the series of the perturbation theory): the infra-red divergences and the incorrect Bose-Einstein condensation temperature.

We get the temperature behaviour of the isothermal sound velosity (c_T) based on the experimental data for the adiabatic sound velocity (c_σ) [17], specific heat at a constant volume (C_ν) and specific heat at a constant pressure (C_p) [32] using the well-known relationship: $c_T = c_\sigma\sqrt{C_\nu/C_p}$.

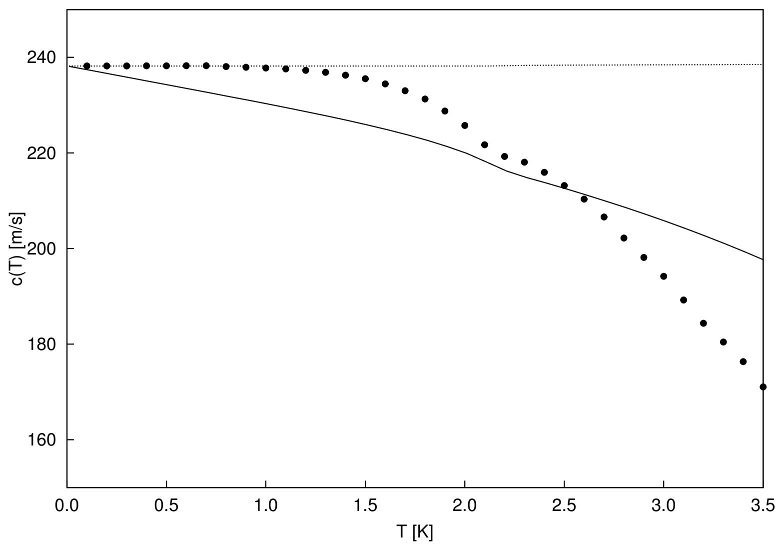

Figure 1. Temperature dependence of the first sound velocity in liquid ^{4}He. Dashed curve — pair correlations approximation, solid curve — post-RPA approximation, points — indirect experimental data (c_T).

5. Conclusions

In this paper we have got a temperature dependence of the first sound velocity in liquid ^{4}He. The received expression for the first sound velocity matches with the well-known results in both low and high temperature limits. As it can be seen in figure 1, the matching of the theoretical results with the experimental data is quite good but not sufficient. In order to improve it, we should take into account the next approximation for the sound velocity. It means that we need to use the expression for a structure factor in “two sums over the wave vector” approximation in our calculations.

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A. Appendix

Two-particle structure factor of the ideal bose gas [33]:

$$S_0(q) = 1 + 2\frac{n_0}{N} n_q + \frac{1}{N}\sum_{\mathbf{p}\neq\mathbf{0}} n_p n_{|\mathbf{p}+\mathbf{q}|}\,. \tag{A.1}$$

Three-particle structure factor of the ideal bose gas:

$$\begin{aligned} S_0^{(3)}(\mathbf{q},\mathbf{k},-\mathbf{q}-\mathbf{k}) &= 2\frac{n_0}{N}\left(n_q n_k + n_q n_{|\mathbf{q}+\mathbf{k}|} + n_k n_{|\mathbf{q}+\mathbf{k}|}\right) + S_0(q) + S_0(k) + S_0(|\mathbf{q}+\mathbf{k}|) - 2 \\ &+ \frac{2}{N}\sum_{\mathbf{p}\neq 0} n_p n_{|\mathbf{p}+\mathbf{q}|} n_{|\mathbf{p}-\mathbf{k}|}\,. \end{aligned} \tag{A.2}$$

Four-particle structure factor of the ideal bose gas after the elimination of the infra-red divergence:

$$\begin{aligned} S_0^{(4)}(\mathbf{q},-\mathbf{q},\mathbf{k},-\mathbf{k}) &= 2\frac{n_0}{N}\left(n_{|\mathbf{q}-\mathbf{k}|} + n_{|\mathbf{q}+\mathbf{k}|}\right) n_k(1+n_k) \\ &+ 2\left[S_0^{(3)}(\mathbf{q},\mathbf{k},-\mathbf{q}-\mathbf{k}) - S_0(q) - S_0(k) + 1\right] \\ &+ \frac{2}{N}\sum_{\mathbf{p}\neq 0} n_p n_{|\mathbf{p}+\mathbf{q}|} n_{|\mathbf{p}+\mathbf{k}|} n_{|\mathbf{p}+\mathbf{q}+\mathbf{k}|}\,. \end{aligned} \tag{A.3}$$

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Швидкість першого звуку в рідкому ^{4}He

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На основі виразу для структурного фактора багатобозонної системи з урахуванням прямих три- і чотиричастинкових кореляцій знайдено температурну поведінку швидкості першого звуку в рідкому ^{4}He в пост-RPA наближенні. У границі як низьких, так і високих температур отриманий вираз переходить у вже відомий. Результати можуть бути застосовані для аналізу внесків три- та чотиричастинкових кореляцій у термодинамічні та структурні функції рідкого ^{4}He.

Ключові слова: *Бозе системи, рідкий ^{4}He, швидкість першого звуку*