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The upper edge-to-vertex detour number of a graph

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Abstract. For two vertices u and v in a graph $G = (V, E)$, the *detour distance* $D(u, v)$ is the length of a longest u–v path in G. A u–v path of length $D(u, v)$ is called a *u–v detour*. For subsets A and B of V, the *detour distance* $D(A, B)$ is defined as $D(A, B) = \min\{D(x, y) : x \in A, y \in B\}$. A u–v path of length $D(A, B)$ is called an *A–B detour* joining the sets $A, B \subseteq V$ where $u \in A$ and $v \in B$. A vertex x is said to lie on an A–B detour if x is a vertex of an A–B detour. A set $S \subseteq E$ is called an *edge-to-vertex detour set* if every vertex of G is incident with an edge of S or lies on a detour joining a pair of edges of S. The *edge-to-vertex detour number* $dn_2(G)$ of G is the minimum order of its edge-to-vertex detour sets and any edge-to-vertex detour set of order $dn_2(G)$ is an *edge-to-vertex detour basis* of G. An edge-to-vertex detour set S in a connected graph G is called a *minimal edge-to-vertex detour set* of G if no proper subset of S is an edge-to-vertex detour set of G. The *upper edge-to-vertex detour number* $dn_2^+(G)$ of G is the maximum cardinality of a minimal edge-to-vertex detour set of G. The upper edge-to-vertex detour numbers of certain standard graphs are obtained. It is shown that for every pair a, b of integers with $2 \leq a \leq b$, there exists a connected graph G with $dn_2(G) = a$ and $dn_2^+(G) = b$.

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1. Introduction

By a *graph* $G = (V, E)$ we mean a finite undirected graph without loops or multiple edges. The *order* and *size* of G are denoted by p and q respectively. We consider connected graphs with at least two vertices. For basic definitions and terminologies we refer to [1, 5]. For vertices u and v in a connected graph G, the *distance* $d(u, v)$ is the length of a shortest u–v path in G. A u–v path of length $d(u, v)$ is called a u–v *geodesic*. For a vertex v of G, the *eccentricity* $e(v)$ is the distance between v and a vertex farthest from v. The minimum eccentricity among the vertices of G is the *radius*, *rad* G and the maximum eccentricity is its *diameter*, *diam* G of G. For vertices u and v in a connected graph G, the *detour distance* $D(u, v)$ is the length of a longest u–v path in G. A u–v path of length $D(u, v)$ is called a u–v *detour*. The *detour eccentricity* $e_D(v)$ of a vertex v in G is the maximum detour distance from v to a vertex of G. The *detour radius*, $rad_D G$ of G is the minimum detour eccentricity among the vertices of G, while the *detour diameter*, $diam_D G$ of G is the maximum detour eccentricity among the vertices of G. It is known that the distance and the detour distance are metrics on the vertex set V. The detour distance was studied by Chartrand et al. in [2,4]. A vertex x is said to lie on a u–v detour P if x is a vertex of P including the vertices u and v. A set $S \subseteq V$ is called a *detour set* if every vertex v in G lies on a detour joining a pair of vertices of S. The *detour number* $dn(G)$ of G is the minimum order of a detour set and any detour set of order $dn(G)$ is called a *detour basis* of G. A vertex v that belongs to every detour basis of G is a *detour vertex* in G. If G has a unique detour basis S, then every vertex in S is a detour vertex in G. These concepts were studied by Chartrand et al. [3]. The detour concepts and colorings are widely used in the Channel Assignment problem in radio technologies [4]. The *connected detour number* of a graph was introduced and studied in [8].

In general, there are graphs G for which there exist edges which do not lie on a detour joining any pair of vertices of V. For the graph G given in Figure 1.1, the edge v_1v_2 does not lie on a detour joining any pair of vertices of V. This motivated us to introduce the concepts of *weak edge detour set of a graph* and also *edge detour graphs* and were studied in [6, 7].

Definition 1.1 ([6])**.** Let $G = (V, E)$ be a connected graph with at least two vertices. A set $S \subseteq V$ is called a *weak edge detour set* of G if every edge in G has both its ends in S or it lies on a detour joining a pair of

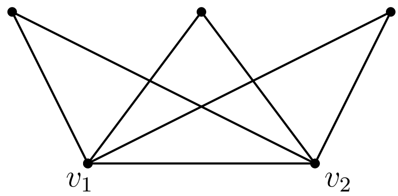

Figure 1.1: G

vertices of S. The *weak edge detour number* $dn_w(G)$ of G is the minimum order of its weak edge detour sets and any weak edge detour set of order $dn_w(G)$ is called a *weak edge detour basis* of G.

Example 1.2. For the graph G given in Figure 1.1, it is clear that the set $S = \{v_1, v_2\}$ is a weak edge detour basis of G so that $dn_w(G) = 2$. For the graph G given in Figure 1.2, it is clear that no two element subset of V is a weak edge detour set of G. The set $S = \{v_1, v_2, v_3\}$ is a weak edge detour basis of G so that $dn_w(G) = 3$. The set $S_1 = \{v_1, v_4, v_5\}$ is another weak edge detour basis of G.

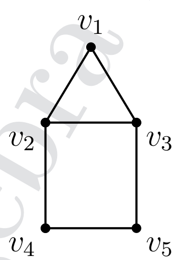

Figure 1.2: G

Definition 1.3 ([7])**.** Let $G = (V, E)$ be a connected graph with atleast two vertices. A set $S \subseteq V$ is called an *edge detour set* of G if every edge in G lies on a detour joining a pair of vertices of S. The *edge detour number* $dn_1(G)$ of G is the minimum order of its edge detour sets and any edge detour set of order $dn_1(G)$ is called an *edge detour basis* of G. A graph G is called an *edge detour graph* if it has an edge detour set.

Example 1.4. For the graph G given in Figure 1.2, it is clear that no two element subset of V is an edge detour set of G. The set $S = \{v_1, v_4, v_5\}$ is a an edge detour basis of G so that $dn_1(G) = 3$ and hence it is an edge detour graph. But the graph G given in Figure 1.1 is not an edge detour graph.

The *edge-to-vertex detour number* of a graph was introduced and studied in [9].

Definition 1.5. [9] Let $G = (V, E)$ be a connected graph with at least three vertices. For subsets A and B of V, the *detour distance* $D(A, B)$ is defined as $D(A, B) = \min\{D(x, y)\colon x \in A,\ y \in B\}$. A u–v path of length $D(A, B)$ is called an *A–B detour* joining the sets A and B, where $u \in A$ and $v \in B$. A vertex x is said to lie on an A–B detour if x is a vertex of an A–B detour. For $A = \{u, v\}$ and $B = \{z, w\}$ with uv and zw edges, we write an A–B detour as uv–zw detour and $D(A, B)$ as $D(uv, zw)$.

Example 1.6. For the graph G given in Figure 1.3, with $A = \{v_1,\ v_2\}$ and $B = \{v_4,\ v_5,\ v_6\}$, v_1, v_2, v_3, v_4 and v_1, v_6, v_5, v_4 are the v_1–v_4 detours, v_1, v_2, v_3, v_4, v_6, v_5 is the v_1–v_5 detour, v_1, v_2, v_3, v_4, v_5,v_6 is the v_1–v_6 detour, v_2, v_1, v_6, v_5, v_4 is the v_2–v_4 detour, v_2, v_1, v_6, v_4, v_5 and v_2, v_3, v_4, v_6, v_5 are the v_2–v_5 detours and v_2, v_3, v_4, v_5, v_6 is the v_2–v_6 detour. Hence $D(A, B) = 3$ and an A–B detour is a v_1–v_4 detour so that v_1, v_2, v_3, v_4 and v_1, v_6, v_5, v_4 are the only two A–B detours.

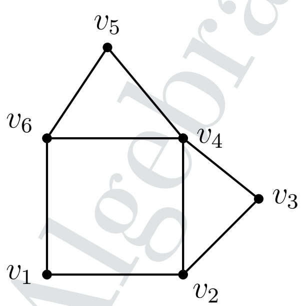

Figure 1.3: G

Definition 1.7. [9] Let $G = (V, E)$ be a connected graph with at least three vertices. A set $S \subseteq E$ is called an *edge-to-vertex detour set* of G if every vertex of G is incident with an edge of S or lies on a detour joining a pair of edges of S. The *edge-to-vertex detour number* $dn_2(G)$ of G is the minimum cardinality of its edge-to-vertex detour sets and any edge-to-vertex detour set of cardinality $dn_2(G)$ is an *edge-to-vertex detour basis* of G.

Example 1.8. For the graph G given in Figure 1.4, the two v_1v_2–v_4v_5 detours are $P : v_2, v_1, v_6, v_5$ and $Q : v_2, v_3, v_4, v_5$, each of length 3 so that $D(v_1v_2, v_4v_5) = 3$. Since the vertices v_6 and v_3 lie on the v_1v_2–v_4v_5 detours P and Q respectively, $S_1 = \{v_1v_2, v_4v_5\}$ is an edge-to-vertex detour basis of G so that $dn_2(G) = 2$. Also $S_2 = \{v_1v_6, v_3v_4\}$ is another edge-to-vertex detour basis of G. Thus there can be more than one edge-to-vertex detour basis for a graph.

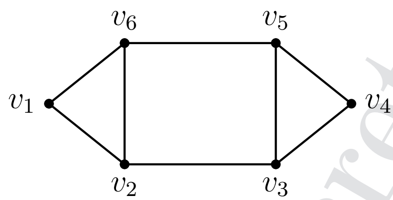

Figure 1.4: G

Throughout this paper G denotes a connected graph with at least three vertices. We need the following theorems in the sequel.

Theorem 1.9. [9] *Every end-edge of a connected graph G belongs to every edge-to-vertex detour set of G. Also if the set S of all end-edges of G is an edge-to-vertex detour set, then S is the unique edge-to-vertex detour basis for G.*

Theorem 1.10. [9] *If T is a tree with k end-edges, then $dn_2(T) = k$.*

2. The upper edge-to-vertex detour number of a graph

Definition 2.1. An edge-to-vertex detour set S in a connected graph G is called a *minimal edge-to-vertex detour set* of G if no proper subset of S is an edge-to-vertex detour set of G. The *upper edge-to-vertex detour number* $dn_2^+(G)$ of G is the maximum cardinality of a minimal edge-to-vertex detour set of G.

Example 2.2. For the graph G given in Figure 2.1, $S_1 = \{uv, xy\}$ and $S_2 = \{uv, vx, vy\}$, are the minimal edge-to-vertex detour sets of G so that $dn_2(G) = 2$ and $dn_2^+(G) = 3$.

It is clear that every minimum edge-to-vertex detour set is a minimal edge-to-vertex detour set. However, the converse is not true. For the graph G given in Figure 2.1, $S_2 = \{uv, vx, vy\}$ is a minimal edge-to-vertex detour

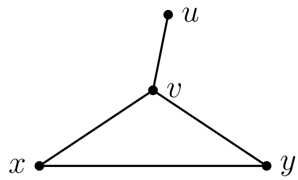

Figure 2.1: G

set of G but not a minimum edge-to-vertex detour set of G. Since any edge-to-vertex detour basis of a graph G is also a minimal edge-to-vertex detour set of G, we have the following theorem.

Theorem 2.3. *For any connected graph G, $2 \leq dn_2(G) \leq dn_2^+(G)$.*

We observe that the bound in Theorem 2.3 is sharp. For any path $P_n(n \geq 3)$, $dn_2(P_n) = dn_2^+(P_n) = 2$. Also for the graph G given in Figure 2.1, $dn_2(G) < dn_2^+(G)$.

Now, we proceed to determine $dn_2(G)$ and $dn_2^+(G)$ for some classes of graphs.

Theorem 2.4. (i) *For the complete graph K_p $(p \geq 4)$, a set S of edges is an edge-to-vertex detour basis if and only if S consists of two independent edges of K_p.*
(ii) *For the complete bipartite graph $K_{m,n}$ $(2 \leq m \leq n)$, a set S of edges is an edge-to-vertex detour basis if and only if S consists of two independent edges of $K_{m,n}$.*

Proof. (i) Let $S = \{e, f\}$ be any set of two independent edges of K_p. Then it is clear that $D(e, f) = p - 1$ and hence it follows that S is an edge-to-vertex detour set of K_p. Now, let S be an edge-to-vertex detour basis of K_p. Let S' be any set consisting of two independent edges. Then as in the first part of this theorem S' is an edge-to-vertex detour basis of K_p. Hence $|S| = |S'| = 2$. Let $S = \{e, f\}$. If e and f are not independent, then $D(e, f) = 0$ and since $p \geq 4$, S can not be an edge-to-vertex detour set of G, which is a contradiction. Thus S consists of two independent edges.
(ii) Let X and Y be the bipartite sets of $K_{m,n}$ $(2 \leq m \leq n)$ with $|X| = m$ and $|Y| = n$ and let $S = \{uv,\ zw\}$ be a set of any two independent edges of $K_{m,n}$ such that $u,\ z \in X$ and $v,\ w \in Y$. We show that S is an edge-to-vertex detour basis of $K_{m,n}$.
Case 1: Let $m = n = 2$. Then $K_{m,n} = C_4$ and it is clear that every vertex

of $K_{m,n}$ is incident with an edge of S so that S is an edge-to-vertex detour basis of $K_{m,n}$.

Case 2: Let $2 \leq m \leq n$ and $n \neq 2$. We consider two subcases:

Subcase 1: Let $m < n$. It is clear that $D(u,z) = 2(m-1)$, $D(u,w) = D(v,z) = 2m-1$, $D(v,w) = 2m$ and so $D(uv, zw) = 2(m-1)$. Let $y \in Y$ be any vertex different from v and w. If $m > 2$, consider any set of $m-2$ vertices $y_1, y_2, \ldots, y_{m-2}$ from $Y - \{v, y, w\}$. Then the vertex y lies on the uv–wz detour $P : u = x_1, y, x_2, y_1, x_3, y_2, \ldots, x_{m-1}, y_{m-2}, x_m = z$, where $x_1, x_2, \ldots, x_m \in X$. If $m = 2$, then y lies on the uv–wz detour $Q : u, y, z$. Since every vertex of X also lies on the same detour P and Q in respective cases, it follows that S is an edge-to-vertex detour basis of $K_{m,n}$ and hence $dn_2(K_{m,n}) = 2$.

Subcase 2: Let $m = n$. It is clear that $D(u,z) = D(v,w) = 2(m-1)$, $D(u,w) = D(v,z) = 2m-1$ and so $D(uv, zw) = 2(m-1)$. Also $P : u, v, x_1, y_1, x_2, y_2, \ldots, x_{m-2}, y_{m-2}, z$, where $u, x_1, x_2, \ldots, x_{m-2}, z \in X$ and $v, y_1, y_2, \ldots, y_{m-2} \in Y$ with $w \neq v_i$ $(1 \leq i \leq m-2)$ is a uv–zw detour containing all vertices of $K_{m,n}$ other than the vertex w. Since w is incident with the edge zw, it follows that S is an edge-to-vertex detour basis of $K_{m,n}$. The proof of the converse is similar to that of Theorem 2.4(i). □

Theorem 2.5. *For the complete graph K_p $(p \geq 3)$, a set S of edges is a minimal edge-to-vertex detour set of K_p if and only if S consists of any two independent edges or S consists of all edges incident at any vertex of K_p.*

Proof. For $p = 3$, it is clear that a set S of edges is a minimal edge-to-vertex detour set of K_3 if and only if S consists of all edges that are incident at a vertex of K_3.

Let $p \geq 4$. If S consists of any two independent edges of K_p, then by Theorem 2.4(i), S is an edge-to-vertex detour basis of K_p so that S is minimal. If S consists of all edges incident at any vertex, say v of K_p, then since every vertex of K_p is incident with an edge of S, it follows that S is an edge-to-vertex detour set of K_p. We show that S is a minimal edge-to-vertex detour set of K_p. If T is a proper subset of S, then there exists at least one edge, say $e = vv_1$ of S such that $e \notin T$. Then it is clear that the vertex v_1 neither lies on any detour joining a pair of edges of T nor is incident with any edge of $T$and so T is not an edge-to-vertex detour set of K_p. Thus S is a minimal edge-to-vertex detour set of K_p.

Conversely, assume that S is a minimal edge-to-vertex detour set of K_p $(p \geq 4)$. If $|S| = 2$, then S is an edge-to-vertex detour basis of G and so by Theorem 2.4(i), it is clear that S contains exactly two independent edges of K_p. Let $|S| = 3$. Since S is minimal, it follows from Theorem 2.4(i) that no two edges of S are independent. Hence it follows that the subgraph induced by S is either K_3 or the star $K_{1,3}$. If it is K_3, then since $p \geq 4$, it follows that S is not an edge-to-vertex detour set of K_p, which is a contradiction. Hence the subgraph induced by S is $K_{1,3}$. Since $p \geq 4$ and S is an edge-to-vertex detour set, it follows that the graph is K_4 and S contains all edges incident at any vertex of K_4.

Let $|S| \geq 4$. We show that the subgraph induced by S can not contain K_3. Suppose that the subgraph induced by S contains K_3. Let v_1, v_2, v_3 be the vertices of K_3. Since $|S| \geq 4$, there is an edge e in S different from the edges of K_3. Since S is minimal, it follows that the edge e is incident with a vertex, say v_1 of K_3. Now the edges e and v_2v_3 are independent and it follows that S is not minimal, which is a contradiction. Thus the subgraph induced by S does not contain K_3. Since S is an edge-to-vertex detour set of K_p, it follows that S contains all edges incident at any vertex of K_p. □

Theorem 2.6. *For the complete bipartite graph $K_{m,n}$ $(2 \leq m \leq n)$, a set S of edges is a minimal edge-to-vertex detour set of $K_{m,n}$ if and only if S consists of any two independent edges.*

Proof. Let S consist of any two independent edges of $K_{m,n}$. Then by Theorem 2.4(ii), S is an edge-to-vertex detour basis of $K_{m,n}$ so that S is minimal.

Conversely assume that S is a minimal edge-to-vertex detour set of $K_{m,n}$. If $|S| = 2$, then S is an edge-to-vertex detour basis of G and so by Theorem 2.4(ii), it is clear that S contains exactly two independent edges of $K_{m,n}$. Let $|S| \geq 3$. Since S is minimal, it follows from Theorem 2.4(ii) that no two edges of S are independent. Since the graph is a bipartite graph, the subgraph induced by S can not contain K_3. Hence it follows that the subgraph induced by S is a star at a vertex, say v. Let v belong to a bipartite set X of $K_{m,n}$. Since m, $n \geq 2$, there exists a vertex $u \in X$ such that $u \neq v$ and it is clear that the vertex u is neither incident with any edge of S nor lies on a detour joining a pair of edges of S. Hence S is not an edge-to-vertex detour set of $K_{m,n}$, which is a contradiction. Thus S consists of two independent edges. □

Theorem 2.7. (i) *If G is the complete graph K_p $(p \geq 3)$, then $dn_2(G) = 2$, $dn_2^+(G) = p - 1$.*
(ii) *If G is the complete bipartite graph $K_{m,n}$ $(2 \leq m \leq n)$, then $dn_2(G) =$*

$dn_2^+(G) = 2$.
(iii) *If G is a tree with k end-vertices, then $dn_2(G) = dn_2^+(G) = k$.*

Proof. (i) This follows from Theorem 2.4(i) and Theorem 2.5.
(ii) This follows from Theorem 2.4(ii) and Theorem 2.6.
(iii) This follows from Theorems 1.9 and 1.10. □

Problem 2.8. Characterize connected graphs G with $dn_2(G) = dn_2^+(G)$.

Theorem 2.9. *For any cycle $G = C_p$ of length $p \geq 3$, we have $dn_2(G) = 2$.*

Proof. For $p = 3$, the result follows from the Theorem 2.7(i). For $p \geq 4$, let $C_p : v_1, v_2, \ldots, v_{p-1}, v_p, v_1$ be the cycle of length $p \geq 4$. Let $S = \{v_1v_2, v_{p-1}v_p\}$. Then S is an edge-to-vertex-detour basis of C_p and so $dn_2(G) = 2$. □

Problem 2.10. Determine $dn_2^+(G)$ for a cycle G.

In view of Theorem 2.3, the following theorem gives a realization result.

Theorem 2.11. *For every pair a, b of integers with $2 \leq a \leq b$, there exists a connected graph G with $dn_2(G) = a$ and $dn_2^+(G) = b$.*

Proof. Let $a = b$. Then by Theorem 2.7(iii), $dn_2(T) = dn_2^+(T) = a$ for any tree T with a end-vertices. Let $2 \leq a < b$. Let G be the graph obtained from the complete graph K_{b-a+2} by adding $a - 1$ new vertices $y_1, y_2, \ldots, y_{a-1}$ and joining them to a vertex, say v of K_{b-a+2}. The graph G is connected and is shown in Figure 2.2. Let $v, v_1, v_2, \ldots, v_{b-a+1}$ be the vertices of K_{b-a+2}, $X = \{vv_1, vv_2, \ldots, vv_{b-a+1}\}$, $Y = \{vy_1, vy_2, \ldots, vy_{a-1}\}$ and Z be the set of edges of K_{b-a+2} which are not incident at v.

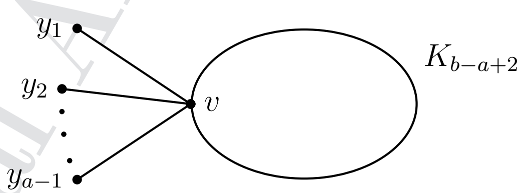

Figure 2.2: G

First, we show that $dn_2(G) = a$. By Theorem 1.9, every edge-to-vertex detour set of G contains Y. Clearly Y is not an edge-to-vertex detour set of G and so $dn_2(G) \geq |Y| + 1 = a$. On the other hand, let $S = Y \cup \{f\}$, where $f \in Z$. Then $D(e, f) = b - a + 1$ for any $e \in Y$ and $f \in Z$ and every vertex of K_{b-a+2} lies on a $e - f$ detour. Hence S is an edge-to-vertex detour set of G and so $dn_2(G) \leq |S| = a$. Therefore $dn_2(G) = a$.

Now, we show that $dn_2^+(G) = b$. Let $S = X \cup Y$. Then every vertex of G is incident with an edge of S and so S is an edge-to-vertex detour set of G. We show that S is a minimal edge-to-vertex detour set of G. Assume, to the contrary, that S is not a minimal edge-to-vertex detour set of G. Then there is a proper subset T of S such that T is an edge-to-vertex detour set of G. Since T is a proper subset of S, there exists an edge $e \in S$ and $e \notin T$. By Theorem 1.9, every edge-to-vertex detour set contains all end-edges of G and so we must have $e = vv_i$ for some $i\,(1 \leq i \leq b-a+1)$. Then it is clear that the vertex v_i neither lies on any detour joining a pair of edges of T nor is incident with any edge of T and so T is not an edge-to-vertex detour set of G, which is a contradiction. Thus S is a minimal edge-to-vertex detour set of G and so $dn_2^+(G) \geq |S| = b-a+1+a-1 = b$. Now, if $dn_2^+(G) > b$, then let M be a minimal edge-to-vertex detour set of G with $|M| > b$. Then there exists at least one edge, say $e \in M$ such that $e \notin S = X \cup Y$. By Theorem 1.9, M contains Y and hence e is an edge of K_{b-a+2} such that $e \neq vv_i\,(1 \leq i \leq b-a+1)$. Thus $e \in Z$ and $S' = Y \cup \{e\}$ is a proper subset of M. It is clear that S' is an edge-to-vertex detour set of G so that M is not a minimal edge-to-vertex detour set of G, which is a contradiction. Therefore, $dn_2^+(G) = b$. □

Remark 2.12. The graph G in Figure 2.2 contains exactly $(b-a+1)C_2+1$ minimal edge-to-vertex detour sets namely $X \cup Y$ and $Y \cup \{e\}$, where $e \in Z$. Hence this example shows that there is no "Intermediate Value Theorem" for minimal edge-to-vertex detour sets, that is, if k is an integer such that $dn_2(G) < k < dn_2^+(G)$, then there need not exist a minimal edge-to-vertex detour set of cardinality k in G.

Using the structure of the graph G constructed in the proof of Theorem 2.11, we can obtain a graph H_n of order n with $dn_2(G) = 2$ and $dn_2^+(G) = n - 1$ for all $n \geq 4$. Thus we have the following.

Theorem 2.13. *There is an infinite sequence $\{H_n\}$ of connected graphs H_n of order $n \geq 4$ such that $dn_2(H_n) = 2$, $dn_2^+(H_n) = n - 1$, $\lim_{n\to\infty} \frac{dn_2(H_n)}{n} = 0$ and $\lim_{n\to\infty} \frac{dn_2^+(H_n)}{n} = 1$.*

Proof. Let H_n be the graph obtained from the complete graph K_{n-1} by adding a new vertex y and joining it to a vertex, say v of K_{n-1}. Clearly the graph H_n is connected and is shown in Figure 2.3.

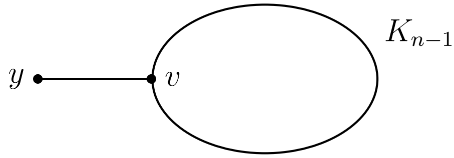

Figure 2.3: H_n

Let $v, v_1, v_2, \dots, v_{n-2}$ be the vertices of K_{n-1}, $X = \{vv_1, vv_2, \dots, vv_{n-2}\}$, $Y = \{vy\}$ and Z be the set of edges of K_{n-1} which are not incident at v. It is clear from the proof of Theorem 2.11 that the graph H_n contains exactly $(n-2)C_2 + 1$ minimal edge-to-vertex detour sets namely $X \cup Y$ and $Y \cup \{e\}$, where $e \in Z$ so that $dn_2(H_n) = 2$ and $dn_2^+(H_n) = n - 1$. Hence the theorem follows. □

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