THE SUPERRADIANCE OF A BUNCH OF ROTATING ELECTRONS

V.M. Kuklin, D.N. Litvinov, A.E. Sporov
V.N. Karazin Kharkiv National University, Kharkov, Ukraine
E-mail: v.m.kuklin@karazin.ua

Equations describing the excitation of a TE wave by a beam of electrons rotating in an external magnetic field in a waveguide in two regimes were considered. In the first regime the interaction of the oscillators – rotating electrons in the magnetic field – was neglected. It was assumed that the beam electrons interact only with the waveguide modes. In the second case, in the superradiance regime, the beam electrons interact with each other due to their spontaneous radiation. In the article the basic features of the description of the gyrotron gain regime are discussed.

PACS: 05.45.Xt, 52.40.Mj

INTRODUCTION

As a rule, in most papers the Larmor rotation radius of beam electrons is small, less than the characteristic size of the transverse field inhomogeneity, and is less than or of the order of the beam thickness (the beam electrons sufficiently uniformly fill a flat or a cylindrical layer). Scientific school of A.V. Gaponov in the former USSR has developed the methods for description of the excitation of eigen oscillations of the waveguides in the presence of an external magnetic field by an electron beam [1, 2], which helped to develop a number of (vacuum) devices and equipment.

Later, it became necessary to study the processes of excitation by charged-particle beams (usually electrons) of cyclotron oscillations in plasma, for its further heating, mainly for controlled thermonuclear fusion purposes. This led to a great cycle of the developments in Kharkov (see, for example, [3 - 5]) in the nonlinear theory of wave excitation in magnetoactive plasma media (and waveguides) by the beams of charged particles, in particular, and with finite values of the Larmor radius [6, 7]. In these works it was also shown that the conservation laws of the elementary effects of anomalous and normal Doppler are also satisfied for more complicated beam systems with stimulated radiation.

Similar original descriptions of nonlinear excitation processes of oscillations by the beams of charged particles, where the main attention was focused on the relativistic effects and the spatial restrictions of the complex resonant systems, were also considered by the authors [8 - 14].

Great scientific interest to one of the most powerful and popular generators – the gyrotron was associated with the need to take into account the plasma medium in its volume. Actually, if a certain value of the gyrotron power is exceeded, the gas extraction from the elements of the structure and its ionization takes place which leads to the appearance of a plasma with comparatively low density. The difficulty that deals with the impossibility to separate waves of different polarizations in a magnetoactive plasma was avoided in the work [15], with the help of introducing the small parameter of their coupling

$$g = \left(\omega^2 \varepsilon_{xy} / c^2 k_\perp{}^2\right) << 1,$$

here $\varepsilon_{xy} = i\omega_{pe}^2 \omega_B / \omega(\omega^2 - \omega_B^2)$ is the transverse component of the dielectric permittivity of a cold magnetoactive plasma. Moreover, the ratio of the longitudinal and transverse components of the wave vector was also small $(k_z / k_\perp) < 1$. Taking into account such low density plasma made it possible to increase the efficiency of the device (see, for example, [16]). But nevertheless, the dynamics of particles at resonances requires further detailed study [17 - 20]. In this work we dwell on the problem of radiation of an group of interaction with each other electrons under the conditions of the cyclotron resonance with a TE wave.

1. EXCITATION OF THE TE-WAVE BY THE NONINTERACTICE WITH EACH OTHER ELECTRONS

The dispersion equation of such wave in metallic waveguide can be written as:

$$D(\omega,\vec{k}) = (k_z^2 + k_\perp^2 - \frac{\omega^2}{c^2})(k_z^2 - \frac{\omega^2}{c^2})^{-1}, \tag{1}$$

at that $D_\omega = -\dfrac{2(k_z^2 + k_\perp^2)^{1/2}}{c}$, $D_{k_z} = 2k_z$,

where $k_\perp$ – is the transversal wavenumber that is defined by the boundary conditions, and the group velocity along the waveguide is expressed as

$$v_g = \frac{\partial(\omega D)/\partial k_z}{\partial(\omega D)/\partial\omega} = k_z c / (k_z^2 + k_\perp^2)^{1/2},$$

the longitudinal wave magnetic fields is written in the form [7]

$$B_z = b \cdot J_m(k_\perp r)\exp\{-i\omega t + im\theta + ik_z z\}, \tag{2}$$

and the field equation has the following form [7]

$$\frac{\partial B}{\partial \tau} + V_g \frac{\partial B}{\partial \xi} - i\Delta_\delta B + \Theta_\delta B = i\frac{1}{N}\sum_{j=1}^{N} a_j J_n^{'}(a_j)\exp\{-2\pi i\zeta_j\}, \tag{3}$$

where $\tau = \delta t$, $\mu = \dfrac{n\omega_{B0}}{2\delta\beta_{\perp 0}^2}$, $R = \dfrac{k_z^2\omega_B}{k_{ms}^2\delta}$, $M = 2\pi k_z d \cdot N_{0b}$,

$\xi = 2\pi k_z z$,

$\delta^2 = 4N_{b0}e^2\omega_B[m_e \cdot c^2 \cdot k_{ms}^2 \cdot r_W^2 \cdot J_m^2(x_{ms}) \cdot$
$(1 - \dfrac{m^2}{x_{ms}^2}) \cdot D_\omega]^{-1} \cdot J_{m-n}^2(k_{ms} r_C),$

$2\pi\zeta_\pm = 2\pi\zeta' \pm k_z z = -\omega \cdot t \pm k_z z + n\omega_B t + m \cdot \Phi_0 +$
$+(n-m)\Phi_{C0} + \pi m/2,$

$B = eb \cdot J_{m-n}(k_{ms} r_C) / m_e c \cdot \delta,$

$V_g = 2\pi\dfrac{k_z^2 c^2}{\omega\delta}$ $\eta = [(k_z v_z - \omega + n\omega_{B0})/\delta] + \mu \cdot (1 - \dfrac{a^2}{a_0^2})$, N_{b0} – is the number of the particles on the unit length of the waveguide. The electrons motion equations that do not interact with each other and interact only with the wave have the following form:

$$2\pi\frac{d\zeta_i}{d\tau}=\eta_i+nB\cdot J_n(a_i)\cdot[1-\frac{n^2}{a_i^2}]\cdot \mathrm{Cos}(2\pi\zeta_i+\varphi_e),$$
$$da_i/d\tau=-n\cdot B\cdot J_n'(a_i)\cdot \mathrm{Sin}(2\pi\zeta_i+\varphi_e), \qquad (4)$$
$$d\eta_i/d\tau=-R\cdot B\cdot a_i\cdot J_n'(a_i)\cdot \mathrm{Sin}(2\pi\zeta_i+\varphi_e).$$

2. SPONTANEOUS RADIATION OT THE SINGLE ELECTRON

The case of the radiation of the single particle (from its total number, that is equal to *N*) it is necessary to consider in the following manner. The equation for the field, that is radiated from a single particle, can be written in the form

$$v_g\frac{\partial B_j}{\partial z}=i\frac{a_j}{N}J_n'(a_j)\exp\{-2\pi i\zeta'_j\}\exp\{-ik_z z_j\}\cdot\delta(z-z_j) \qquad (5)$$

or $\frac{\partial B_j}{\partial z}=\lambda\cdot\delta(z-z_j)$, where

$$\lambda=i\frac{a_j}{Nv_g}J_n'(a_j)\exp\{-2\pi i\zeta'_j\}\exp\{-ik_z z_j\}$$

its solution has the form $B_j=C+\lambda\cdot\theta(z-z_j)$, where C – is a constant which must be defined.

Since for the wave that is radiated by the oscillator the equation $D(\omega,k)=0$ with the following roots

$$k_{z1,2}=\pm\mathrm{Re}\,D(1+i\,\mathrm{Im}\,D/\mathrm{Re}\,D)\approx\pm(\frac{\omega^2}{c^2}-k_\perp^2)^{1/2}(1+i0)$$

is valid, so for the wave that propagates in the $z>z_j$ direction, the wavenumber $k_z=k_{z1}>0$ and the constant C must be chosen to be equal to zero for avoiding field unlimited growth at the infinity. For the wave, that propagates in the $z<z_j$ direction, the wavenumber $k_z=k_{z2}<0$ and the value of the constant C for the same reasons must be chosen to be equal to $-\lambda$. At the same time the field amplitude can be expressed as

$$B_j(\xi)=i\frac{a_j}{NV_g}J_n'(a_j)\exp\{-2\pi i\zeta'_j\}[\exp\{2\pi i(\xi-\xi_j\}\cdot U(\xi-\xi_j)+\exp\{-2\pi i(\xi-\xi_j\}\cdot U(\xi_j-\xi)], \qquad (6)$$

here $U(z)=1$ when $z\geq 0$ and $U(z)=0$ when $z<0$. It is necessary to mention that the direction of the longitudinal component of the magnetic-field strength vector at that case does not depend on the wave propagation direction. Such behavior is caused by the suppression by them the eigen magnetic field of the rotating electron while the wave radiates in both directions.

3. THE EQUATIONS OF ELECTRONS BEAM SUPERRADIANCE

It is obvious that for the system of N oscillators the equation for the field can be written in the form

$$B(\xi)=i\frac{1}{2N\vartheta}\sum_{j=1}^{N}a_jJ_n'(a_j)\exp\{-2\pi i\zeta'_j\}[\exp\{2\pi i(\xi-\xi_j\}\cdot U(\xi-\xi_j)+\exp\{-2\pi i(\xi-\xi_j\}\cdot U(\xi_j-\xi)], \qquad (7)$$

where $\vartheta=2V_gN_{0b}/M=2v_g/d\cdot\delta$ – is the maximum increment δ to the damping decrement due to radiation from the ends of the system $2v_g/d$ ratio. It is necessary to note, that in such notations wave energy on the system length $\xi_M=2\pi k_z d$ to the particles energy ratio is expressed as such expression $\int_0^{\xi_M}|B(\xi)|^2d\xi/\frac{1}{N}\sum_{i=1}^{N}a_{i0}^2$.

Since the ratio of the energy radiated from the system to the total field energy at the time of order $1/\delta$ in the system is equal to ϑ, so the efficiency of the system (if the quantity $1/\delta$ is chosen as the time unit) may be evaluated as $\frac{\vartheta}{\xi_M}\int_0^{\xi_M}|B(\xi)|^2d\xi/\frac{1}{N}\sum_{i=1}^{N}a_{i0}^2$.

The beam electrons motion equation may be written as:

$$2\pi\frac{d\zeta_i}{d\tau}=\eta_i+\frac{n}{\vartheta}\left(1-\frac{n^2}{a^2}\right)J_n(a_i)\frac{1}{2N}\sum_{j=1}^{N}a_jJ_n'(a_j)\cdot[\mathrm{Sin}\{2\pi(\zeta_{i+}-\zeta_{j+})\}\cdot U(\zeta_{i+}-\zeta_{j+})+\mathrm{Sin}\{2\pi(\zeta_{i-}-\zeta_{j-})\}\cdot U(\zeta_{j-}-\zeta_{i-})];$$
$$\frac{da_i}{d\tau}=-n\frac{J'_n(a_i)}{2N\vartheta}\sum_{j=1}^{N}a_jJ_n'(a_j)[\mathrm{Cos}\{2\pi(\zeta_{i+}-\zeta_{j+})\}\cdot U(\zeta_{i+}-\zeta_{j+})+\mathrm{Cos}\{2\pi(\zeta_{i-}-\zeta_{j-})\}\cdot U(\zeta_{j-}-\zeta_{i-})]; \qquad (8)$$
$$\frac{d\eta_i}{d\tau}=-\frac{RJ'_n(a_i)}{2N\vartheta}\sum_{j=1}^{N}a_jJ_n'(a_j)[\mathrm{Cos}\{2\pi(\zeta_{i+}-\zeta_{j+})\}\cdot U(\zeta_{i+}-\zeta_{j+})+\mathrm{Cos}\{2\pi(\zeta_{i-}-\zeta_{j-})\}\cdot U(\zeta_{j-}-\zeta_{i-})].$$

It is necessary to mention that the field values are not needed for the calculations they are already included in the right-hand parts of (8).

Nevertheless it is possible to calculate the value of the longitudinal magnetic field (7), with the help of which it can possible to restore all other wave field components at any point as inside waveguide as outside it.

4. THE REGIME OF THE GYROTRON SUPERRADIANCE

For the small values of the Larmor radius, for $A_i=a_i\cdot\exp\{i\zeta_{i+}/n\}$ and for the exact resonance $\eta_i=0$, assuming that Bessel functions arguments are small $J_n(x)\approx(x/2)^n(\frac{1}{n}); J'_n(x)=\frac{1}{x}(x/2)^n$ let obtain the equation for the longitudinal magnetic wave field

$$B(\xi)=i\frac{1}{2N\vartheta}\sum_{j=1}^{N}(\frac{a_j}{2})^n\exp\{-2\pi i\zeta'_j\}\cdot[\exp\{2\pi i(\xi-\xi_j\}\cdot U(\xi-\xi_j)]. \qquad (9)$$

For the case of the system of electrons that rotate in the magnetic field and practically do not shift along the system let show the motion equations

$$a_i\frac{d\zeta_{i+}}{nd\tau}=(\mu/n)\cdot a_i(1-\frac{a_i^2}{a_{i0}^2})-\frac{na_i}{4\vartheta}\left(a_i/2\right)^{n-2}\cdot\frac{1}{2N}\sum_{j=1}^{N}(a_j/2)^n[\mathrm{Sin}\{2\pi(\zeta_{i+}-\zeta_{j+})\}\cdot U(\zeta_{i+}-\zeta_{j+})]; \qquad (10)$$
$$\frac{da_i}{d\tau}=-na_i\frac{\left(a_i/2\right)^{n-2}}{8N\vartheta}\sum_{j=1}^{N}(a_j/2)^n[\mathrm{Cos}\{2\pi(\zeta_{i+}-\zeta_{j+})\}\cdot U(\zeta_{i+}-\zeta_{j+})].$$

The similar approach was used by the authors of [1, 2] to create the theory of gyrotron. However, the gyrotron theory was based on the neglecting of the electrons interaction with each other due to their spontaneous radiation. These effects were neglected, considering that the electrons interact only with waveguide modes. In the dissipative regimes of generation $\Theta_\delta >> 1$, as was noted earlier (see also the article of the authors in this proceedings), the superradiance of interacting electrons and the dissipative generation regime under the conditions of neglecting of their interaction nevertheless lead to the same characteristic times of the process development and to the comparable generation intensities. However, these are different processes and they have a number of features, so we give the equations for the superradiation of a gyrotron in the gyrotron traditional gain regime. Moreover with the notation and terms those are similar to the papers [1, 2]. Thus for example, equations (10) can be written in a complex form

$$\frac{dA_i}{dZ} = i\Delta + i(|A|^2 - 1)\}A_i - GA_i \frac{\left(|A_i|/2\right)^{n-2}}{N} \cdot \tag{11}$$
$$\cdot \sum_{j=1}^{N} (\frac{|A_j|}{2})^n \exp\{2\pi i(\zeta_i - \zeta_j\} \cdot U(\zeta_i - \zeta_j)];$$

where the following quantities were used $\omega_b^2 = 4\pi e^2 N_{0b} / Sm_e$ – averaged over the volume, the Langmuir frequency of the beam particles, $S = \pi r_W^2$ – waveguide cross-section, $Z = \omega_B \delta^2 \beta_{\perp 0}^2 z / 2v_z$,

$$\Delta = \frac{2(\omega - n\omega_{B0})}{n\omega_{B0} \cdot \beta_{\perp 0}^2},\ A_i = a_i \cdot \exp\{i\zeta_{i+} / n\}$$

$$\beta_\perp^2 = (v_\perp / c)^2,\ G = \frac{\omega_b^2 nV \cdot J_{m-n}^2(k_{ms} r_C) a_0^{2n-1}}{4\pi\beta_{\perp 0}^2 \cdot c^2 \cdot J_m^2(x_{ms}) \cdot (x_{ms}^2 - m^2) \cdot D_k}.$$

In this approach the only one complex equation (11) where the values of the fields interacting with the particles have already been taken into account is completely sufficient to calculate the gain regime. The calculation details can be found in the book [21].

REFERENCES

1. A.V. Gaponov, M.I. Petelin, V.K. Yulpatov. The induced radiation of excited classical oscillators and its use in high-frequency electronics // *Radiophysics and Quantum Electronics*. 1967, v. 10, № 9-10, p. 794-813.
2. V.A. Flyagin, A.V. Gaponov, M.I. Petelin, V.K. Yulpatov. The Gyrotron // *IEEE Transactions on Microwave Theory and Techniques*. 1977, v. 25, № 6, p. 514-521.
3. I.P. Panchenko, V.I. Sotnikov. Nonlinear theory of the excitation of monochromatic waves by an electron beam in a plasma placed in a magnetic field // *Plasma Physics*. 1976, v. 2, № 6, p. 945-952.
4. Kh.D. Aburdzhaniya, A.B. Kitsenko, I.M. Pankratov. Nonlinear stage of interaction of a stream of charged particles with a plasma in a magnetic field // *Plasma Physics*. 1978, v. 4, № 1, p. 227-234.
5. V.A. Balakirev, N.I. Karbushev, A.O. Ostrovsky, Yu.V. Tkach. *Theory of Cherenkov's Amplifiers and Generators on Relativistic Beams*. Kiev: “Naukova dumka”, 1993, p. 208.
6. A.N. Kondratenko, J. Krusha, V.M. Kuklin. On the features of the development of beam-plasma instability under the conditions of the anomalous Doppler effect // *Beitr. Plasmaphys*. 1979, v. 19, № 4, p. 201-209.
7. O.V. Kuklina, V.M. Kuklin. On the mechanisms of saturation of cyclotron instabilities of an electron beam in waveguides // *Electromagnetic phenomena*. 2001, v. 2, № 4(8), p. 490-497.
8. N.S. Ginzburg. A nonlinear theory of amplification and generation on the anomalous Doppler effect // *Izv. vuzov. Radiophysics*. 1979, v. 22, № 4, p. 470-479.
9. V.L. Granatstein, I. Alexeff. *High-Pouwer Microwave Source*. Boston: “Arlech House”, 1987, p. 564.
10. K.R. Chu, A.T. Lin. Gain and Bandwidth of the Gyro-TWT and CARM Amplifiers // *IEEE Transactions on Plasma Science*. 1988, v. 16, № 2, p. 90-104.
11. M.V. Kuzelev, A.A. Rukhadze. *Electrodynamics of dense electron beams in a plasma*. M.: “Nauka”, 1990, p 336.
12. A.G. Zagorodni, P.I. Fomin, A.P. Fomina. Superradiation of electrons in a magnetic field and a nonrelativistic gyrotron // *Ext. NAS of Ukraine*. 2004, № 4, p. 75-80; P.I. Fomin, A.P. Fomina. Dicke Superradiance on Landau Levels // *Problems of Atomic Science and Technology*. 2001, № 6, p. 45-48.
13. C.J. Edgcombe. *Gyrotron Oscillators: Their Principles and Practice*. London, U.K.: “Taylor and Francic”, 1993.
14. N.S. Ginzburg, N.A. Zavolsky, G.S. Nusinovich Dynamics of Gyrotrons with a Nonfixed Axial Structure of RF Field // *Soviet Journal of Commun. Technology and Electron.* 1987, v. 32, № 9, p. 132-139.
15. V.M. Kuklin. *Report PST EV N 978763*. Hamburg: «NATO Science Program Cooperative Science and Technology Sub-Program», 2002.
16. V.M. Kuklin, S.Yu. Puzyrkov, K. Schunemann, G.I. Zaginaylov. Influence of low-density Plasma on Gyrotron Operation // *UFJ*. 2006, v. 51, № 4, p. 358-366.
17. S.S. Moiseev, V.A. Buts, N.S. Erokhin. Peculiarities of Charged Particle Dynamics under Cyclotron Resonance Conditions // *Plasma Physics Reports*. 2016, v. 42, № 8, p. 761-768.
18. V.A. Butz, V.V. Kuzmin, A.P. Tolstoluzhsky. Features of the dynamics of particles and fields in cyclotron resonances // *JETP*. 2017, v. 152, № 4(10), p. 767-780.
19. A.F. Alisov, A.M. Artamoshkin, I.A. Zagrebelny, N.M. Zemlyansky, V.I. Karas`, et. al. Experimental study of propagation microwave radiation with stochastic jumping phase in overdense plasmas // *Problems of Atomic Science and Technology. Series “Plasma Electronics and New Acceleration Methods*”. 2003, № 4, p. 69-73.
20. V.V. Yanovskii, A.V. Tur, Yu.N. Maslovsky. Charged structurally-complex particle in a constant electric field // *Theoretical and Mathematical Physics*. 2013, v. 175, № 2, p. 247-278.
21. V.M. Kuklin. *Selected Chapters (Theoretical Physics)*. Kharkiv: “V.N. Karazin Kharkiv National University”, 2018, p. 212.

Article received 04.06.2018

СВЕРХИЗЛУЧЕНИЕ СГУСТКА ВРАЩАЮЩИХСЯ ЭЛЕКТРОНОВ

В.М. Куклин, Д.Н. Литвинов, А.Е. Споров

Рассмотрены уравнения, описывающие возбуждение пучком вращающихся во внешнем магнитном поле электронов ТЕ-волны в волноводе в двух режимах. В первом режиме взаимодействием излучателей – вращающихся в магнитном поле электронов, пренебрегаем. Полагаем, что электроны пучка взаимодействуют только с волноводными модами. Во втором случае, в режиме сверхизлучения, электроны пучка взаимодействуют друг с другом за счет их спонтанного излучения. Обсуждаются особенности описания режима усиления гиротрона.

НАДВИПРОМІНЮВАННЯ ЗГУСТКА ЕЛЕКТРОНІВ, ЩО ОБЕРТАЮТЬСЯ

В.М. Куклін, Д.М. Литвинов, О.Є. Споров

Розглянуто рівняння, що описують збудження пучком електронів, що обертаються в зовнішньому магнітному полі ТЕ-хвилі у хвилеводі в двох режимах. У першому режимі нехтуємо взаємодією випромінювачів – електронів, що обертаються в магнітному полі. Вважаємо, що електрони пучка взаємодіють лише з хвилеводними модами. У другому випадку, у режимі надвипромінювання, електрони пучка взаємодіють один з одним за рахунок їх спонтанного випромінювання. Обговорюються особливості опису режиму підсилення гіротрона.

ISSN 1562-6016. ВАНТ. 2018. №4(116)