ANOMALOUS DIFFUSION OF PLASMA IN THE LOWER HYBRID CAVITIES OBSERVED IN THE TERRESTRIAL IONOSPHERE

N.A. Azarenkov, D.V. Chibisov

V.N. Karazin Kharkiv National University, Kharkiv, Ukraine

E-mail: dmitriychibisov@karazin.ua

The problem of the disappearance of the lower hybrid cavities, which are observed in the terrestrial ionosphere, is considered. As a possible mechanism which leads to the disappearance of the cavities the anomalous diffusion of plasma, which occurs due to a drift turbulence of plasma is considered. The frequency and the characteristic time of development of the drift instability in the cavity, the saturation level of the instability and the anomalous diffusion coefficient of plasma in the cavity are obtained. The cavity lifetime is also estimated, which can be greater or about 1 second.

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INTRODUCTION

In the plasma of terrestrial ionosphere the axially symmetric regions elongated along the geomagnetic field are observed, which are characterized by a reduced density of plasma in comparison with the environment as well as an increased level of oscillations in the range of the lower hybrid frequency [1]. Such regions called the lower hybrid cavities (LHC) and another name is the lower hybrid solitary structures (LHSS), have transverse dimensions of a few to many thermal ion gyroradii usually from tens to hundreds of meters. The registration and measurements of the LHC are carried out by satellites as well as sounding rockets, and it occurs only when they pass through these structures. Because of the relatively small transverse dimensions of LHC and the high velocities of the spacecraft, the time of measurement is up to tens of milliseconds. However during this time the cavity does not change significantly, which indicates that LHC is sufficiently stable formation. Although there are a number of works on the explanation of this phenomenon which are given in the review [1], the mechanisms for occurrence of LHC, as well as their stability, are not completely clear. There are also no estimates of the time of their existence and the explanations of their disappearance.

This paper is devoted to the problem of disappearance of LHC. As a possible mechanism for the disappearance of LHC we consider the anomalous diffusion of inhomogeneous plasma across the magnetic field. It is known that in magnetized plasma the excitation of various instabilities due to the radial inhomogeneity of the plasma density is possible. One of them is the drift lower hybrid instability due to which an increased level of low hybrid oscillations in the LHC is believed to occur [1]. In addition, it is possible the excitation of a drift instability with a frequency much less than the ion cyclotron frequency, which can lead to the lower frequency drift turbulence of plasma. In turn, due to the drift turbulence, an anomalous diffusion of plasma across the magnetic field occurs, which should lead to the filling of the LHC with plasma and its disappearance. Since the cavities have axial symmetry, then an analysis of the development of turbulence as well as diffusion processes in plasma of LHC in the present work is considered using the model of small-scale cylindrical waves [2-5]. We considered the linear as well as the nonlinear stages of drift instability for cavities conditions, found the level of turbulence in the LHC, and also obtained the diffusion coefficient of plasma and solved the diffusion equation for given initial conditions. We also estimated the lifetime of the cavity.

1. DRIFT WAVE TURBULENCE IN THE LOWER HYBRID CAVITIES

In homogeneous magnetized plasma, we consider the axially symmetric cavity whose axis coincides with z – directed magnetic field and the dependence of the plasma density on the radius as follows

$$n(r) = n_0\left(1 - a\exp\left(-\frac{r^2}{2r_0^2}\right)\right), \qquad (1)$$

which is the "inverse" Gaussian distribution. In (1) n_0 is the plasma density in the environment, a is the constant which determines the depth of the cavity, r_0 is the length of the plasma density inhomogeneity. Note that the dependence (1) was obtained by satellite measurements [1]. The measurements showed that the magnitude of a varies up to 0.4, at altitudes of 600…1000 km, is up to 0.2 at altitudes of 1500…13000 km and does not exceed 0.05 at altitudes of 20000…35000 km. The velocity distribution for the components of plasma is assumed to be Maxwellian, which is confirmed by observations. The distribution function for the plasma in this case is [6]

$$F_{0\alpha} = \frac{n_0}{(2\pi)^{3/2}\mathrm{v}_{T\alpha}^3}\left(1 - a\exp\left(-\frac{R_\alpha^2}{2R_{0\alpha}^2}\right)\right)\exp\left(-\frac{\rho_\alpha^2}{2\rho_{T\alpha}^2} - \frac{\mathrm{v}_z^2}{2\mathrm{v}_{T\alpha}^2}\right), \qquad (2)$$

where the subscript α denotes ions (i) or electrons (e), R_α, ρ_α and v_z, are the radial coordinate of the guiding center, Larmor radius and velocity along the magnetic field of the particles correspondingly, $R_{0\alpha}$ is the length of the inhomogeneity of the radial distribution of the guiding centers of particles, $\rho_{T\alpha} = \mathrm{v}_{T\alpha}/\omega_{c\alpha}$ is the thermal Larmor radius $\mathrm{v}_{T\alpha}$ is the thermal velocity, $\omega_{c\alpha}$ is the cyclotron frequency. Plasma is assumed to slightly inhomogeneous, so that the inequality $R_{0\alpha} > \rho_{T\alpha}$ holds. The last condition yield $R_{0i} \approx R_{0e} \approx r_0$.

Measurements have shown that the temperature of the plasma components in the cavities exceeds their temperature in the environment which is due to lower hybrid oscillations; in fact, it means that the temperatures of the electrons and ions inside the plasma cavity decrease with increasing of radius. Then assuming the dependence $\rho_{T\alpha}(r_0)$ an arbitrary, we are sure that inequality $\nabla\rho_{T\alpha} < 0$ hold.

For the analysis of drift instability in the cavity, we use the dispersion relation for the linear stage of the small-scale, $k_\perp r_0 |m| >> 1$, low-frequency, $\omega << \omega_{ci}$, oscillations in axially-symmetric plasma with arbitrary dependence of the density and temperature of the plasma components on the radius, which was obtained in [5]:

$$\varepsilon(k,\omega,r_f) = 1 + \frac{1}{k^2\lambda_{Di}^2(r_f)}\left[k_\perp^2\rho_{Ti}^2 + \frac{m\omega_{i*}}{\omega}(1 - k_\perp^2\rho_{Ti}^2 \times \right.$$
$$\left.\times(1+\eta_i(r_f)))\right] + \frac{1}{k^2\lambda_{De}^2(r_f)} \times$$
$$\times\left[1 + i\sqrt{\frac{\pi}{2}}\frac{\omega - m\omega_{e*}(r_f)(1-\eta_e(r_f)/2)}{k_z v_{Te}}\right] = 0\,. \quad (3)$$

Here $k_\perp$, m and k_z are the transverse, azimuthal and longitudinal wave numbers, respectively, $k = (k_\perp, k_z)$, $\eta_\alpha = d\ln T_\alpha(r_f)/d\ln n_\alpha(r_f)$, T_α is the temperature, $\lambda_{D\alpha}$ is the Debye length,

$$\omega_{\alpha*} = \omega_{c\alpha}\rho_{T\alpha}^2 \left.\frac{d\ln n(r)}{rdr}\right|_{r=r_f}$$

is the drift frequency. For distribution of the plasma density in the cavity (1), we obtain approximately

$$\omega_{\alpha*} \approx a\omega_{c\alpha}\frac{\rho_{T\alpha}^2}{r_0^2}\exp\left(-\frac{r_f^2}{2r_0^2}\right)$$

with the fulfillment of inequality $|\omega_{\alpha*}| << |\omega_{c\alpha}|$. Additionally in (3) the assumptions $T_e > T_i$, $k_\perp\rho_{Ti} < 1$, are made.

Equation (3) determines in the short wavelength asymptotic limit $k_\perp r_0 |m| >> 1$ the dispersion properties of cylindrical waves analytically expressed by the Bessel functions $J_m(k_\perp r)$. The values of the plasma parameters in Eq. (3) are determined at the point $r_f = |m|/k_\perp$ which is approximately the radial coordinate of the first maximum of the Bessel function, for which this equation is written. The solution of the equation (3) yield the frequency and the growth rate of the drift oscillations in LHC

$$\omega_m(k) = m\omega_{e*}\frac{1 - k_\perp^2\rho_{Ti}^2(1+\eta_i)}{1 + k_\perp^2\rho_s^2}\,, \quad (4)$$

$$\gamma_m(k) = \omega_m(k)\frac{\sqrt{\pi}z_{e0}\exp(-z_{e0}^2)}{1 + k_\perp^2\rho_s^2}\,, \quad (5)$$

where $\rho_s^2 = \rho_{Ti}^2(T_e/T_i)$, $z_{e0} = \omega_m(k)/\sqrt{2}k_z v_{Te}$. We note that the increase in the drift oscillations does not depend on the sign of the density gradient and is due the Cherenkov interaction of resonant electrons with drift waves. Considering also that under the conditions of the cavity the inequalities $\nabla\rho_{Ti} < 0, \nabla n > 0$ hold we have $\eta_i < 0$. Assuming also that the lengths of the inhomogeneity of the plasma density and the ion temperature are approximately equal, we get $\eta_i \approx -1$, so the frequency of the drift oscillations in LHC is

$$\omega_m(k) = \frac{m\omega_{e*}}{1 + k_\perp^2\rho_s^2}\,. \quad (6)$$

Let us estimate the value of the frequency of the drift oscillations (6) which are excited in the lower hybrid cavities. At a height of up to 1000 the main ion component is the singly ionized oxygen, which here amounts to 90 %. The ion cyclotron frequency of oxygen ions is approximately 34 Hz, the ratio $r_0/\rho_{Ti} \approx 3$, $a = 0.4$. In this case, the frequency of the drift oscillations is of the order of 3…4 Hz. A similar estimate of the drift frequency at altitudes of 1500…2000 km, where the main ion component is protons, is of the order of 8 Hz. we now evaluate the instability development time t_{inst}, which is equal to $\gamma_m^{-1}(k)$. For the given plasma parameters in the cavity we obtain *t_{inst} ~0.5…1.5 s*.

Nonlinear evolution of the drift instability in the cavity is determined by the induced scattering of cylindrical waves by ions [2-5]. The kinetic equation for the spectral intensity $I_m(k)$ of small-scale cylindrical waves in plasma with arbitrary dependences on the radius of density and temperature is [5]:

$$\frac{1}{2}\frac{\partial I_m(k)}{\partial t} = (\gamma_m(k) + \Gamma_m(k))I_m(k)\,, \quad (7)$$

where $\Gamma_m(k)$ is the nonlinear growth rate of drift waves

$$\Gamma_m(k) = \left(\frac{\partial\,\mathrm{Re}\,\varepsilon}{\partial\omega_m(k)}\right)^{-1}\sum_{m_1}\int dk_1 I_{m_1}(k_1)B(k_\perp, m|k_{\perp1}, m_1)$$
$$\times\,\mathrm{Im}U_i(k, m, \omega_m(k)|k_1, m_1\omega_{m_1}(k_1)). \quad (8)$$

In (6) the value ε is given by (3), $B(k_\perp, m|k_{\perp1}, m_1)$ is the factor of nonlinear interaction of cylindrical waves [2-4]

$$B(k_\perp, m|k_{\perp1}, m_1) = \begin{cases} \dfrac{1}{\pi m|\cos\alpha_0|}\dfrac{k_\perp}{k_{\perp1}}, & m_1 < m_{10} - m_{10}^{1/3} \\ O(m^{-2/3}), & m_{10} - m_{10}^{1/3} < m_1 < m_{10} \\ O(m^{-2}), & m_1 > m_{10}, \end{cases}$$

where $m_{10} = mk_{\perp1}/k_\perp$, $\cos^2\alpha_0 = 1 - r_{1f}^2/r_f^2$, $r_{1f} = |m_1|/k_{\perp1}$; $\mathrm{Im}U_i$ is the matrix element of the induced scattering of drift waves by ions which is equal to

$$\mathrm{Im}U_i \approx \frac{\pi}{k^2\lambda_{Di}^2}\frac{e^2}{T_i^2}(k_\perp k_{1\perp}\rho_{Ti}^2)^2\frac{(m_1 - m)\omega_{i*}(r_{f2})}{\omega_m^2(k_\perp)}\cos^2\alpha_0 \times$$
$$\times\,\delta(\omega_m(k_\perp) - \omega_{m_1}(k_{\perp1})), \quad (9)$$

where [5]

$$\omega_{i*}(r_{f2}) = \omega_{ci}\rho_{Ti}^2\frac{d\ln n(r)}{rdr}\left[1 - (k_\perp^2 + k_{\perp1}^2)\rho_{Ti}^2(1+\eta_i)\right]\Big|_{r=r_{f2}}\,,$$

$r_{f2} = |m_2|/k_{\perp2}$ is the coordinate of the first maximum of a cylindrical “beat wave” of two cylindrical waves with wave numbers $(k_\perp, m)$ and $(k_{\perp1}, m_1)$. The wave numbers of the beat wave are determined by [2-4]

$$m_2 = m - m_1,\ k_{\perp 2} = \sqrt{k_\perp^2 - 2k_\perp k_{\perp 1}\sin\alpha_0 + k_{\perp 1}^2}\ .$$

In cavity conditions when (1) hold, we have approximately

$$\omega_{i*}(r_{f2}) \approx a\omega_{ci}\frac{\rho_{Ti}^2}{r_0^2}\exp\left(-\frac{r_{f2}^2}{2r_0^2}\right).$$

Apply the equation (7) for the analysis of the nonlinear stage of drift turbulence in the conditions of the lower hybrid cavities. First by using the condition $\omega_m(k_\perp) = \omega_{m_1}(k_{\perp 1})$ we determine the radial wave number $k_{\perp 1}$ of the cylindrical wave $J_{m_1}(k_{\perp 1}r)$ interacting with the wave $J_m(k_\perp r)$:

$$k_{\perp 1}^2\rho_s^2 \approx \frac{m_1}{m} + \frac{m_1}{m}k_\perp^2\rho_s^2 - 1\,. \tag{10}$$

Using (10) we get for $\cos^2\alpha_0$

$$\cos^2\alpha_0 = \frac{m_1 - m}{m^2 k_{\perp 1}^2\rho_s^2}\left(m - m_1 k_\perp^2\rho_s^2\right).$$

The requirement $\cos^2\alpha_0 > 0$ leads to systems of inequalities

$$\begin{cases} m > m_1, \\ m < m_1 k_\perp^2\rho_s^2 \end{cases} \tag{11}$$

or

$$\begin{cases} m < m_1, \\ m > m_1 k_\perp^2\rho_s^2. \end{cases} \tag{12}$$

The system (11) has a solution when $k_\perp^2\rho_s^2 > 1$:

$$\frac{m}{k_\perp^2\rho_s^2} < m_1 < m\,. \tag{13}$$

In this case, only long waves $J_{m_1}(k_{\perp 1}r)$ with azimuthal wave numbers $m_1 < m$ interact with the wave $J_m(k_\perp r)$. Then the nonlinear growth rate in equation (7), which is proportional to the sum $\Gamma_m(k) \propto \sum_{m_1}(m_1 - m)$, is negative and the wave $J_m(k_\perp r)$ turns out to be nonlinearly damped. Thus in LHC the part of the drift oscillating spectrum with $k_\perp^2\rho_s^2 > 1$ damped nonlinearly and disappears finally.

The system (12) has a solution for the long wavelength part of the spectrum when $k_\perp^2\rho_s^2 < 1$:

$$m < m_1 < \frac{m}{k_\perp^2\rho_s^2}\,. \tag{14}$$

In this case the wave $J_m(k_\perp r)$ interact only with the shorter waves $J_{m_1}(k_{\perp 1}r)$ for which $m_1 > m$. Then the nonlinear growth rate $\Gamma_m(k) \propto \sum_{m_1}(m_1 - m)$, is positive and the wave $J_m(k_\perp r)$ grows nonlinearly. The saturation analysis of the drift turbulence in LHC at $k_\perp^2\rho_s^2 < 1$ requires going beyond the frame of weak turbulence theory. However, in [3], the saturation level of the drift turbulence is estimated in the case of a homogeneous temperature distribution and the Gaussian dependence of the plasma density on the radius

$$\frac{W}{n_0 T_i} \approx \frac{\rho_s^2}{r_0^2}\,. \tag{15}$$

We believe that this level of saturation is also reached for LHC.

2. ANOMALOUS DIFFUSION OF PLASMA IN THE LOWER HYBRID CAVITIES

As a result of the drift turbulence, a change in the averaged distribution function of plasma components occurs in the cavity plasma. The evolution of the distribution function of electrons is governed by the quasilinear equation [3]

$$\frac{\partial F_{e0}}{\partial t} = \pi\frac{e^2}{m_e}\sum_m\int dk I_m(k)\delta(\omega_m(k) - k_z v_z)\left[\frac{m}{\omega_{ce}}\frac{1}{R_e}\frac{\partial}{\partial R_e} + k_z\frac{\partial}{\partial v_z}\right]J_m^2(k_\perp R_e)\left[\frac{m}{\omega_{ce}}\frac{1}{R_e}\frac{\partial F_{e0}}{\partial R_e} + k_z\frac{\partial F_{e0}}{\partial v_z}\right]. \tag{16}$$

Integrating (16) with respect to ρ_e and v_z, and also considering that $R_e \approx r$, we obtain the diffusion equation for resonant electrons

$$\frac{\partial n_e}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(rD_{e\perp}\frac{\partial n_e}{\partial r}\right), \tag{17}$$

where

$$D_{e\perp} = \frac{c^2}{B_0^2}\sum_m\int dk\frac{m^2}{r^2}J_m^2(k_\perp R_e)I_m(k)\frac{\gamma_m(k)}{\omega_m^2(k)}$$

is the electron diffusion coefficient across the magnetic field. For the level of turbulence (15) we have

$$D_{e\perp} = \frac{cT_e}{eB_0}\frac{z_e\rho_s}{r_0}\,. \tag{18}$$

For the most rapidly growing part of the instability spectrum, we have $z_e \sim 1$. The diffusion of ions across the magnetic field occurs as a result of ion scattering by random pulsations of the electric field of drift turbulence [3]. The evolution of the ion distribution function is determined by the equation

$$\frac{\partial n_i}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(rD_{i\perp}\frac{\partial n_i}{\partial r}\right) \tag{19}$$

with the diffusion coefficient $D_{i\perp}$ which equal to the electron diffusion coefficient (18) $D_{i\perp} = D_{e\perp} = D_\perp$, so that the diffusion is ambipolar. The equations (17) and (19) give diffusion of plasma in the direction of the center of cavity, since the plasma density increases from the center.

The solution of the diffusion equation (17) or (19) with initial condition (1) is

$$n(r,t) = n_0\left(1 - \frac{a}{\sigma^2(t)}\exp\left(-\frac{r^2}{2r_0^2\sigma^2(t)}\right)\right) \tag{20}$$

where

$$\sigma^2(t) = 1 + \frac{2D_\perp}{r_0^2}t\,.$$

Equation (20) shows that the cavity depth $a(t)$ decreases with time as

$$a(t) = a\left(1 + \frac{2D_\perp}{r_0^2}t\right)^{-1}. \quad (21)$$

In addition to decreasing the depth of the cavity, an increase in the transverse dimension of the cavity occurs, the mean square root of the cavity is determined by

$$r_0\sigma(t) = r_0\sqrt{1 + \frac{2D_\perp}{r_0^2}t}.$$

Now we evaluate the time of change in the cavity depth by k times. Equating (21) to a/k we get

$$t_k = \frac{(k-1)r_0^2}{2D_\perp}.$$

Respectively, transverse dimension of the cavity increases by $\sqrt{k}$ times. Substitution of the diffusion coefficient (18) yields

$$t_k = \frac{(k-1)r_0^3 eB_0}{2cT_e\rho_s}. \quad (22)$$

For the parameters of cavities in the ionosphere r_0/ρ_s^3, $r_0 \sim 50$ *m*, $B_0 \sim 0.2$ *Gs*, $T_e \sim 0.3$ *eV* and assuming that the depth of the cavity decreases, for example, by 10 times, that is $k = 10$, we have $t_{10} \sim 10^{-3}$ s. A comparison of the time of change in the depth of the cavity and the time of development of the drift instability, $t_{inst} \sim 0.5...1.5$ s, shows that the lifetime t_{lt} of the cavity is determined basically by the time of development of the drift instability in the cavit that is $t_{lt} \sim 0.5...1.5$ s.

CONCLUSIONS

Radial inhomogeneity of the plasma density and temperature in the plasma of LHC leads to the development of the drift instability and drift turbulence of plasma in the cavity. In turn, drift turbulence causes an anomalous diffusion of the plasma across the magnetic field, which leads to the disappearance of the cavity.

It was found that the time of anomalous plasma transport across the magnetic field is much smaller than the characteristic time of the development of the drift instability, so that the lifetime of the cavity is determined by the growth rate of the instability, which is of the order of or more than 1 s.

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АНОМАЛЬНАЯ ДИФФУЗИЯ ПЛАЗМЫ В НИЖНЕГИБРИДНЫХ ПОЛОСТЯХ, НАБЛЮДАЕМЫХ В ЗЕМНОЙ ИОНОСФЕРЕ

Н.А. Азаренков, Д.В.Чибисов

Рассмотрена проблема исчезновения нижнегибридных полостей, наблюдаемых в земной ионосфере. В качестве возможного механизма, приводящего к исчезновению полостей, рассматривается аномальная диффузия плазмы, возникающая из-за её дрейфовой турбулентности. Получены частота и характерное время развития дрейфовой неустойчивости в полости, уровень насыщения неустойчивости и коэффициент аномальной диффузии плазмы в полости. Также оценивается время жизни полости, которое может составлять больше или порядка 1 с.

АНОМАЛЬНА ДИФУЗІЯ ПЛАЗМИ В НИЖНЬОГІБРИДНИХ ПОРОЖНИНАХ, ЩО СПОСТЕРІГАЮТЬСЯ В ЗЕМНІЙ ІОНОСФЕРІ

М.О. Азарєнков, Д.В. Чібісов

Розглянуто проблему зникнення нижньогібридних порожнин, які спостерігаються в земній іоносфері. В якості можливого механізму, який призводить до зникнення порожнин, розглядається аномальна дифузія плазми, що виникає внаслідок її дрейфової турбулентності. Отримано частоту і характерний час розвитку дрейфової нестійкості в порожнині, рівень насичення нестійкості і коефіцієнт аномальної дифузії плазми в порожнині. Також оцінюється час життя порожнини, який може становити більше або порядку 1 с.