CHERENKOV RADIATION OF THE LASER PULSE IN A DIELECTRIC WAVEGUIDE

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Theoretical studies of the excitation of the Cherenkov radiation by a superluminal laser pulse in a dielectric waveguide, similar to the radiation of an electron moving uniformly in a slowing-down medium with a superluminal velocity, are presented. The nonlinear polarization of the medium caused by the ponderomotive force of a laser pulse is obtained. The structure of excited fields, including wakefields, is found. Frequency dispersion of the dielectric permittivity is considered to provide superluminal group velocity of the laser pulse.

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INTRODUCTION

A charged particle moving in a dielectric medium with a superluminal velocity radiates electromagnetic waves called Cherenkov radiation [1]. The electric field of a moving charge polarizes the atoms (molecules) of the dielectric medium, which in turn coherently re-emit electromagnetic waves. Cherenkov radiation excited in dielectric waveguide by an intense relativistic electron bunch or a train of bunches can be used for elaboration of high gradient dielectric wakefield accelerators [2-5].

A similar effect occurs when a short high-power laser pulse propagates in a dielectric [6,7]. In the linear approximation in the field, the effect of polarization of the medium at the field frequency leads only to a change in the phase and group velocities of the laser pulse. So to observe the Mach cone of Cherenkov radiation the transversal inhomogeneity of the permittivity should be introduced for obtaining superluminal group velocity of the laser pulse [8]. In the nonlinear approximation, the pulsed ponderomotive force that propagates in a medium with the velocity equal to the group velocity of the laser pulse also acts quadratically with respect to the field on the coupled electrons of the dielectric medium. This force, in turn, leads to the polarization of the dielectric medium. When the Cherenkov synchronism condition between the ponderomotive force of the laser pulse and the slow electromagnetic waves of the medium is satisfied, it causes the excitation of electromagnetic Cherenkov radiation.

Thus, the effect of Cherenkov radiation of a laser pulse is quite similar to the Cherenkov radiation of a charged particle with the only difference that the role of the electric field of a charged particle is played by the ponderomotive force of a laser pulse.

The wake Cherenkov radiation of a powerful ultrashort laser pulse in a dielectric medium can be used to accelerate charged particles like an analogous acceleration method using wakefields driven in plasma by a laser pulse or a train of bunches [9-11].

In this paper we formulate a system of nonlinear equations of macroscopic electrodynamics that describes the process of excitation of Cerenkov radiation by a laser pulse in a dielectric medium. On the basis of these equations, the effect of Cherenkov radiation of a laser pulse in a dielectric waveguide (optical fiber) is investigated.

1. FORMULATION OF THE PROBLEM. BASIC EQUATIONS

In a homogeneous dielectric medium, a laser pulse (wave packet) propagates with components of the electromagnetic field

$$\vec{E}_L(\vec{r},t)=\frac{1}{2}\vec{E}_0(\vec{r},t)e^{i\psi}+\text{c.c.}$$

$$\vec{H}_L(\vec{r},t)=\frac{1}{2ik_0}\text{rot}\left[\vec{E}_0(\vec{r},t)e^{i\psi}\right]+\text{c.c.} \qquad (1)$$

$\psi=\vec{k}\vec{r}-\omega_L t$, $\vec{k}$ is the wave vector, $k_0=\omega_L/c$, ω_L is the carrier frequency of the laser pulse, $\vec{E}_0(\vec{r},t)$ is of the laser pulse envelope slowly changing in the space and time.

Under the action of the ponderomotive force (HF-pressure force) quadratic in the laser field (1), the polarization, slow in the scale of the carrier frequency, arises in the dielectric, which in turn is the source of the electromagnetic field (i.e. Cherenkov radiation) excited by the laser pulse.

The system of Maxwell equations describing the electromagnetic field excited by the polarization induced by the laser pulse has the form

$$\text{rot}\vec{E}=-\frac{1}{c}\frac{\partial\vec{H}}{\partial t},\ \text{rot}\vec{H}=\frac{1}{c}\frac{\partial\vec{E}}{\partial t}+\frac{4\pi}{c}\frac{\partial\vec{P}}{\partial t},$$

$$\text{div}\vec{E}=-4\pi\,\text{div}\vec{P},\ \text{div}\vec{H}=0, \qquad (2)$$

where $\vec{P}$ is vector of electric polarization.

2. EQUATION FOR NONLINEAR POLARIZATION

The next step of the theory is the determination of the polarization $\vec{P}$ caused by the action on certain atoms of the condensed dielectric medium both an electric field which is in the Maxwell equations (2) and the ponderomotive force from side of the laser pulse (1). For this, a simple but adequate model of an elementary dipole located in the crystal lattice point is necessary. Note that beginning from the microwave range of radiation wavelengths and moreover in the optical range, the orientational (dipole) and induced ionic polarization mechanisms do not play an appreciable role due to the high inertia of the ions. In these wavelength ranges, the induced electron polarization of atoms is dominant [12]. Electronic polarization is due to the displacement of the shell from the bound electrons of the atom relatively to the nucleus under the action of the electric field.

The induced electronic polarization can be described in the framework of the following model [13]. The atom is represented as a point nucleus of charge Z|e|, surrounded by a smeared electron cloud with the charge -Z|e|. The electron cloud will be considered as a spherically symmetric homogeneous charged region of radius R_0. When the electron cloud is shifted as a whole with respect to the nucleus, the dipole moment of the atom $\vec{p} = -eZ\vec{r}$ arises, where $\vec{r}$ is the radius vector of the center of the electron cloud. Accordingly, the following dipole returning force will act on the electron cloud

$$\vec{F}_e = -\frac{(Ze)^2}{R_0^3}\vec{r} ,$$

which leads to harmonic dipole oscillations of an atom with eigen frequency

$$\omega_0 = \sqrt{\frac{Ze^2}{mR_0^3}} .$$

In a condensed medium, each atom is in a local (acting) electric field $\vec{E}_{loc}$, which can differ greatly from the macroscopic field $\vec{E}$ contained in Maxwell's equations (2). The local electric field includes both the external field and the total electric field of the induced dipoles surrounding the taken atom. In a crystalline medium with a cubic crystal lattice, the local electric field is described by the Lorentz formula [5, 6]

$$\vec{E}_{loc} = \vec{E} + \frac{4\pi}{3}\vec{P} .$$

The equation for the nonlinear polarization $\vec{P}_L$ in the field of the laser pulse (1) is obtained in the following view [14]

$$\frac{d^2\vec{P}}{dt^2} + \omega_d^2\vec{P} = \frac{Ze^2N}{m}\vec{E} - \frac{eN\alpha_L}{4m}\frac{\varepsilon_L + 2}{3}\vec{F} , \qquad (3)$$

where $\omega_d^2 = \omega_0^2 - \omega_p^2/3, \quad \omega_p^2 = \frac{4\pi Ze^2 N}{m}$ – plasma frequency, N is the number of atoms per unit volume of the dielectric, $\varepsilon_L \equiv \varepsilon(\omega_L),\ \varepsilon(\omega) = 1 + \frac{\omega_p^2}{\omega_d^2 - \omega^2}$ is the permittivity of the medium, $\alpha_L = \frac{Ze^2}{m}\frac{1}{\omega_o^2 - \omega_L^2}$ is the polarizability of an individual atom. In the quasistatic approximation $\omega_0^2 >> \omega_L^2$, the expression for the polarizability is simplified and does not depend on the frequency $\alpha_L = \frac{Ze^2}{m\omega_0^2}$,

$$\vec{F} = \nabla\left|\vec{E}_0\right|^2 + \frac{\varepsilon_L - 1}{3}\left[\left(\vec{E}_0^*\nabla\right)\vec{E}_0 + \left(\vec{E}_0\nabla\right)\vec{E}_0^*\right]. \qquad (4)$$

The first term in (4) describes the gradient force of HF pressure. The second term occurs only in the case of a crystalline medium and is due to the difference between the local electric field $\vec{E}_L^{loc}$ in the crystal and the electric field of the laser pulse. In dielectric media, where the acting field coincides with the external field, for example, in a gas dielectric, this term is absent.

The Maxwell equations (2) together with the equation for the polarization (3) describe the electromagnetic Cerenkov radiation of a laser pulse in a condensed dielectric medium.

3. CHERENKOV RADIATION OF A LASER PULSE IN A DIELECTRIC WAVEGUIDE

Let us consider a dielectric waveguide made in the form of a homogeneous dielectric cylinder whose lateral surface is covered with an ideally conducting metal film. Along the axis of the waveguide, a circularly polarized laser pulse propagates with the components of the electric field

$$E_{0x} = \sqrt{\frac{I_0}{2}}\psi\left(r, t - z/v_g\right),\ E_{0y} = iE_{0x} ,$$

$$\psi = \left[R(r)T\left(t - z/v_g\right)\right]^{1/2} .$$

The function $R(r)$ describes the radial intensity profile of the laser pulse, $I_0 = \left|\vec{E}_0\right|^2$, $R(0) = 1$, $R(r = b) = 0$, b is the radius of the waveguide, the function $T(\tau)$ describes the longitudinal profile, $\tau = t - z/v_g$, v_g is the group velocity, $max T(\tau) = 1$, I_0 is the maximum intensity.

Let us solve system of Maxwell's equations (2), together with equation for polarization (3) by the Fourier transform. From these equations follows the wave equation for the longitudinal Fourier component of the Cherenkov electric field

$$\Delta E_{z\omega} + k_0^2\varepsilon_{ch}(\omega)E_{z\omega} = 4\pi\left(\frac{1}{\varepsilon_{ch}(\omega)}\frac{\partial\rho_{pol\omega}}{\partial z} - i\frac{k_0}{c}j_{pol\omega}\right). \quad (5)$$

The Fourier components of the polarization charges and currents are determined by expressions

$$\rho_{pol\omega} = \mu\left[\Delta I_\omega(r,z) + \frac{\varepsilon_L - 1}{6}\Delta_\perp I_\omega(r,z)\right],$$

$$j_{zpol\omega} = i\omega\mu\frac{\partial}{\partial z}I_\omega(r,z),\ I_\omega(r,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} I(r,\tau)e^{i\omega t}dt .$$

The longitudinal Fourier component of the electrical field should be found in the form of a series of Bessel functions

$$E_{z\omega} = \sum_{n=0}^{\infty} C_n(z,t)J_0\left(\lambda_n\frac{r}{b}\right),$$

where λ_n are roots of the Bessel function $J_0(x)$. From equation (5) we can obtain the equation for the coefficients C_n.

As a result, we find the following expression for the Fourier component of the longitudinal component of the electric field

$$E_{z\omega}(r) = \Pi T(\omega)G(r,\omega) .$$

Here

$$G(r,\omega)=i\frac{[\varepsilon_{ch}(\omega)-1]}{\varepsilon_{ch}(\omega)}e^{ik_g z}\Big[R(r)- \quad (6)$$
$$-\frac{\varepsilon_L-1}{6}\sum_{n=1}^{\infty}\rho_n\frac{J_0(\lambda_n r/b)}{\Delta_n(\omega)}\Big].$$
$$\Pi=\frac{I_0(\varepsilon_L-1)}{16\pi v_g ZeN},\ \rho_n=\frac{\lambda_n^2}{b^2}\frac{\alpha_n}{N_n},$$
$$\Delta_n(\omega)=k_0^2\varepsilon_{ch}-\frac{\lambda_n^2}{b^2}-k_g^2,\ \ \varepsilon_{ch}(\omega)=1-\frac{\omega_p^2}{\omega^2-\omega_d^2},$$
$$N_n=\frac{b^2}{2}J_1^2(\lambda_n),\quad \alpha_n=\int_0^b R(r)J_0\left(\lambda_n\frac{r}{b}\right)rdr,$$
$$T_\omega=\frac{1}{2\pi}\int_{-\infty}^{\infty}T(\tau)e^{i\omega\tau}d\tau\ .$$

Accordingly, the longitudinal component of the excited electric field can be represented as a convolution

$$E_z(r,\tau)=\frac{1}{2\pi}\Pi\int_{-\infty}^{\infty}T(\tau_0)G(r,\tau-\tau_0)d\tau_0, \quad (7)$$

where

$$G(r,\tau-\tau_0)=\int_{-\infty}^{\infty}G(r,\omega)e^{-i\omega(\tau-\tau_0)}d\omega. \quad (8)$$

The expression for the Green's function (8), taking into account expression (6), is conveniently written as follows

$$G(r,\vartheta)=G_1(r,\vartheta)+G_2(r,\vartheta),$$
$$G_1(r,\vartheta)=R(r)S_0(\vartheta),$$
$$G_2(r,\vartheta)=-\frac{\varepsilon_L-1}{6}\sum_{n=1}^{\infty}\rho_n J_0(\lambda_n\frac{r}{b})S_n(\vartheta),$$

where

$$S_0(\vartheta)=i\int_{-\infty}^{\infty}\omega d\omega e^{-i\omega\vartheta}\frac{\varepsilon_{ch}(\omega)-1}{\varepsilon_{ch}(\omega)}, \quad (9)$$

$$S_n(\vartheta)=i\int_{-\infty}^{\infty}\omega d\omega e^{-i\omega\vartheta}\frac{\varepsilon_{ch}(\omega)-1}{\varepsilon_{ch}(\omega)\Delta_n(\omega)},\ \vartheta=\tau-\tau_0. \quad (10)$$

The integrand in (9) has simple poles $\omega=\pm\omega_n-i0$ located in the lower half-plane of the complex variable ω, $\omega_g=\sqrt{\omega_p^2+\omega_d^2}$ – is the frequency of the eigen longitudinal oscillations of the dielectric medium, which nullifies the dielectric permittivity $\varepsilon_{ch}(\omega)=0$.

Calculating the residues at these poles, we obtain an expression for the integral $S_0(\vartheta)$ and, accordingly, for the first term of the Green's function

$$G_1(r,\vartheta)=-2\pi R(r)\omega_p^2\chi(\vartheta)\cos\omega_g\vartheta,$$

where $\chi(\tau-\tau_0)$ is unit Heaviside function. As for the Fourier integral (10), its integrand, in addition to the poles listed above, has additional poles $\Delta_n(\omega)=0$. This equation can be given in a more convenient form for analysis

$$\Delta_n(\omega)=-\frac{1}{v_g^2\gamma_g^2}\frac{1}{\omega^2-\omega_d^2}\left(\omega^2-\omega_{chn}^2\right)\left(\omega^2+\nu_{stn}^2\right),$$

where

$$\gamma_g^2=(1-\beta_g^2)^{-1},\beta_g=v_g/c,\ \omega_{chn}=\omega_d x_{chn},\ \nu_{stn}=\omega_d x_{stn},$$

$$x_{chn}^2=-\frac{1}{2}(b_0+y_n^2)+\sqrt{\frac{1}{4}(b_0+y_n^2)^2+y_n^2},$$
$$x_{stn}^2=\frac{1}{2}(b_0+y_n^2)+\sqrt{\frac{1}{4}(b_0+y_n^2)^2+y_n^2},$$
$$b_0=\gamma_g^2(\beta_g^2\varepsilon_0-1),\ \ y_n^2=\frac{\lambda_n^2c^2}{\omega_d^2b^2}\beta_g^2\gamma_g^2.$$

The poles $\omega=\pm\omega_{chn}-i0$ are also located in the lower half-plane of the complex variable ω near the real axis. They correspond to the eigen electromagnetic waves of the dielectric waveguide, which are in Cerenkov synchronism with the laser pulse. Since these poles are always real, the Cherenkov radiation of the laser pulse in the dielectric waveguide takes place for all values of the group velocity of the laser pulse and the parameters of the dielectric waveguide (in our case, the values of the static dielectric constant ε_0 and the radius of the waveguide b). Besides of the real Cherenkov poles, there is also a pair of complex conjugate poles $\omega=\pm i\nu_{stn}$ in the integrand in (10) located on the imaginary axis. These poles correspond to a quasi-static electromagnetic field localized in the laser pulse region. Calculating the residues at all poles of the integrand (10), we may find the second term of the Green's function $G_2(r,\tau-\tau_0)$ and in a result the final expression for the whole Green's function

$$G(r,\vartheta)=-\omega_p^2\left[R(r)+\frac{\varepsilon_L-1}{6}\Phi(r)\right]\chi(\vartheta)\cos\omega_g\vartheta-$$
$$-\sigma\sum_{n=1}^{\infty}L_n(r)\left[\frac{1}{\varepsilon_{chn}}\chi(\vartheta)\cos\omega_{chn}\vartheta-\frac{1}{2\varepsilon_{stn}}sign(\vartheta)e^{-\nu_{stn}|\vartheta|}\right], \quad (11)$$

where $\sigma=\frac{\varepsilon_L-1}{6}\omega_p^2v_g^2\gamma_g^2$, $L_n(r)=\frac{\rho_n J_0(\frac{\lambda_n}{b}r)}{\omega_{chn}^2+\nu_{stn}^2}$,

$\varepsilon_{chn}\equiv\varepsilon_{ch}(\omega_{chn}),\ \varepsilon_{stn}\equiv\varepsilon_{ch}(i\nu_{stn})$,

$$\Phi(r)=\sum_{n=1}^{\infty}\frac{\rho_n J_n(\lambda_n r/b)}{k_g^2+\lambda_n^2/b^2}.$$

The Green's function contains potential monochromatic wakefield, caused by the excitation of polarization oscillations of the dielectric medium. The electromagnetic part of the Green's function contains a wakefield in the form of a superposition of eigenmodes of a dielectric waveguide, as well as a set of bipolar electromagnetic pulses.

After substituting the Green's function (11) into the expression for the total field of the laser pulse in the dielectric waveguide (7), we obtain the following expression for the longitudinal component of the wakefield of a laser pulse with an arbitrary profile

$$E_z(r,\tau)=E_z^{(pol)}(r,\tau)+E_z^{(em)}(r,\tau), \quad (12)$$

where

$$E_z^{(pol)}(r,\tau)=-\omega_p^2\Pi F(r)Z(\omega_g\tau), \quad (13)$$
$$F(r)=R(r)+\frac{\varepsilon_L-1}{6}\Phi(r),$$

$$E_z^{(em)} = -\sigma\Pi\sum_{n=1}^{\infty} L_n(r)\left[\frac{1}{\varepsilon_{chn}} Z(\omega_{chn}\tau) - \frac{1}{\varepsilon_{stn}} X(\nu_{stn}\tau)\right]. \quad (14)$$

Function

$$Z(\omega\tau) = \int_{-\infty}^{\tau} T(\tau_0)\cos\omega(\tau-\tau_0)d\tau_0$$

describes the longitudinal distribution of the wakefield polarization field $\omega = \omega_g$, as well as the wakefield electromagnetic field of the corresponding radial harmonic $\omega = \omega_{chn}$ of the dielectric waveguide.

Function

$$X(\nu_{stn}\tau) = \int_{-\infty}^{\infty} T(\tau_0) sign(\tau-\tau_0) e^{-\nu_{stn}|\tau-\tau_0|} d\tau_0$$

describes the longitudinal component of a set of bipolar quasi-static electromagnetic pulses.

Behind the laser pulse $\tau > 0$, the electric field is the superposition of electromagnetic monochromatic waves and polarization oscillations, and the quasistatic field that disappears with distance from the laser pulse $\tau \to \infty$. Before the pulse $\tau < 0$, there is only a quasistatic field, that also decreases with distance from the pulse.

Let us investigate the expression for the electric field (12)-(14) for a number of model transverse and longitudinal intensity profiles of the laser pulse. Firstly we consider the model longitudinal profile of the laser pulse

$$T(\tau_0) = \exp(-|\tau_0|/t_L),$$

where t_L is the laser pulse duration.

For the longitudinal component of the electric field excited by the laser pulse, we obtain the expression

$$E_z(r,\tau) = -\Pi\frac{\omega_p^2 t_L}{\omega_g^2 t_L^2+1}F(r)\left[2\chi(\tau)\cos\omega_g\tau + t_L\frac{dT(\tau)}{d\tau}\right] -$$
$$-\sigma\Pi\sum_{n=1}^{\infty}\frac{\rho_n J_0(\lambda_n r/b)}{\omega_{chn}^2+\nu_{stn}^2}\left\{\frac{1}{\varepsilon_{chn}}\frac{t_L}{\omega_{chn}^2 t_L^2+1}\left[2\chi(\tau)\cos\omega_{chn}\tau + \right.\right.$$
$$\left.\left. + t_L\frac{dT(\tau)}{d\tau}\right] + \frac{1}{\varepsilon_{stn}}\frac{2t_L}{\nu_{stn}^2 t_L^2-1}\left[e^{-\nu_{stn}|\tau|} + t_L\frac{dT(\tau)}{d\tau}\right]\right\}. \quad (15)$$

It follows from this expression that the polarization electric field, along with a monochromatic wakefield wave, contains a solitary pulse whose longitudinal profile completely repeats the electrical polarization profile of the dielectric and, accordingly, of the ponderomotive force. The width of this pulse $\Delta\tau_g \sim t_L$ is close to the width of the laser pulse and does not depend on the parameters of the dielectric waveguide. Each radial electromagnetic harmonic of the dielectric waveguide has an analogous structure. In addition, there is a set of electromagnetic pulses. The width of each of them $\Delta\tau_{stn} \sim 1/\nu_{stn}$ is determined by the parameters of the dielectric waveguide and does not depend on the duration of the laser pulse.

Let us investigate the expression for the electromagnetic field (15) in the quasistatic approximation $\omega_d^2 >> \omega^2$. In this approximation, the permittivity $\varepsilon_{ch} = \varepsilon_0$ is independent of frequency.

Consider the most interesting case $b_0 > 0$ or $v_g > c/\sqrt{\varepsilon_0}$. In this case we have

$$\omega_{chn} = \frac{\omega_d}{\sqrt{b_0}} = \frac{\lambda_n v_g}{b\sqrt{\beta_g^2\varepsilon_0 - 1}},\ \nu_{stn} = \omega_d\gamma_g\sqrt{\beta_g^2\varepsilon_0 - 1} \equiv \nu_{st}.$$

Accordingly, the expression for the electric field (15) becomes

$$E_z(r,\tau) = -\Gamma q_g F(r)\left[2\chi(\tau)\cos\omega_g\tau + t_L\frac{dT(\tau)}{d\tau}\right]$$
$$-\Gamma\frac{\varepsilon_L - 1}{6}\sum_{n=1}^{\infty}\frac{\alpha_n}{N_n}q_n J_0(\rho_n)\left[2\chi(\tau)\cos\omega_{chn}\tau + t_L\frac{dT}{d\tau}\right]$$
$$-\beta_g^2\Gamma\frac{\varepsilon_0(\varepsilon_L - 1)}{3}\left[e^{-\nu_{st}|\tau|} + t_L\frac{dT(\tau)}{d\tau}\right]S(r), \quad (16)$$

$$\rho_n = \frac{\lambda_n r}{b},\quad \Gamma = \Pi\frac{1}{t_L}\frac{\varepsilon_0 - 1}{\varepsilon_0},$$

$$q_g = \frac{\omega_g^2 t_L^2}{\omega_g^2 t_L^2 + 1}, q_n = \frac{\omega_{chn}^2 t_L^2}{\omega_{chn}^2 t_L^2 + 1},\ S(r) = \sum_{n=1}^{\infty}\frac{\omega_{chn}^2}{\nu_{st}^2}\alpha_n J_0(\rho_n).$$

The width of all electromagnetic pulses is the same $\Delta\tau_{stn} \sim 1/\nu_{st}$ and does not depend on the number of the radial harmonic. The level of the field of these pulses is small. The electromagnetic part of the field (16) contains a sequence of bipolar antisymmetric pulses, as well as a wake monochromatic electromagnetic wave of a small amplitude.

Behind the laser pulse $\tau >> t_L$ Cherenkov electromagnetic field is a superposition of electromagnetic eigenwaves of the dielectric waveguide. In the case of Gaussian transverse profile $R(r) = \exp(-r^2/r_L^2)$, where r_L is the characteristic transverse size of the laser pulse, $r_b << b$, the expression for electromagnetic wakefield has the form

$$E_z(r,\tau) = -E_w\sum_{n=1}^{\infty}\frac{\Psi(\eta_n)}{J_1^2(\lambda_n)}J_0(\rho_n)\cos\omega_{chn}\tau,$$

where

$$E_w = \frac{2\pi^2}{3}a_0^2\frac{e}{\lambda_L^2}\frac{\omega_*^2}{\omega_d^2}\frac{ct_L}{\beta_g r_{cl}}\frac{(\varepsilon_L - 1)^2}{\varepsilon_0},\ a_0 = \frac{eE_0}{mc\omega_L},$$

$\Psi(\eta_n) = \eta_n\exp(-\eta_n),\ \eta_n = \frac{\lambda_n^2}{4}\frac{r_L^2}{b^2}$, $r_{cl}\ \frac{e^2}{mc^2}$, λ_L is the laser pulse wavelength. The function $\Psi(\eta_n)$ has maximum $\Psi_{max} = 1/2.72$ for $\eta_n = 1$. If condition $b >> a$ is fulfilled, this means that the radial harmonic with the number $\lambda_n \approx 2b/r_L$ or $n \approx 4b/\pi r_L$ is most effectively excited.

CONCLUSIONS

In this paper the process of excitation of the wake Cherenkov radiation by a laser pulse in a dielectric waveguide is investigated. The nonlinear polarization of the dielectric medium, induced by the ponderomotive force from the laser pulse, is determined. It is shown that the excited electric field consists of a potential field of polarization oscillations excited by a potential component of a ponderomotive force and a set of eigen wakefield electromagnetic waves of a dielectric waveguide. The latter are excited by polarization charges and currents induced by the vortex component of the ponderomotive force.

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ЧЕРЕНКОВСКОЕ ИЗЛУЧЕНИЕ ЛАЗЕРНОГО ИМПУЛЬСА В ДИЭЛЕКТРИЧЕСКОМ ВОЛНОВОДЕ

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Представлены теоретические исследования возбуждения черенковского излучения сверхсветовым лазерным импульсом в диэлектрической среде, аналогичного излучению электрона, равномерно движущегося в замедляющей среде со сверхсветовой скоростью. Получена нелинейная поляризация среды, вызванная пондеромоторной силой лазерного импульса. Найдена структура возбуждаемых полей, в том числе кильватерных. Рассмотрена частотная дисперсия диэлектрической проницаемости, обеспечивающая сверхсветовую групповую скорость лазерного импульса.

ЧЕРЕНКОВСЬКЕ ВИПРОМІНЮВАННЯ ЛАЗЕРНОГО ІМПУЛЬСУ В ДІЕЛЕКТРИЧНОМУ ХВИЛЕВОДІ

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Представлено теоретичні дослідження збудження черенковського випромінювання надсвітловим лазерним імпульсом у діелектричному хвилеводі, аналогічного випромінюванню електрона, що рівномірно рухається в уповільнюючому середовищі з надсвітловою швидкістю. Отримана нелінійна поляризація середовища, викликана пондеромоторною силою лазерного імпульсу. Знайдена структура збуджуваних полів, в тому числі кільватерних. Розглянута частотна дисперсія діелектричної проникності, що забезпечує надсвітлову групову швидкість лазерного імпульсу.