

# A NUMERICAL MODEL OF RADIO-FREQUENCY WALL CONDITIONING FOR STEADY-STATE STELLARATORS

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A new self-consistent model of the radio-frequency (RF) plasma production in stellarators in the ion-cyclotron (IC) and electron-cyclotron (EC) frequency ranges is described in this work. This model will be used for RF and ECRH start-up and for numerical analysis of the plasma discharge for the vacuum chamber wall conditioning in stellarator type machines. The self-consistent model includes the system of the particle and energy balance equations and the boundary problem for the Maxwell's equations. A new feature of this model is account of molecular ions,  $H_2^+$  and  $H_3^+$ , in the particle balance equations and possibility to model the ECRH breakdown.

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## INTRODUCTION

Wall conditioning in magnetic fusion devices is used after opening the vacuum chamber to air or after usage of vacuum locks. During such operations, some substances are brought to the vacuum chamber and adsorbed at its wall. The goal of the discharge wall conditioning is to turn adsorbed species to volatile state so that they may be pumped out of the vacuum chamber.

An advantage of an RF discharge is that it could be arranged in steady magnetic field. This is important for the devices with superconducting magnets at which switching on/off the magnetic field is much more complicated than in devices using copper magnets.

To be efficient in generating the neutral atoms and to provide uniform wall cleaning, the RF discharge should preferably be volumetric and its plasma should have low temperature.

RF start-up is of interest for target plasma production for further neutral beam injection and in those cases when the conditions for electron-cyclotron (EC) heating are not met and this heating cannot produce plasma. Then it is possible to heat the plasma at different steady magnetic fields.

In this paper a model of RF plasma production in stellarators in the ion cyclotron and electron-cyclotron frequency ranges is presented.

New model will be used for simulation of plasma start-up and for numerical analysis of the plasma discharge for the vacuum chamber wall conditioning [1] in stellarator type machines.

## NUMERICAL MODEL

As a prototype for the whole code, the models for atomic gas [2] and for molecular hydrogen [3] are used, which are developed by the research group earlier.

The developed earlier model for atomic hydrogen can describe the final stage of plasma production. In the model for molecular hydrogen, only electron-hydrogen molecular collisions are accounted for. The particle balance is determined by ionization of the hydrogen

molecule. This model is suitable for low plasma densities.

There is a need for a model which incorporated all the collision processes and is valid at all stages of plasma production. Note here that the lower dimensionality 0D model for all sorts of hydrogen and helium is described in [4].

A newly developed model as well as the previous models includes the system of the particle and energy balance equations for the electrons and the boundary problem for the Maxwell's equations. A new feature is account of molecular ions,  $H_2^+$  and  $H_3^+$ , in the particle balance equations, and the radio-frequency module of the code is modified accordingly. The contribution of ions and collision frequencies is also taken into account in the dielectric tensor.

Neutral gas is assumed to consist of molecular and atomic hydrogen.

The stellarator plasma column is modeled as a straight plasma cylinder of lengths  $2\pi R_{tor}$  ( $R_{tor}$  is the major radius of the torus) with identical electric fields at its ends. The plasma is assumed to be axisymmetric, radially non-uniform and uniformly distributed along the plasma column.

The self-consistent model has a part that calculates particle and heat transport. This part of the code accounts for the neoclassical diffusion, the turbulent transport, and the elementary atomic and molecular collision processes of plasma interaction with the neutral gas Table.

On the base of this numerical model, a one-dimensional numerical code is developed. In this code, input power to electrons and ions is calculated by the RF module.

The RF power is calculated by solving the boundary problem for Maxwell's equations. Maxwell's equations are solved for the current profiles of the plasma density and plasma temperature at each time step and RF power deposition radial distribution is the output. Maxwell's equations are solved in a one-dimensional approximation by using the Fourier series along the azimuthal and longitudinal coordinates.

Elementary processes in hydrogen plasma

Processes	Reactions	Reaction Rates
Ionization of atom H	$e + H \rightarrow 2e + H^+$	$\langle \sigma_{i,H} v \rangle$
Ionization of molecule H <sub>2</sub>	$e + H_2 \rightarrow 2e + H_2^+$ $e + H_2 \rightarrow 2e + H + H^+$	$\langle \sigma_{i,H_2} v \rangle$
Recombination of atom H	$e + H^+ \rightarrow H + hv$	$\langle \sigma_{rec,H^+} v \rangle$
Dissociative recombination of molecular ion H <sub>2</sub> <sup>+</sup>	$e + H_2^+ \rightarrow 2H$	$\langle \sigma_{rec,H_2^+} v \rangle$
Dissociative recombination of molecular ion H <sub>3</sub> <sup>+</sup>	$e + H_3^+ \rightarrow 3H$	$\langle \sigma_{rec,H_3^+} v \rangle$
Three-body recombination	$2e + H^+ \rightarrow e + H$	$\langle \alpha_{rec} v \rangle$
Vibrational excitation of atom H <sub>2</sub>	$e + H_2 \rightarrow e + H_2^*$ $e + H_2 \rightarrow e + H_2 + hv$	$\langle \sigma_{vibr,H_2} v \rangle$
Vibrational excitation of molecular ion H <sub>2</sub> <sup>+</sup>	$e + H_2^+(v) \rightarrow e + H_2^+(v')$	$\langle \sigma_{vibr,H_2^+} v \rangle$
Vibrational excitation of molecular ion H <sub>3</sub> <sup>+</sup>	$e + H_3^+(v_3) \rightarrow e + H_3^+(v_3')$	$\langle \sigma_{vibr,H_3^+} v \rangle$
Dissociation of molecule H <sub>2</sub>	$e + H_2 \rightarrow e + 2H$	$\langle \sigma_{d,H_2} v \rangle$
Dissociation of molecular ion H <sub>2</sub> <sup>+</sup>	$e + H_2^+ \rightarrow e + H + H^+$	$\langle \sigma_{d,H_2^+} v \rangle$
Dissociation of molecular ion H <sub>3</sub> <sup>+</sup>	$e + H_3^+ \rightarrow e + H^+ + 2H$ $e + H_3^+ \rightarrow e + H^+ + H_2$ $e + H_3^+ \rightarrow e + H + H_2^+$	$\langle \sigma_{d,H_3^+} v \rangle$
Electron excitation of molecule H <sub>2</sub>	$H^+ + H_2 \rightarrow H^+ + H_2^*$ $e + H_2 \rightarrow e + H(1s) + H$	$\langle \sigma_{e,H_2} v \rangle$
Electron excitation of molecular ion H <sub>2</sub> <sup>+</sup>	$e + H_2^+(v) \rightarrow e + H_2^+(v')$	$\langle \sigma_{e,H_2^+} v \rangle$
H <sub>3</sub> <sup>+</sup> ion formation	$H_2^+ + H_2 \rightarrow H_3^+ + H$	$\langle \sigma_{tr,H_2^+} v \rangle$

The RF module uses 3rd order finite elements in radial direction. Antenna is specified as prescribed electric currents.

In addition to the RF module, the numerical code is supplemented with a module for EC heating. A new module that calculates second harmonic electron cyclotron heating in case of weak wave damping is created and incorporated into the code. The code can work using either RF module or EC.

The ECRH module takes into account that power deposition is proportional to the plasma density and electron Larmor radius square. Besides, it is proportional to the width of the ion cyclotron zone which is narrow when the electron temperature is low. The width, in turn, is proportional to the square root of the temperature. The power deposition formula reads:

$$P_{ECRH} = P_0 f_{ECRH}(r) (n_e / n_0) (T_e / T_0)^{3/2}, \quad (1)$$

where  $P_{ECRH}$  is the power deposition density,  $f_{ECRH}$  is the power deposition shaping function which depends on electron cyclotron zone position and the ECRH ray focusing,  $n_e$  is the plasma density,  $T_e$  is the electron temperature and the quantities indexed with zero are normalizing constants.

The system of the balance equations of particles and energy is presented below.

Energy balance equation is:

$$\begin{aligned} \frac{3}{2} \frac{\partial (k_B n_e T_e)}{\partial t} = & P_{RF} - k_B [\varepsilon_{vibr,H_2} \langle \sigma_{vibr,H_2} v \rangle n_e n_{H_2} + \\ & + \varepsilon_{vibr,H_2^+} \langle \sigma_{vibr,H_2^+} v \rangle n_e n_{H_2^+} + \varepsilon_{vibr,H_3^+} \langle \sigma_{vibr,H_3^+} v \rangle n_e n_{H_3^+} + \\ & + \varepsilon_{d,H_2} \langle \sigma_{d,H_2} v \rangle n_e n_{H_2} + \varepsilon_{d,H_2^+} \langle \sigma_{d,H_2^+} v \rangle n_e n_{H_2^+} + \\ & + \varepsilon_{d,H_3^+} \langle \sigma_{d,H_3^+} v \rangle n_e n_{H_3^+} + \varepsilon_{i,H_2} \langle \sigma_{i,H_2} v \rangle n_e n_{H_2} + \\ & + \varepsilon_{i,H_2^+} \langle \sigma_{i,H_2^+} v \rangle n_e n_{H_2^+} + \varepsilon_{i,H_3^+} \langle \sigma_{i,H_3^+} v \rangle n_e n_{H_3^+} + \\ & + \varepsilon_{e,H_2} \langle \sigma_{e,H_2} v \rangle n_e n_{H_2} + \varepsilon_{e,H_2^+} \langle \sigma_{e,H_2^+} v \rangle n_e n_{H_2^+} + \\ & + \varepsilon_{e,H_3^+} \langle \sigma_{e,H_3^+} v \rangle n_e n_{H_3^+}] - \frac{3}{2} k_B T_e \langle \sigma_{rec,H^+} v \rangle n_e n_{H^+} - \\ & - \frac{3}{2} k_B T_e \langle \sigma_{rec,H_2^+} v \rangle n_e n_{H_2^+} - \frac{3}{2} k_B T_e \langle \sigma_{rec,H_3^+} v \rangle n_e n_{H_3^+} - \\ & - \frac{3}{2} C_{rec} k_B T_e \langle \alpha_{rec} v \rangle n_e^2 n_{H^+} - (C_a + 1) \frac{k_B n_e T_e}{\tau_n} - \\ & - \frac{k_B}{r} \frac{\partial}{\partial r} r \left( q_e + n_e V_e T_e - \chi n_e \frac{\partial T_e}{\partial r} \right) - e n_e V_e E_r, \end{aligned}$$

Balance equation for electrons is:

$$\begin{aligned} \frac{\partial n_e}{\partial t} = & \langle \sigma_{i,H} \mathbf{v} \rangle n_e n_H + \langle \sigma_{i,H_2} \mathbf{v} \rangle n_e n_{H_2} + \langle \sigma_{d,H_2^+} \mathbf{v} \rangle n_e n_{H_2^+} + \\ & + \langle \sigma_{d,H_3^+} \mathbf{v} \rangle n_e n_{H_3^+} - \langle \sigma_{rec,H^+} \mathbf{v} \rangle n_e n_{H^+} - \langle \sigma_{rec,H_2^+} \mathbf{v} \rangle n_e n_{H_2^+} - \\ & - \langle \sigma_{rec,H_3^+} \mathbf{v} \rangle n_e n_{H_3^+} - \langle \alpha_{rec} \mathbf{v} \rangle n_e^2 n_{H^+} - \frac{n_e}{\tau_n} - \frac{1}{r} \frac{\partial}{\partial r} (m_e n_e V_e), \end{aligned}$$

Balance equations for molecular ions are:

$$\begin{aligned} \frac{\partial n_{H^+}}{\partial t} = & \langle \sigma_{i,H} \mathbf{v} \rangle n_e n_H + \langle \sigma_{d,H_2^+} \mathbf{v} \rangle n_e n_{H_2^+} + \\ & + (1 - C_{dis}) \langle \sigma_{d,H_3^+} \mathbf{v} \rangle n_e n_{H_3^+} - \langle \sigma_{rec,H^+} \mathbf{v} \rangle n_e n_{H^+} - \\ & - \langle \alpha_{rec} \mathbf{v} \rangle n_e^2 n_{H^+} - \frac{n_{H^+}}{\tau_n} - \frac{1}{r} \frac{\partial}{\partial r} (m_{H^+} V_e), \end{aligned}$$

$$\begin{aligned} \frac{\partial n_{H_2^+}}{\partial t} = & \langle \sigma_{i,H_2} \mathbf{v} \rangle n_e n_{H_2} + C_{dis} \langle \sigma_{d,H_3^+} \mathbf{v} \rangle n_e n_{H_3^+} - \\ & - \langle \sigma_{d,H_2^+} \mathbf{v} \rangle n_e n_{H_2^+} - \langle \sigma_{rec,H_2^+} \mathbf{v} \rangle n_e n_{H_2^+} - \\ & - \langle \sigma_{ir,H_2^+} \mathbf{v} \rangle n_{H_2} n_{H_2^+} - \frac{n_{H_2^+}}{\tau_n} - \frac{1}{r} \frac{\partial}{\partial r} (m_{H_2^+} V_e), \end{aligned}$$

$$\begin{aligned} \frac{\partial n_{H_3^+}}{\partial t} = & \langle \sigma_{ir,H_3^+} \mathbf{v} \rangle n_{H_2} n_{H_3^+} - \langle \sigma_{d,H_3^+} \mathbf{v} \rangle n_e n_{H_3^+} - \\ & - \langle \sigma_{rec,H_3^+} \mathbf{v} \rangle n_e n_{H_3^+} - \frac{n_{H_3^+}}{\tau_n} - \frac{1}{r} \frac{\partial}{\partial r} (m_{H_3^+} V_e), \end{aligned}$$

Balance equations for neutral gas are:

$$\begin{aligned} \frac{\partial n_H}{\partial t} = & 2 \langle \sigma_{d,H_2} \mathbf{v} \rangle n_e n_{H_2} + \langle \sigma_{ir,H_2^+} \mathbf{v} \rangle n_{H_2} n_{H_2^+} + \\ & + \langle \sigma_{rec,H} \mathbf{v} \rangle n_e n_H + 2 \langle \sigma_{rec,H_2^+} \mathbf{v} \rangle n_e n_{H_2^+} + \langle \alpha_{rec} \mathbf{v} \rangle n_e^2 n_{H^+} + \\ & + 3 \langle \sigma_{rec,H_3^+} \mathbf{v} \rangle n_e n_{H_3^+} - \langle \sigma_{i,H} \mathbf{v} \rangle n_e n_H - C_H \frac{S}{V_V} \langle v_H \rangle n_H, \end{aligned}$$

$$\begin{aligned} \frac{\partial n_{H_2}}{\partial t} = & I_{puff} + C_H \frac{S}{2V_V} \langle v_H \rangle n_H - \langle \sigma_{i,H_2} \mathbf{v} \rangle n_e n_{H_2} - \\ & - \langle \sigma_{d,H_2} \mathbf{v} \rangle n_e n_{H_2} - \langle \sigma_{ir,H_2^+} \mathbf{v} \rangle n_{H_2} n_{H_2^+}. \end{aligned} \quad (2)$$

Here, the averaged quantities are calculated as

$$\bar{A} = 2 \int_0^{r_{wall}} A r dr / r_{wall}^2, \quad n_H, n_{H_2} \text{ are the densities of atom}$$

and molecule hydrogen,  $n_{H^+}, n_{H_2^+}, n_{H_3^+}$  are the densities of atomic and molecular hydrogen ions,  $P_{RF}$  is the RF power density of electron heating,  $k_B$  is the Boltzmann constant,  $C_{rec}$  is the relative part of the energy which is spent by the electrons in the recombination process,  $C_{dis}$  is the probability appearance of  $H_2^+$  ion in dissociation of  $H_3^+$  ion,  $C_H$  is the coefficient of reflection of atomic hydrogen from the chamber wall,  $nV_e$  is the particle flux,  $\chi$  is the turbulent transport,  $I_{puff}$  is the neutral gas puff rate,  $\tau_n$  is the particle confinement time,  $V_V$  is the

vacuum chamber volume, and  $C_a = e\Phi_a/T_e \approx 3.5$  is the ratio of the electron energy in the ambipolar potential to the electron thermal energy. Only electrons with energies higher than the potential energy  $e\Phi_a$  leave the plasma.

The finite difference method is used for discretization in space. After this, the system of equations (2) could be written as

$$\frac{d\mathbf{y}}{dt} = \hat{D}(\mathbf{x}, \mathbf{y})\mathbf{y} + \hat{R}(\mathbf{x}, \mathbf{y})\mathbf{y}, \quad (3)$$

where  $\mathbf{x}$  is the vector containing coordinates of all discretization points in space,  $\mathbf{y}$  is the vector of the values of all unknowns at the points of discretization,  $D$  is the discretized differential (diffusion) operator and  $R$  is discretized residual terms in (3) (sources and sinks). For discretization in time the running mesh is chosen with variable time step  $\Delta t$  and  $i$  as an index enumerating time moments. The Crank-Nicolson method provides numerically stable solution of the system of equations. The difficulty comes from the non-linearity in the operators  $D$  and  $R$ . The first step

$$\frac{\mathbf{y}_s^{i+1} - \mathbf{y}^i}{\Delta t} = \hat{D}(\mathbf{x}, \mathbf{y}^i)(\mathbf{y}_s^{i+1} + \mathbf{y}^i)/2 + \hat{R}(\mathbf{x}, \mathbf{y}^i)\mathbf{y}^i. \quad (4)$$

Does not give the necessary accuracy since operators  $D$  and  $R$  are calculated not in the central point of time segment. Anyway one can use Runge-Kutta iteration to approximate the central point value. It is calculated approximately using the obtained solution:

$$\mathbf{y}^{i+1/2} = (\mathbf{y}_s^{i+1} + \mathbf{y}^i)/2. \quad (5)$$

The final step of the calculations is solution of the equation:

$$\frac{\mathbf{y}^{i+1} - \mathbf{y}^i}{\Delta t} = \hat{D}(\mathbf{x}, \mathbf{y}^{i+1/2})(\mathbf{y}^{i+1} + \mathbf{y}^i)/2 + \hat{R}(\mathbf{x}, \mathbf{y}^{i+1/2})\mathbf{y}^{i+1/2}. \quad (6)$$

## THE CODE

The numerical code is written within the framework of the Fortran90 language standard, which makes it possible to compile and run it on various computers.

Operation of the numerical code is organized as follows. On first call it makes reading and storing calculation parameters and, after this, makes memory management. All parameters are available to all subprograms of the code through the module units.

Next step is the formation of two meshes. The first mesh is used by the balance equations and the second one is for Maxwell's equations. Both meshes could be non-uniform. In Maxwell's equations mesh the mesh nodes are adapted to the placement of the antenna. If the code uses ECRH module, only one mesh is used.

In a separate block, initial profiles for plasma densities and temperatures are calculated.

Next to these the code core is executed. It is main loop is over time. Inside this loop, the steps of the Crank-Nicolson – Runge-Kutta algorithm are

performed. This algorithm required calculations of collision rates of the elementary processes. The cross-sections for these processes are taken from the literature, interpolated and digitized and then incorporated to the code. Crank-Nicolson algorithm needs also to solve systems of linear equations. This is performed using LU-decomposition of the resulting matrix and solving the algebraic equations.

After the step of the Crank-Nicholson algorithm is completed, the conditional output is made, the value of the time step is controlled and, when the conditions for termination of the code are met, the code is terminated.

## DISCUSSION

A new self-consistent model of RF plasma production in stellarators in the ion cyclotron and electron-cyclotron frequency ranges is presented.

A newly developed model as the previous models includes the system of the particle and energy balance equations for the electrons and the boundary problem for the Maxwell's equations.

A new feature of this model as compared with previous models is account of molecular ions,  $H_2^+$  and  $H_3^+$ , in the particle balance equations. Neutral gas is assumed to consist of molecular and atomic hydrogen. The radio-frequency module of the code is modified accordingly. The code uses the neoclassical diffusion,

turbulent transport, and elementary atomic and molecular collision processes.

In the balance of neutral gas, the hydrogen retention and recombination at the wall surface are taken into account.

On the base of this model, a one-dimensional numerical code is developed.

A new module that calculates second harmonic electron cyclotron heating in case of weak wave damping is created and incorporated into the code.

This model will be used for start-up and for numerical analysis of the plasma discharge for the vacuum chamber wall conditioning in stellarator type machines.

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## САМОСОГЛАСОВАННОЕ МОДЕЛИРОВАНИЕ ВОЗРАСТАНИЯ ПЛОТНОСТИ ПЛАЗМЫ С ВЫСОКОЧАСТОТНЫМ НАГРЕВОМ

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Представлена новая самосогласованная модель высокочастотного (ВЧ) создания плазмы в стеллараторах в ионно-циклотронном (ИЦ) и электронно-циклотронном (ЭЦ) диапазонах частот. Эта модель будет использоваться для ВЧ- и ИЦ-создания плазмы, а также для численного анализа плазменного разряда для чистки стенок вакуумной камеры в установках стеллараторного типа. Самосогласованная модель включает в себя систему уравнений баланса частиц и энергии для электронов и краевую задачу для уравнений Максвелла. Новой особенностью модели является учет молекулярных ионов  $H_2^+$  и  $H_3^+$  в уравнениях баланса частиц и возможность моделировать ИЦ-пробой.

## САМОУЗГОДЖЕНЕ МОДЕЛЮВАННЯ ЗРОСТАННЯ ГУСТИНИ ПЛАЗМИ З ВИСОКОЧАСТОТНИМ НАГРІВОМ

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Представлено нову самоузгоджену модель високочастотного (ВЧ) створення плазми в стеллараторах в іонно-циклотронному (ИЦ) і електронно-циклотронному (ЭЦ) діапазонах частот. Ця модель буде використовуватися для моделювання ВЧ- та ІЦ-створення плазми, а також для числового аналізу плазмового розряду для очищення стінок вакуумної камери в установках стеллараторного типу. Самоузгоджена модель включає в себе систему рівнянь балансу частинок та енергії для електронів і крайову задачу для рівнянь Максвелла. Новою особливістю моделі є врахування молекулярних іонів  $H_2^+$  і  $H_3^+$  у рівняннях балансу частинок та можливість моделювати ІЦ-пробій.