ELECTROMAGNETIC FIELD ENERGY AND RADIATION INTENSITY IN A MEDIUM WITH TEMPORAL AND SPATIAL DISPERSION OUTSIDE THE TRANSPARENCY DOMAIN

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Calculation of electromagnetic field energy in a medium with temporal and spatial dispersion outside the transparency domain is discussed. It is shown that charged particle contribution to the energy of electromagnetic perturbations in the general case can be described in terms of a bilinear combination of the dielectric polarizability of the medium. The explicit form of such contribution is found. The relations obtained are used to generalize the Planck law and Kirchhoff law to the case of an absorptive medium with spatial dispersion.

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INTRODUCTION

It is well known that the energy density of an electromagnetic wave in a medium with spatial and temporal dispersion can be consistently defined only in the transparency domain [1-5]. This problem has been discussed in the literature during decades. After the pioneer Brillouin result for the electromagnetic wave energy in dispersive transparent media [6, 7] a lot of papers has been published on this subject and many attempts to generalize the Brillouin's approach have been made to consider absorptive properties of medium. Nevertheless, the results known from the literature do not concern the general solution of the problem, but only various particular cases.

As is known, the energy of an electromagnetic perturbation in a matter contains the "pure"' electromagnetic energy and the kinetic energy of charge carriers obtained due to their motion in the electromagnetic field [2, 3, 8-10]. If neutral particles (i.e. atoms or molecules) are present, the additional potential energy acquired by bound electrons in such field also should be added [10-16]. Beside that in the case of absorptive medium some part of electromagnetic energy is converted into a heat [10, 12, 16]. Thus, the problem arises to describe consistently all these quantities. This introduces the principal difficulties to generalize the Brillouin formula to the case of dispersive absorptive medium since in such a case the macroscopic Maxwell equations generate a Pointing-like equation that does not provide the possibility to identify explicitly the contribution of the electromagnetic perturbation energy and the heat production to the total energy transferred to the medium by the electromagnetic field, in contrast to the case of an transparent medium for which the total energy of the field is well defined and the heat production is absent.

In order to avoid the above-mentioned difficulties, it is possible to calculate all constituents of the electromagnetic field energy directly and express them in terms of dielectric susceptibilities as it was done for the case of dissipative medium without spatial dispersion [2, 10]. This approach can be justified using the energy balance equation which follows from the combination of the Maxwell equations and the kinetic equation for charge carriers. Such energy balance equation for the first time was formulated by V. Ginzburg for a plasma medium [8, 9]. In spite of the fact that the general ideas of electromagnetic field energy description were formulated many years ago it was not yet applied to the case of absorptive medium with spatial dispersion.

The purpose of the present contribution is to derive a general relation for the energy of electromagnetic perturbation in the medium with temporal and spatial dispersion outside the transparency domain.

1. BASIC SET OF EQUATIONS AND STATEMENT OF THE PROBLEM

We start from the Maxwell equations for the electromagnetic field in a medium in the form that is often used in the plasma theory [3, 4, 20, 21]

$$\begin{aligned}
&\mathbf{rot}\,\mathbf{E}(\mathbf{r},t) = -\frac{1}{c}\frac{\partial \mathbf{B}(\mathbf{r},t)}{\partial t},\\
&\mathrm{div}\,\mathbf{B}(\mathbf{r},t) = 0,\\
&\mathbf{rot}\,\mathbf{B}(\mathbf{r},t) = \frac{1}{c}\frac{\partial \mathbf{D}(\mathbf{r},t)}{\partial t} + \frac{4\pi}{c}\mathbf{J}^e(\mathbf{r},t),\\
&\mathrm{div}\,\mathbf{D}(\mathbf{r},t) = 4\pi\rho^e(\mathbf{r},t),
\end{aligned} \tag{1}$$

where $\mathbf{J}^e(\mathbf{r},t)$ and $\rho^e(\mathbf{r},t)$ are the external sources, if present. In the case under consideration $\mathbf{H}(\mathbf{r},t) \equiv \mathbf{B}(\mathbf{r},t)$, and thus the total medium response to the electromagnetic field is described by the dielectric permittivity tensor $\varepsilon_{ij}(\mathbf{r},\mathbf{r}';t-t')$,

$$D_i(\mathbf{r},t) = E_i(\mathbf{r},t) + \int_{-\infty}^{t} dt' J_i(\mathbf{r},t')$$

$$= \int_{-\infty}^{t} dt' \int d\mathbf{r}' \varepsilon_{ij}(\mathbf{r},\mathbf{r}';t-t') E_j(\mathbf{r}',t'),$$

where $J_i(\mathbf{r},t)$ is the total induced current that includes all kinds of responses and can be expressed in terms of the conductivity tensor $\sigma_{ij}(\mathbf{r},\mathbf{r}';t-t')$ [21]

$$J_i(\mathbf{r},t) = \int_{-\infty}^{t} dt' d\mathbf{r}' \sigma_{ij}(\mathbf{r},\mathbf{r}';t-t')E) j(\mathbf{r}',t'). \quad (2)$$

Thus

$$\varepsilon_{ij}(\mathbf{r},\mathbf{r}';t-t') = \delta_{ij}\delta(\mathbf{r}-\mathbf{r}')\delta(t-t') + 4\pi\int_{t'}^{t} dt''\sigma_{ij}(\mathbf{r},\mathbf{r}';t''-t'). \quad (3)$$

We need also equations describing the interaction of electromagnetic fields with the medium. In what follows we illustrate the possibility to calculate the energy of electromagnetic perturbation using a plasma-like medium. So, we supplement Eqs. (1), (2) with the kinetic equation for plasma particles

$$\left\{\frac{\partial}{\partial t}+\mathbf{v}\frac{\partial}{\partial \mathbf{r}}+\frac{e_\alpha}{m_\alpha}\mathbf{F}^{\text{ext}}+\frac{e_\alpha}{m_\alpha}\left[\mathbf{E}(\mathbf{r},t)+\frac{\mathbf{v}}{c}\times\mathbf{B}(\mathbf{r},t)\right]\frac{\partial}{\partial \mathbf{v}}\right\}\cdot f_\alpha(\mathbf{r},\mathbf{v},t) = \mathrm{I}_\alpha, \quad (4)$$

where $f_\alpha(\mathbf{r},\mathbf{v},t)$ is the distribution function of particles of α species, I_α is the collision term, $\mathbf{F}^{\text{ext}}$ is the external force field, if present, other notation is traditional.

Eq. (4) is valid in the case of classical plasma-like medium. The appropriate calculations for the case of a combined plasma-molecular medium can be performed using the model of bound particles (see, for instance, Refs. [10, 11, 16-18]). Quantum description of both plasma and plasma-molecular systems is also possible [17, 19]. However, since the formulation of the general approach does not require the explicit form of the response function (except for the calculation of specific examples) as is shown below we need to know only the general relation between the induced macroscopic currents $\mathbf{J}(\mathbf{r},t)$ and the self-consistent electric field $\mathbf{E}(\mathbf{r},t)$ given by Eq. (2).

Using Eqs. (1) we obtain the well-known equation

$$\frac{1}{4\pi}\left\{\mathbf{E}\frac{\partial \mathbf{D}}{\partial t}+\mathbf{B}\frac{\partial \mathbf{B}}{\partial t}\right\}+\mathbf{J}^{\text{ext}}\,\mathbf{E} = -\frac{c}{4\pi}\,\mathrm{div}[\mathbf{EB}], \quad (5)$$

that reduces to the Pointing equation in the case of a nondispersive medium. It can be also used to calculate the energy W_ω of the quasi-monochromatic field in the case of a weakly absorbing homogeneous medium [3, 10, 13], to recover the well-known Brillouin formula [1, 6, 7].

In order to obtain the general relations we derive an equation for the energy balance that takes into account the particle energy explicitly [8-10]. To do this it is necessary to multiply the kinetic equation (4) by $n_\alpha m_\alpha v^2/2$ (n_α is the density of particles of α species) and to integrate over the velocity $\mathbf{v}$. The result is

$$\frac{\partial}{\partial t}\int d\mathbf{v}\frac{n_\alpha m_\alpha v^2}{2} f_\alpha(X,t)+\frac{\partial}{\partial \mathbf{r}}\int d\mathbf{v}\,\mathbf{v}\frac{n_\alpha m_\alpha v^2}{2} f_\alpha(X,t) + \int d\mathbf{v}\frac{n_\alpha e_\alpha v^2}{2}\left[\mathbf{E}+\frac{v}{c}\times\mathbf{B}\right]\frac{\partial f_\alpha(X,t)}{\partial v} = \int d\mathbf{v}\frac{m_\alpha v^2}{2}\,\mathrm{I}_\alpha. \quad (6)$$

We note that $\int d\mathbf{v}\frac{m_\alpha v^2}{2}\,\mathrm{I}_\alpha = 0$, employ the equality

$$\sum_\alpha\int d\mathbf{v}\frac{n_\alpha e_\alpha v^2}{2}\left[\mathbf{E}+\frac{\mathbf{v}}{c}\times\mathbf{B}\right]\frac{\partial f_\alpha(X,t)}{\partial \mathbf{v}} = -e_\alpha n_\alpha\int d\mathbf{v}\mathbf{v}\,\mathbf{E}f_\alpha(X,t) = -\mathbf{EJ}, \quad (7)$$

and combine Eqs. (6), (7) with Eq. (5).

Thus we obtain an equation for the energy balance [8-10]

$$\frac{\partial}{\partial t}\left\{\frac{1}{8\pi}\left(\mathbf{E}^2(\mathbf{r},t)+\mathbf{B}^2(\mathbf{r},t)\right)+\sum_\alpha\int d\,\mathbf{v}\frac{n_\alpha m_\alpha c v^2}{2} f_\alpha(X,t)\right\} + \frac{\partial}{\partial \mathbf{r}}\left\{\frac{c}{4\pi}\left[\mathbf{E}(\mathbf{r},t)\times\mathbf{B}(\mathbf{r},t)\right]+\sum_\alpha\int d\mathbf{v}\,\mathbf{v}\frac{n_\alpha m_\alpha c v^2}{2} f_\alpha(X,t)\right\} + \mathbf{J}^{\text{ext}}(\mathbf{r},t)\mathbf{E}(\mathbf{r},t)=0, \quad (8)$$

where the terms responsible for the particle energy and energy flux are present in the explicit form. We see that there is no need to extract the particle energy term from the quantity $\mathbf{E}(\partial\mathbf{D}/\partial t)$ as it is done in the case of a weakly absorptive medium [3, 10].

2. ENERGY DENSITY OF THE ELECTROMAGNETIC FIELD PERTURBATION WITH REGARD TO THE PARTICLE ENERGY ACQUIRED UNDER THE ACTION OF THE FIELD

In the zero-order approximation on the gas-dynamic parameter ($l/L \ll 1$, where l is the mean free path, L is the size of the system) the solution of the kinetic equation (4) may be written in the form of the local Maxwellian distribution [17]

$$f_\alpha(X,t) = \frac{n_\alpha(\mathbf{r},t)}{n_\alpha}\left(\frac{m_\alpha}{2\pi T_\alpha(\mathbf{r},t)}\right)^{3/2}\exp\left[-\frac{m_\alpha(\mathbf{v}-\mathbf{u}_\alpha(\mathbf{r},t))}{2T_\alpha(\mathbf{r},t)}\right], \quad (9)$$

where

$$n_\alpha(\mathbf{r},t) = n_\alpha\int d\mathbf{v}f_\alpha(X,t),$$
$$u_\alpha(\mathbf{r},t) = \frac{n_\alpha\int \mathbf{v}d\mathbf{v}f_\alpha(X,t)}{n_\alpha(\mathbf{r},t)}, \quad (10)$$
$$T_\alpha(\mathbf{r},t) = \frac{n_\alpha\int d\mathbf{v}(m_\alpha/2)(\mathbf{v}-\mathbf{u}_\alpha(\mathbf{r},t))^2 f_\alpha(X,t)}{3n_\alpha(\mathbf{r},t)}.$$

Within such an approximation we can present the full energy density as given by

$$W = W_F + W_T + W_K, \quad (11)$$

where the field W_F, thermal W_T and kinetic W_T energies respectively are given by

$$W_F = \frac{1}{8\pi}\left(\mathbf{E}^2(\mathbf{r},t)+\mathbf{B}^2(\mathbf{r},t)\right),$$
$$W_T = \sum_\alpha\frac{3}{2}n_\alpha(\mathbf{r},t)T_\alpha(\mathbf{r},t), \quad (12)$$
$$W_K = \sum_\alpha n_\alpha(\mathbf{r},t)\frac{m_\alpha u_\alpha^2(\mathbf{r},t)}{2}.$$

Since W_T is the heat produced by the perturbation we can treat the energy associated with the electromagnetic field as the sum of W_F and W_K.

Restricting ourselves by the second order approximation in the perturbation, we can rewrite the part of energy W_K as

$$W_K = \sum_\alpha \frac{n_\alpha m_\alpha \ \mathbf{u}_\alpha^2(\mathbf{r},t)}{2} = \sum_\alpha \frac{m_\alpha}{2e_\alpha^2 n_\alpha} J_\alpha^2(\mathbf{r},t). \qquad (13)$$

Here $J_\alpha(\mathbf{r},t)$ is the partial contribution of the particle of α species to the induced current $J(\mathbf{r},t) = \sum_\alpha J_\alpha(\mathbf{r},t)$.

It should be noted that Eq. (13) directly follows from the transparent physical reasoning: the kinetic energy acquired by particles under the action of the electromagnetic field can be directly expressed in terms of the averaged induced velocity. Namely this approach was used to estimate the energy density of particles in the case of cold plasmas [10, 13]. However, as is seen Eq. (13) does not require such restrictions.

The generalization of the results obtained in [10, 12, 13] can be achieved using the relation between the induced current and the electric field (2). In terms of the generalized dielectric polarizability $\chi_{ij}^{(\alpha)}(\mathbf{k},\omega) \equiv (4\pi i/\omega)\sigma_{ij}^{(\alpha)}(\mathbf{k},\omega)$ (where $\sigma_{ij}^{(\alpha)}(\mathbf{k},\omega)$ is the partial contribution of particles of α species to the conductivity tensor of the system $\sigma_{ij}^{(\alpha)}(\mathbf{k},\omega) = \sum_\alpha \sigma_{ij}^{(\alpha)}(k,\omega)$) one obtains the following expression for the electromagnetic perturbation energy

$$W = \frac{1}{8\pi}\int\frac{d\mathbf{k}}{(2\pi)^3}\int\frac{d\mathbf{k}'}{(2\pi)^3}\int\frac{d\omega}{2\pi}\int\frac{d\omega'}{2\pi} e^{i(\mathbf{k}-\mathbf{k}')\mathbf{r}} e^{-i(\omega-\omega')t} \cdot \left\{\frac{k_i k_j'}{\mathbf{k}\mathbf{k}'} + \left(1+\frac{c^2}{\omega\omega'}\mathbf{k}\mathbf{k}'\right)\left(\delta_{ij} - \frac{k_i k_j'}{\mathbf{k}\mathbf{k}'}\right) + \sum_{\alpha=e,i}\frac{\omega^2}{\omega_{p\alpha}^2}\chi_{ki}^{(\alpha)}(\mathbf{k},\omega)\chi_{\mathbf{k}j}^{(\alpha)*}(\mathbf{k}',\omega')\right\} E_{i\mathbf{k}\omega}E^*_{j\mathbf{k}'\omega'}, \qquad (14)$$

where $\omega_{p\alpha}^2 = 4\pi e_\alpha^2 n_\alpha / m_\alpha$.

This is the general relation for a plasma-like medium.

It should be noted that Eq. (13) can be also used to estimate the kinetic energy of bound electrons in atoms and molecules. However, in this case the energy of electromagnetic perturbation contains along with the kinetic energy of electrons also the potential energy of bound electrons in the fields of ions with which they are bound. In the case of the classical model of the atom-oscillator [10-15] such energy can be estimated as

$$W_U = n_m \frac{\omega_0^2 r_m^2(\mathbf{r},t)}{2}.$$

Here n_m is the density of bound electrons, ω_0 is the eigenfrequency of the oscillator, $\mathbf{r}_m(\mathbf{r},t)$ is the reduced coordinate of the bound electron. Since $\mathbf{u}_m(\mathbf{r},t) = (d\mathbf{r}_m(\mathbf{r},t)/(dt))$, the energy W_U may be expressed in terms of the mean velocity $\mathbf{u}_m(\mathbf{r},t)$, i.e. in terms of the induced current of the bound electrons.

Thus,

$$W_U = \frac{1}{8\pi}\int\frac{d\mathbf{k}}{(2\pi)^3}\int\frac{d\mathbf{k}'}{(2\pi)^3}\int\frac{d\omega}{2\pi}\int\frac{d\omega'}{2\pi}\mathbf{e}^{i(\mathbf{k}-\mathbf{k}')\mathbf{r}} \cdot \mathbf{e}^{-i(\omega-\omega')t}\frac{\omega_0^2}{\omega_{pm}^2}\chi_{ki}^{(m)}(\mathbf{k},\omega)\chi_{lj}^{(m)*}(\mathbf{k}',\omega')E_{i\mathbf{k}\omega}E^*_{j\mathbf{k}'\omega'}, \qquad (15)$$

where $\chi_{ij}^{(m)}(\mathbf{k},\omega)$ in the case of the classical model of an atom-oscillator is given by [18]

$$\chi_{ij}^{(m)}(\mathbf{k},\omega) = -\delta_{ij}\int d\mathbf{v}\frac{\omega_{pm}^2 f_{0m}(\mathbf{v})}{(\omega-\mathbf{k}\mathbf{v})^2 - \omega_0^2 + i\gamma(\omega-\mathbf{k}\mathbf{v})}, \quad \omega_{pm}^2 = \frac{4\pi e_b^2 n_m}{m_b}, \qquad (16)$$

$f_{0m}(\mathbf{v})$ is the distribution function of bound particles (atoms, or molecules), e_b and m_b are the effective charge and the reduced mass of a bound electron.

So, in the case of a plasma-molecular system the energy of a perturbation may be written as

$$W = W_F + W_K + W_U = \frac{1}{8\pi}\int\frac{d\mathbf{k}}{(2\pi)^3}\int\frac{d\mathbf{k}'}{(2\pi)^3}\int\frac{d\omega}{2\pi}\int\frac{d\omega'}{2\pi} e^{i(\mathbf{k}-\mathbf{k}')\mathbf{r}} e^{-i(\omega-\omega')t} \cdot \left\{\left(\delta_{ij} - \frac{k_i k_j'}{\mathbf{k}\mathbf{k}'}\right)\left(1+\frac{c^2\mathbf{k}\mathbf{k}'}{\omega^2}\right) + \frac{k_i k_j'}{\mathbf{k}\mathbf{k}'} + \sum_{\alpha=e,i}\frac{\omega^2}{\omega_{p\alpha}^2}\chi_{ki}^{(\alpha)}(\mathbf{k},\omega)\chi_{kj}^{(\alpha)*}(\mathbf{k}',\omega') + \frac{\omega^2+\omega_0^2}{\omega_{pm}^2}\chi_{ki}^{(m)}(\mathbf{k},\omega)\chi_{kj}^{(m)*}(\mathbf{k}',\omega')\right\} \cdot E_{i\mathbf{k}\omega}E^*_{j\mathbf{k}'\omega}. \qquad (17)$$

This equation remains valid in the case of quantum description provided the polarizabilities $\chi_{ij}^{(\alpha)}(\mathbf{k},\omega)$ ($\alpha = e,i,m$) are calculated appropriately (see, for example, Ref. [17,19,20,22]).

In the case of the monochromatic field $\mathbf{E}(\mathbf{r},t) = (1/2)\{\mathbf{E}(\mathbf{r})e^{-i\omega t} + \mathbf{E}^*(\mathbf{r})\, e^{i\omega t}\}$, after the averaging over the oscillation period $T = 2\pi/\omega$ and the volume of the system *V*, Eq. (17) in the absence of spatial dispersion is reduced to

$$\bar{W} \equiv \bar{W}_E + \bar{W}_B, \qquad (18)$$

where

$$\bar{W}_E = \frac{1}{16\pi}\left\{\delta_{ij} + \sum_{e,i,m}\frac{\omega^2+\omega_{0\alpha}^2}{\omega_{p\alpha}^2}\chi_{ki}^{(\alpha)}(\omega)\chi_{kj}^{(\alpha)*}(\omega)\right\}\overline{E_i E_j^*},$$

$$\bar{W}_B = \frac{1}{16\pi}\overline{|\mathbf{B}|^2}, \quad \overline{|\mathbf{B}|^2} = \frac{1}{V}\int d\mathbf{r}\,|\mathbf{B}(\mathbf{r})|^2, \qquad (19)$$

$$\overline{E_i E_j^*} = \frac{1}{V}\int d\mathbf{r} E_i(\mathbf{r})E_j^*(\mathbf{r}), \quad \omega_{0\alpha} = 0, \quad \alpha = e,i.$$

Using (18), (19) it is easy to recover the results obtained in Refs. [10,11] for the electric field energy density outside the transparency domain. For example, in the case of a cold molecular system

$$\chi_{ij}^{(m)}(\omega) = -\delta_{ij}\frac{\omega_{pm}^2}{\omega^2 - \omega_0^2 + i\gamma\omega}, \qquad (20)$$

that leads to

$$\bar{W}_E = \frac{1}{16\pi}\overline{|\mathbf{E}|^2}\left[1+\frac{\omega_{pm}^2(\omega^2+\omega_0^2)}{(\omega^2-\omega_0^2)^2+\gamma^2\omega^2}\right]. \quad (21)$$

In the case of a cold plasma

$$\chi_{ij}^{(e)}(\omega) = -\delta_{ij}\frac{\omega_{pe}^2}{\omega(\omega+i\nu_e)}, \quad (22)$$

where ν_e is the effective collision frequency, that gives

$$\bar{W}_E = \frac{1}{16\pi}\left[1+\frac{\omega_{pe}^2}{\omega^2+\nu_e}\right]\overline{|\mathbf{E}|^2}. \quad (23)$$

Eqs. (21) and (23) are in agreement with the well-known Brillouin formulas only in the case of non-dissipative systems ($\gamma = 0$ and $\nu = 0$).

3. ENERGY DENSITY OF THE ELECTROMAGNETIC FIELD FLUCTUATIONS

Within the context of the theory of electromagnetic fluctuations it is easy to show that Eq. (17) may be also applied to the description of the energy density of fluctuations. The statistical averaging of Eq. (17) yields

$$\langle W\rangle = \frac{1}{8\pi}\int\frac{d\mathbf{k}}{(2\pi)^3}\int\frac{d\omega}{2\pi}\left\{\frac{k_ik_j}{k^2}+\left(1+\frac{c^2k^2}{\omega^2}\right)\left(\delta_{ij}-\frac{k_ik_j}{k^2}\right)\right. \quad (24)$$
$$\left.+\sum_{\alpha=e,i,m}\frac{\omega^2+\omega_{0\alpha}^2}{\omega_{p\alpha}^2}\chi_{ki}^{(\alpha)}(\mathbf{k},\omega)\chi_{kj}^{(\alpha)*}(\mathbf{k},\omega)\right\}\langle\delta E_i\delta E_j\rangle_{\mathbf{k}\omega}.$$

When deriving Eq. (24) we take into account that

$$\langle\delta E_{i\mathbf{k}\omega}\delta E^*_{j\mathbf{k}'\omega'}\rangle = (2\pi)^4\delta(\mathbf{k}-\mathbf{k}')\delta(\omega-\omega')\langle\delta E_i\delta E_j\rangle_{\mathbf{k}\omega},$$

where

$$\langle\delta E_i\delta E_j\rangle_{\mathbf{k}\omega} = \int d\mathbf{R}\,\mathrm{e}^{-i\mathbf{k}\mathbf{R}}\int d\omega\,\mathrm{e}^{+i\omega\tau}\langle\delta E_i(\mathbf{r},t)\delta E_j(\mathbf{r}',t')\rangle_{\mathbf{k}\omega},$$
$$\mathbf{R} = \mathbf{r}-\mathbf{r}', \qquad \tau = t-t'.$$

In the case of an equilibrium system $\langle\delta E_i\delta E_j\rangle_{\mathbf{k}\omega}$ is given by the fluctuation dissipation theorem (see, for example, [3,4])

$$\langle\delta E_i\delta E_j\rangle_{\mathbf{k}\omega} = \frac{4\pi i}{\omega}\theta(\omega)\left\{\Lambda_{ij}^{-1}(\mathbf{k},\omega)-\Lambda_{ji}^{-1*}(\mathbf{k},\omega)\right\}. \quad (25)$$

Here

$$\theta \equiv \frac{\hbar\omega}{2}\mathrm{cth}\frac{\hbar\omega}{2T}, \quad \Lambda_{ij}(\mathbf{k},\omega) = \varepsilon_{ij}(\mathbf{k},\omega)-\frac{k^2c^2}{\omega^2}\left(\delta_{ij}-\frac{k_ik_j}{k^2}\right).$$

Further simplification of (24) can be done in the case of an isotropic system for which

$$\varepsilon_{ij}(k,\omega) = \varepsilon_{\mathrm{T}}(\mathbf{k},\omega)\left(\delta_{ij}-\frac{k_ik_j}{k^2}\right)+\varepsilon_{\mathrm{L}}(\mathbf{k},\omega)\frac{k_ik_j}{k^2},$$

where $\varepsilon_{\mathrm{T}}(\mathbf{k},\omega)$ and $\varepsilon_{\mathrm{L}}(\mathbf{k},\omega)$ are the transverse and longitudinal parts of the dielectric permittivity tensor.

Substituting (25) into (24) yields

$$\langle W\rangle = \int_0^\infty\langle W\rangle_\omega d\omega, \quad (26)$$

where for the general case of the non-transparent medium we have

$$\langle W\rangle_\omega = \frac{\theta(\omega)}{2\pi^3\omega}\int_0^\infty dk\,k^2$$
$$\cdot\left\{\frac{\mathrm{Im}\,\varepsilon_{\mathrm{L}}(k,\omega)}{|\varepsilon_{\mathrm{L}}(k,\omega)|^2}\cdot\left[1+\sum_{e,i,m}\frac{\omega^2+\omega_{0\alpha}^2}{\omega_{p\alpha}^2}|\chi_{\mathrm{L}}^{(\alpha)}(\mathbf{k},\omega)|^2\right]\right.$$
$$+\frac{2\,\mathrm{Im}\,\varepsilon_{\mathrm{T}}(k,\omega)}{|\varepsilon_{\mathrm{T}}(k,\omega)-(k^2c^2)/(\omega^2)|^2} \quad (27)$$
$$\left.\cdot\left[1+\frac{k^2c^2}{\omega^2}+\sum_{e,i,m}\frac{\omega^2+\omega_{0\alpha}^2}{\omega_{p\alpha}^2}|\chi_{\mathrm{T}}^{(\alpha)}(\mathbf{k},\omega)|^2\right]\right\}$$

that describes the contribution of both longitudinal and transverse electromagnetic fields.

In the case of negligible dissipation we can use the approximation of the type

$$\frac{\mathrm{Im}\,\varepsilon_{\mathrm{T}}}{|\varepsilon_{\mathrm{T}}(k,\omega)-(k^2c^2)/(\omega^2)|^2} \simeq \pi\delta\left(\mathrm{Im}\,\varepsilon_{\mathrm{T}}(k,\omega)-\frac{k^2c^2}{\omega^2}\right).$$

In the case of cold plasma for $\omega \gg \nu$ we have

$$\langle W\rangle_\omega = \frac{\omega^2\theta(\omega)}{\pi^2c^3}\sqrt{\varepsilon(\omega)} \qquad (\omega > \omega_p). \quad (28)$$

This relation is in agreement with the well-known result for the energy density in the dispersive transparent medium [23] and reproduces the energy density for transparent plasmas [24].

In the case of a molecular medium ($\gamma \to 0$)

$$\langle W\rangle_\omega = \frac{\omega^2\theta(\omega)}{2\pi^2c^3}\sqrt{\varepsilon_b(\omega)}\left[1+\varepsilon_b(\omega)+\frac{\omega_{pm}^2(\omega^2+\omega_0^2)}{(\omega^2-\omega_0^2)^2}\right]. \quad (29)$$

For $\omega \gg \omega_0$ we come back to the equation of the type (27).

For $\omega \ll \omega_0$ the frequency dispersion can be neglected, and we obtain the result for nondispersive transparent medium [23]

$$\langle W\rangle_\omega = \frac{\omega^2\theta(\omega)}{2\pi^2c^3}\tilde{\varepsilon}^{3/2}, \quad \text{where} \quad \tilde{\varepsilon} = \lim_{\omega\to 0}\varepsilon_b(\omega). \quad (30)$$

In the general case Eq. (27) may be rewritten in the form of the Planck formula modified by the presence of the medium, i.e.,

$$\langle W\rangle_\omega = \frac{\hbar\omega^3}{\pi^2c^3}\left\{\frac{1}{2}+\frac{1}{\mathrm{e}^{(\hbar\omega)/T}-1}\right\}S(\omega), \quad (31)$$

where $S(\omega)$ is the function describing the influence of the medium

$$S(\omega) = \frac{c^3}{2\pi\omega^3}\int_0^\infty dk\,k^2$$
$$\cdot\left\{\frac{\mathrm{Im}\,\varepsilon_{\mathrm{L}}(k,\omega)}{|\varepsilon_{\mathrm{L}}(k,\omega)|^2}\left[1+\sum_{e,i,m}\frac{\omega^2+\omega_{0\alpha}^2}{\omega_{p\alpha}^2}|\chi_{\mathrm{L}}^{(\alpha)}(\mathbf{k},\omega)|^2\right]\right.$$
$$+\frac{2\,\mathrm{Im}\,\varepsilon_{\mathrm{T}}(k,\omega)}{|\varepsilon_{\mathrm{T}}(k,\omega)-\frac{k^2c^2}{\omega^2}|^2} \quad (32)$$
$$\left.\cdot\left[1+\frac{k^2c^2}{\omega^2}+\sum_{e,i,m}\frac{\omega^2+\omega_{0\alpha}^2}{\omega_{p\alpha}^2}|\chi_{\mathrm{T}}^{(\alpha)}(\mathbf{k},\omega)|^2\right]\right\}.$$

It should be noted that Eq. (8) gives also an explicit presentation of the energy flux in terms of the contributions of electromagnetic field and particle components. In particular, the field part of the flux will be described by the term

$$\langle \mathbf{S}(\mathbf{r},t)\rangle = \frac{c}{4\pi}\left\langle[\mathbf{E}(\mathbf{r},t)\times\mathbf{B}(\mathbf{r},t)]\right\rangle,$$

that in the case of an isotropic system leads to the radiation intensity given by

$$\mathrm{I}_{\omega} = \frac{c^2\theta(\omega)}{2\pi^4\omega^2}\int dk\,k^3\,\frac{\mathrm{Im}\,\varepsilon_{\mathrm{T}}(k,\omega)}{|\,\varepsilon_{\mathrm{T}}(k,\omega)-\dfrac{k^2c^2}{\omega^2}\,|^2} = \mathrm{I}_{0\omega}\tilde{S}(\omega), \quad (33)$$

where

$$\mathrm{I}_{0\omega} = \frac{\omega^2\theta(\omega)}{4\pi^3c^2}, \qquad \tilde{S}(\omega) = \frac{2c^4}{\pi\omega^4}\int dk\,k^3\,\frac{\mathrm{Im}\,\varepsilon_{\mathrm{T}}}{|\,\varepsilon_{\mathrm{T}}-\dfrac{k^2c^2}{\omega^2}\,|^2}.$$

CONCLUSIONS

Thus, in the present contribution we derive the general relations for the electromagnetic-field energy density in an absorptive medium with temporal and spatial dispersion. The treatment is based on the assumption that the energy density of an electromagnetic perturbation contains both the electromagnetic field energy and the particle energy acquired in the perturbation field. The results obtained provide a possibility to generalize the Planck law and the Kirchhoff law to the case of an absorptive dispersive medium. The detailed description of both effects in specific media will be a matter of further research.

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ЭНЕРГИЯ ЭЛЕКТРОМАГНИТНОГО ПОЛЯ И ИНТЕНСИВНОСТЬ ИЗЛУЧЕНИЯ В СРЕДЕ С ВРЕМЕННОЙ И ПРОСТРАНСТВЕННОЙ ДИСПЕРСИЯМИ ВНЕ ОБЛАСТИ ПРОЗРАЧНОСТИ

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Рассчитаны энергии электромагнитного поля в среде с временной и пространственной дисперсиями вне области прозрачности. Показано, что в общем случае вклад энергии частиц среды в энергию электромагнитного возмущения описывается в терминах билинейных комбинаций диэлектрической поляризуемости среды. Найден явный вид такого вклада. Полученные результаты использованы для обобщения закона Планка и закона Кирхгофа для поглощающей среды с пространственной дисперсией.

ЕНЕРГІЯ ЕЛЕКТРОМАГНІТНОГО ПОЛЯ ТА ІНТЕНСИВНІСТЬ ВИПРОМІНЮВАННЯ В СЕРЕДОВИЩІ З ЧАСОВОЮ І ПРОСТОРОВОЮ ДИСПЕРСІЯМИ ПОЗА ОБЛАСТЮ ПРОЗОРОСТІ

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Розраховано енергію електромагнітного поля в середовищі з часовою та просторовою дисперсіями поза областю прозорості. Показано, що в загальному випадку внесок енергії частинок середовища в енергію електромагнітного збурення описується в термінах білінійних комбінацій діелектричної поляризованості середовища. Знайдено явний вигляд такого внеску. Отримані результати використано для узагальнення закону Планка і закону Кірхгофа для поглинального середовища з просторовою дисперсією.