THE EFFECT OF NONRECIPROCITY ON THE DYNAMICS OF COUPLED OSCILLATORS AND COUPLED WAVES

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The dynamics of nonreciprocally coupled oscillators and coupled waves is studied. Such coupling can lead to converting the energy of low-frequency (LF) oscillations to the energy of high-frequency (HF) oscillations. The influence of the resonant properties of the coupling elements on the conditions of the energy conversion is carried out. It is shown that this conversion can be realized when either the amplitude or the phase of the coupling coefficient is modulated at a low-frequency. By the example of waves in a rare magnetoactive plasma, it is shown for the first time that the discussed energy conversion takes also place in a system of interacting waves. Results of analytical and numerical studies illustrating the conditions for the excitation of high-frequency oscillations due to the energy conversion are presented.

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INTRODUCTION

In the papers [1, 2], it was described that the introduction of a non-reciprocal coupling between oscillating systems opens a possibility of the energy converting from low-frequency (LF) oscillations to high-frequency (HF) oscillations. It was also noted that the most interesting direction of using the discovered mechanism of the energy conversion is related with the development of novel types of sources of electromagnetic radiation, including sources of terahertz radiation. It should be noted that in this frequency range, the coupling elements themselves have usually resonant properties. However, in [1, 2] these elements were considered as frequency-independent elements. Therefore, it seems important to investigate the influence of resonant properties of the coupling elements on the dynamics of coupled oscillators. The corresponding analysis is presented in the next section, where each of the coupling elements is also considered as a separate oscillator coupled with other oscillators.

So far, the case was considered [1, 2] when the nonreciprocity of the coupling and the LF modulation of the coupling coefficient were provided by introducing an amplitude modulation of the coupling coefficients. It should be borne in mind that the amplitude modulation is not always practical, therefore, we also consider in this paper a possibility of applying a LF phase modulation to realize the conversion of LF oscillations into HF ones. The results of this study are presented in the third section of the paper.

In our previous works [1, 2], we considered systems where HF oscillations are excited as a result of three-frequency interaction under the following resonance conditions

$$\omega_{n1} - \omega_{n2} = \pm\omega,$$

where ω_{n1} and ω_{n2} are normal frequencies of a system of two coupled oscillators, ω is the frequency of LF modulation, which is much smaller than ω_{n1}, ω_{n2}. The case was mainly considered when the partial frequencies ω_{p1}, ω_{p2} of the oscillators are equal. In the fourth section of this paper it is shown that, with an appropriate choice of the coupling between the oscillators, the excitation of HF oscillations is possible even if the following condition is satisfied

$$\omega_{p1} - \omega_{p2} = \pm\omega$$

that is, with unequal partial frequencies of the interacting oscillators.

In the fifth section, it is shown that the mechanism of the energy conversion found in a system of interacting oscillators can be also realized when waves interact. This result is demonstrated by analyzing the propagation of two high-frequency transverse electromagnetic waves in a rare magnetoactive plasma.

The last section summarizes the results presented in the paper.

1. INFLUENCE OF RESONANT PROPERTIES OF COUPLING ELEMENTS

As mentioned above, at high frequencies coupling elements can have resonant properties and should also be regarded as oscillatory systems. In this case, the simplest model of coupled oscillators can be represented as a ring of four oscillators, as shown in Fig. 1.

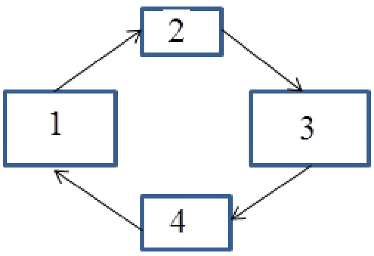

Fig. 1. Scheme of the oscillatory system. Arrows indicate the direction of connection

Numbers 1 and 3 show high-frequency oscillation systems (resonators). The natural frequencies of these resonators are equal to ω_0. Numbers 2 and 4 denote nonreciprocal coupling elements. The eigenfrequencies of these elements coincide and are equal to ω_1. Arrows indicate the direction of the wave propagation in this ring. A set of equations that describes the dynamics of such a system can be represented in the form:

$$\ddot{x}_1 + \omega_0^2 x_1 = \mu x_4;$$
$$\ddot{x}_2 + \omega_1^2 x_2 = \mu x_1; \quad (1)$$
$$\ddot{x}_3 + \omega_0^2 x_3 = \mu x_2;$$
$$\ddot{x}_4 + \omega_1^2 x_4 = \mu x_3.$$

It is easy to determine that this oscillatory system has the following normal frequencies:

$$\Omega_N = \omega_0 \pm \mu^2 / 2\omega_0 \left(\omega_1^2 - \omega_0^2\right).$$

Assuming that the coupling coefficients are small ($\mu << \omega_{0,1}^2$), from (1), one can find the following averaged equations for determining the complex amplitudes of the coupled oscillators:

$$\dot{a}_1 = \frac{\mu}{2i\omega_0} a_4;$$
$$\dot{a}_2 = \frac{\mu}{2i\omega_1} a_1; \quad (2)$$
$$\dot{a}_3 = \frac{\mu}{2i\omega_0} a_2;$$
$$\dot{a}_4 = \frac{\mu}{2i\omega_1} a_3.$$

Note that this set of equations is not changed if some coupling coefficients are slow varying functions of time. Let, for example, the coupling coefficients in the third and fourth equations of (2) are such functions of time $\mu = \mu_1(t)$. Then from (2), we find the following equation describing the dynamics of the amplitudes of the first and third oscillators (resonators):

$$\frac{d^2}{dt^2}[a_1 + a_3] + \Omega^2 [a_1 + a_3] = 0, \quad (3)$$

where $\Omega^2 = (\mu_1(\text{t}))^2 / 4\omega_0\omega_1$.

Let us assume that the function $(\mu_1(\text{t}))^2$ has the following form $(\mu_1(\text{t}))^2 = \omega_2^4 (1 + \varepsilon \cos(2\tau))$, where $\tau = t \cdot \omega_2^2 / 2\sqrt{\omega_0\omega_1}$ and $\omega_2^2 << \omega_0\omega_1$. Then (3) can be reduced to Mathieu equation:

$$\frac{d^2}{d\tau^2}[a_1 + a_3] + (1 + \varepsilon \cos 2\tau)[a_1 + a_3] = 0. \quad (4)$$

From this equation, it follows that the presence of the resonant properties of the coupling elements does not prevent the energy transformation from LF oscillations into the energy of HF oscillations. One should only take into account that the eigenfrequencies of the coupling elements are essential parameters of the entire oscillatory system, and their parameters should be appropriately selected for the realization of the considered energy transformation.

2. MODULATION OF THE PHASE OF THE COUPLING COEFFICIENTS

In the papers [1, 2], an amplitude modulation of the coupling coefficients was considered. For a number of practical applications, it is more convenient to use a phase modulation of the coupling coefficient. To describe the dynamics of a system of two coupled identical oscillators, in which the phase of the coupling elements is a function of time, we use the following system of equations:

$$\ddot{x}_1 + x_1 = \mu(t)[\exp(i\varphi(t))] x_2;$$
$$\ddot{x}_2 + x_2 = \mu_0 x_1. \quad (5)$$

Here $\mu(t)$ *and* $\varphi(t)$ are real slow varying functions of time. We look for a solution of (5) in the following form:

$$x_1 = [A_1 \cdot \exp(it) + A_2 \cdot \exp(-it)]$$
$$x_2 = [B_1 \cdot \exp(it) + B_2 \cdot \exp(-it)]. \quad (6)$$

Here the amplitudes A_k and B_k (k=1, 2) are slow varying functions of time. Applying the averaging technique to (6), we obtain to the following set of equations:

$$i\dot{A}_1 = \mu(t)[\exp(i\varphi(t))] B_1,$$
$$i\dot{B}_1 = \mu_0 A_1, \quad (7)$$
$$-i\dot{A}_2 = \mu(t)[\exp(i\varphi(t))] B_2,$$
$$-i\dot{B}_2 = \mu_0 A_2.$$

Here the amplitudes A_k and B_k are complex functions: $A_k = A_k' + iA_k''$; $B_k = B_k' + iB_k''$. To find the real and imaginary components, we come to the following system of equations:

$$\ddot{B}_1' + [\mu_0 \mu(t) \cos\varphi] B_1' = [\mu_0 \mu(t) \sin\varphi] B_1'',$$
$$\ddot{B}_1'' + [\mu_0 \mu(t) \cos\varphi] B_1'' = -[\mu_0 \mu(t) \sin\varphi] B_1'. \quad (8)$$

An analogous system can also be obtained for the function $A_k = A_k' + iA_k''$. The sets of equations (5) and (8) were solved numerically. In Fig. 2 typical build-up of the amplitude of high-frequency oscillations is shown. This example illustrate that the energy conversion can be also realized when the phase of the coupling coefficient is modulated at a low-frequency.

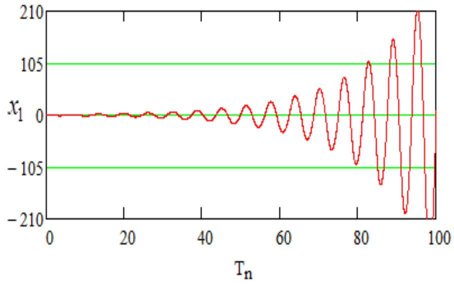

Fig. 2. The characteristic dependence of the amplitudes of the oscillators on time $T_n \equiv (\mu_0 \mu) \cdot t$ at $\varphi = \sin\tau$; $x_1(0) = 1$

3. EXCITATION OF HF OSCILLATIONS WHEN THE PARTIAL FREQUENCIES DO NOT COINCIDE

In the works [1, 2], the partial frequencies of the interacting oscillators were considered to be equal. Under this condition, a three-frequency interaction and the excitation of HF oscillations were realized when the difference of the normal frequencies was approximately equal to the frequency of the LF modulation.

In this section, we show that with a certain method of oscillators coupling, HF oscillations are excited also when the partial frequencies do not coincide. This case is realized in the absence of constant in time coupling between the oscillators. However, a nonreciprocity of the coupling, as in the previous case, is needed. A set of equations that describes such a coupled oscillatory system can be represented as:

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$$\ddot{q}_1 + \omega_1^2 \cdot q_1 = \mu q_2 \cos(\omega \cdot t)$$
$$\ddot{q}_2 + \omega_2^2 \cdot q_2 = -\mu q_1 \cos(\omega \cdot t). \quad (9)$$

Here $\omega = \omega_2 - \omega_1$ is the low frequency modulation of the coupling between the high-frequency oscillators.

We look for the solution of (9) in the form:

$$q_k = A_k(t)\exp(i\omega_k t) + B_k(t)\exp(-i\omega_k t).$$

To find equations for slowly varying amplitudes, we at first come from (9) to the following system of equations:

$$\left[\dot{A}_1(t)\exp(i\omega_1 t) - \dot{B}_1(t)\exp(-i\omega_1 t)\right] =$$
$$= \frac{\mu}{2i\omega_1}\left[A_2\exp(i\omega_2 t) + B_2\exp(-i\omega_2 t)\right]\cos(\omega t)$$
$$\left[\dot{A}_2(t)\exp(i\omega_2 t) - \dot{B}_2(t)\exp(-i\omega_2 t)\right] =$$
$$= -\frac{\mu}{2i\omega_2}\left[A_1\exp(i\omega_1 t) + B_1\exp(-i\omega_1 t)\right]\cos(\omega t). \quad (10)$$

From this equations, it is easy to determine the following relations between the complex amplitudes

$$\dot{A}_1 = \frac{\mu}{4i\omega_1}A_2;$$
$$\dot{A}_2 = -\frac{\mu}{4i\omega_2}A_1; \quad (11)$$
$$\ddot{A}_1 - \frac{\mu^2}{16\omega_1\omega_2}A_1 = 0.$$

From (11), it immediately follows that the excitation of high-frequency oscillations can also occur in such a system as illustrated in Fig. 3. This figure shows the solution of the system of equations (9) at such parameters: $\mu = 0.2$; $\omega = 0.01$; $\omega_1 = 1$; $\omega_2 = 1.01$; $q_1(0) = 0.1$.

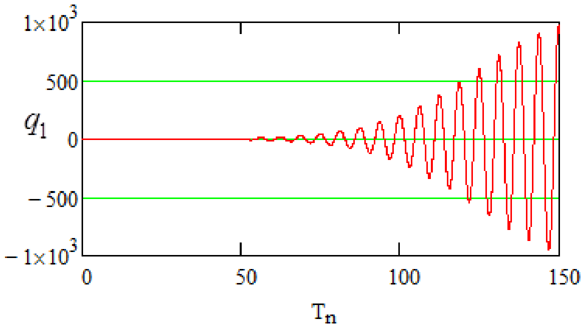

Fig. 3. The excitation of oscillations of two coupled, different high-frequency oscillators (see system (9)). $T_n \equiv t$

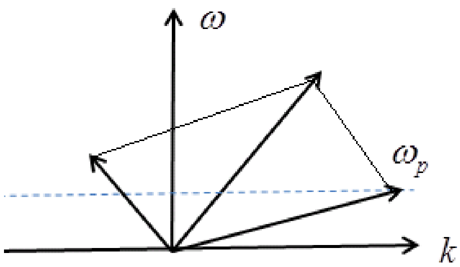

Fig. 4. Dispersion diagram of the waves participating in the interaction

4. THE ROLE OF NONRECIPROCITY IN THE DYNAMICS OF COUPLED WAVES

In the above sections, we discussed the existence in coupled oscillators of LF- to HF energy transfer channel. It can be expected that a similar channel can exist in systems with interacting waves. Below, considering an example of coupling of transverse high-frequency waves to plasma waves, it is shown that such a channel does exist. However, as before, it exists only in the presence of a non-reciprocal coupling between interacting waves.

We consider the following problem. Suppose that there are two high-frequency transverse electromagnetic waves that propagate through rare magnetoactive plasma ($\omega_p^2 << (\omega)^2 \sim \omega_H^2$). The frequencies of these waves are large, and the difference of these frequencies is close to the plasma frequency ($\omega_2 - \omega_1 \approx \omega_p$). For simplicity, we consider a one-dimensional motion, i.e. all waves propagate and interact with each other only along the axis z and in time. A diagram of a possible interaction of the waves is shown in Fig. 4. It can be seen from diagram 4 that a three-wave interaction occurs. In this interaction, two transverse high-frequency waves and a plasma wave are involved. The structure of these waves and their dispersion characteristics are well known (see, for example, [4, 5]). The plasma wave is longitudinal with a large amplitude. We assume that this wave is given. In this case, the plasma frequency can be represented only by its wave characteristic:

$$\omega_p = \left(\sqrt{4\pi e^2 n / m}\right) exp\left[i\kappa z - i\omega_p t\right] + k.c. \quad (12)$$

The equation for the electrical component of the field of the transverse electromagnetic waves can be obtained from the Maxwell equations:

$$\Delta\vec{E} - \frac{1}{c^2}\frac{\partial^2(\vec{D})}{\partial t^2} = 0. \quad (13)$$

Here, $\vec{D} = \hat{\varepsilon}\vec{E}$ and $\hat{\varepsilon}$ is the plasma permittivity tensor.

We look for the components of the electric field of transverse waves in the form

$$E_{x,y}(z,t) = A_{2,1}(t) exp\left[ik_{2,1}z - i\omega_{2,1}t\right] + k.c. \quad (14)$$

Then it is convenient to represent equation (13) in the form of the following set of equations with respect to the amplitudes $A_{1,2}$:

$$\frac{\partial^2 A_2}{\partial t^2} + \omega_2^2 A_2 = -i\varepsilon_+\omega_1^2 A_1;$$
$$\frac{\partial^2 A_1}{\partial t^2} + \omega_1^2 A_1 = i\varepsilon_-\omega_2^2 A_2. \quad (14)$$

Here $\varepsilon_\pm = \mp\omega_H\omega_p^2 / \omega_\pm\left(\omega_\pm^2 - \omega_H^2\right)$ are off-diagonal components of the permittivity tensor; $k_2^2 = \omega_2^2 / c^2$, $k_1^2 = \omega_1^2 / c^2$.

The upper sign in these expressions corresponds to a wave propagating along the magnetic field $\omega_+ = \omega_2$; the lower sign belongs to a wave propagating in the opposite direction $\omega_- = \omega_1$. When obtaining (14), the condition of spatial synchronism $\kappa = k_2 + |k_1|$ has been used. We note that the first wave ω_1, k_1 propagates in the direction opposite to the direction of the external magnetic field.

For an effective interaction of the waves, it is necessary that together with the spatial synchronism condition, the time synchronism should be satisfied: $\omega_2 - \omega_1 - \omega_p = 0$.

We look for a solution of (14) in the form

$$A_{2,1} = a_{1,2} exp(-i\omega_{2,1}t) + k.c. \qquad (15)$$

Substituting (15) into (14), we obtain the following equations with respect to the slowly varying amplitudes $a_{2,1}$:

$$\frac{\partial^2 a_{2,1}}{\partial t^2} - \frac{|\varepsilon_+ \varepsilon_-|}{4}\omega_1\omega_2 a_{2,1} = 0 \quad . \qquad (16)$$

It can be seen from this equation that the amplitudes of the transverse waves increase exponentially with the increment:

$$\Gamma \approx \omega_H \omega_p^2 / \left(2\left|\omega^2 - \omega_H^2\right|\right). \qquad (17)$$

In this expression, it is taken into account that the frequencies of the HF waves are close to each other $\omega_2 - \omega_1 = \omega_p << \omega_1$. High-frequency transverse electromagnetic waves receive energy from the LF Langmuir waves excited in the plasma.

CONCLUSIONS

We note the most important results presented in the paper. There are two basic scientific results. The first is that the conversion of the energy of LF oscillations to the energy of HF frequency oscillations is a rather "strong" effect, in the sense that it can be realized in very different ways (see Sections 3 and 4), and also in the presence of significant perturbations, like, for example, additional resonances in the system (section 2).

The second result is that the availability of nonreciprocity can create a channel for converting the energy of LF oscillations to HF oscillations not only in systems with coupled oscillators, but also in systems with coupled waves (see Section 5). In latter case, the presence of a nonreciprocity leads to a qualitatively new dynamics of the three-wave interaction. Indeed, it is well known (see, for example, [4 - 5]) that if at the initial instant of time the low-frequency wave has the largest amplitude in a system with a three-wave interaction, then practically no dynamics with energy exchange can occur in such system. In the case considered above, the presence of a nonreciprocity leads to the excitation of HF waves, in spite of the fact that at the initial moment only a low-frequency Langmuir wave exists.

In our paper, there are also several results of practical importance. At first, it is shown that when considering the excitation of high-frequency oscillations (for example, in terahertz frequency range), it is necessary to take into account the oscillatory properties of the coupling elements (see Section 1). Secondly, it is proved that a phase modulation of the coupling coefficient can be used as well as an amplitude modulation to realize the energy transfer. At third, it is shown that oscillatory systems with different partial frequencies can be used for the conversion of LF- to HF oscillations. The only requirement in this case is that the modulation frequency of the coupling coefficients of these systems should be approximately equal to the difference between the partial frequencies.

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РОЛЬ НЕВЗАИМНОСТИ В ТЕОРИИ СВЯЗАННЫХ КОЛЕБАНИЙ И СВЯЗАННЫХ ВОЛН

В.А. Буц, Д.М. Ваврив

Исследована динамика связанных осцилляторов и волн при наличии невзаимной связи между ними, которая приводит к возможности преобразования энергии низкочастотных колебаний в энергию высокочастотных колебаний. Проведен анализ влияния резонансных свойств элементов связи на условия преобразования энергии. Показано, что преобразование энергии можно реализовать как при низкочастотной амплитудной, так и при низкочастотной фазовой модуляции коэффициента связи. На примере анализа распространения волн в редкой магнитоактивной плазме впервые показана возможность преобразования энергии и при взаимодействии волн. Приведены результаты аналитического и численного исследований, иллюстрирующие условия возбуждения высокочастотных колебаний и их свойства.

РОЛЬ НЕВЗАЄМНОСТІ В ТЕОРІЇ ПОВ'ЯЗАНИХ КОЛИВАНЬ І ПОВ'ЯЗАНИХ ХВИЛЬ

В.О. Буц, Д.М. Ваврів

Досліджено динаміку зв'язаних осциляторів і хвиль при наявності невзаємного зв'язку між ними, який призводить до можливості перетворення енергії низькочастотних коливань в енергію високочастотних коливань. Проведено аналіз впливу резонансних властивостей елементів зв'язку на умови перетворення енергії. Показано, що перетворення енергії можна реалізувати як при низькочастотній амплітудній, так і при низькочастотній фазовій модуляції коефіцієнта зв'язку. На прикладі аналізу поширення хвиль у рідкій магнітоактивній плазмі вперше показана можливість перетворення енергії і при взаємодії хвиль. Наведено результати аналітичного і чисельного дослідження, що ілюструють умови збудження високочастотних коливань і їх властивості.