RADIATION OF A CHARGED PARTICLE IN THE IDEALLY CONDUCTING METAL WAVEGUIDE FILLED WITH A SPATIALLY PERIODIC LAYERED DIELECTRIC

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The parametric Cherenkov radiation of a uniformly moving particle in an ideally conducting metal waveguide filled with a spatially periodic layered dielectric is investigated analytically and numerically for the case of wavelengths comparable with the inhomogeneity period. Fields and spectra of parametric Cherenkov radiation are described. The particle's average energy losses on the period of the structure and energy fluxes of the fields are determined.

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INTRODUCTION

In [1], for the first time, a general expression was obtained for the energy losses of a uniformly moving charged particle in an unbounded layered medium and in a waveguide filled with a layered dielectric. The main attention was paid to the energy losses of the charged particle for the case of wavelengths exceeding the period of the dielectric structure. Here the spectral distribution of the polarization losses as well as the losses to the parametric Cherenkov radiation due to the specificity of the interaction of waves in a layered dielectric is studied in detail.

For the case of wavelengths comparable with the inhomogeneity period, the energy losses of an oscillating charge moving with a nonrelativistic velocity in a periodically changing medium are considered in [2, 3]. In the present paper we continue the investigation of the parametric Cherenkov radiation of a uniformly moving particle in a layered dielectric for the case of wavelengths comparable to the inhomogeneity period.

The fields of the parametric Cerenkov radiation and the spectra of this radiation are obtained, the energy loss of the particle are averaged over the period of the structure and the energy fluxes of the fields are determined.

However, it should be noted that the conclusions of [1] are based not on calculating the radiation obtained for the spectral distribution, but on the basis of the transition to an equivalent anisotropic dielectric. Such transition is possible in the case when the wavelength of the radiation considerably exceeds the period of the structure. However, such limitation on the wavelength of the radiation is not always justified.

Therefore, it is of interest to consider the parametric Cherenkov radiation of a uniformly moving particle in an ideally conducting metal waveguide filled with a spatially periodic layered dielectric for the case of the wavelengths comparable with the period of inhomogeneity.

Let's consider the radiation of a charged particle moving along the axis of an ideally conducting metal waveguide filled with a spatially periodic layered dielectric. Let us determine the spectrum of its parametric Cherenkov radiation.

1. OBTAINING EQUATIONS DESCRIBING PARTICLE RADIATION IN A SPATIALLY PERIODIC LAYERED DIELECTRIC

To solve the stated problem let's start from the system of Maxwell equations describing the interaction of a uniformly moving particle with the electromagnetic waves of a given medium [1]:

$$-\frac{\partial H_\phi}{\partial z} = \frac{\hat{\varepsilon}(z)}{c}\cdot\frac{\partial E_r}{\partial t}, \quad (1)$$

$$\frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r} = -\frac{\hat{\mu}(z)}{c}\cdot\frac{\partial H_\phi}{\partial t}, \quad (2)$$

$$\frac{1}{r}\cdot\frac{\partial}{\partial r} rH_\phi = \frac{\hat{\varepsilon}(z)}{c}\cdot\frac{\partial E_z}{\partial t} + \frac{4\pi}{c} qv\cdot\delta(vt-z)\cdot\frac{\delta(r)}{2\pi r}. \quad (3)$$

Here the operators $\hat{\varepsilon}, \hat{\mu}$, are defined as

$$\hat{\varepsilon}(z)\cdot e^{i\omega t} = \varepsilon(\omega, z)\cdot e^{i\omega t},\ \hat{\mu}(z)\cdot e^{i\omega t} = \mu(\omega, z)\cdot e^{i\omega t}, \quad (4)$$

q is the charge and v is the velocity of the particle.

In the considered case of a charged particle's motion along the axis of a waveguide filled with a dielectric, the equation for determining the Fourier component of the longitudinal component of the electric induction $D_{z,\omega n}(z)$ can be represented as:

$$\varepsilon(z,\omega)\frac{\partial}{\partial z}\left(\frac{1}{\varepsilon_{z,\omega}}\frac{\partial D_{z,\omega n}(z)}{\partial z}\right) + D_{z,\omega n}(z)\left(k^2\mu(z,\omega)\varepsilon(z,\omega) - \frac{\lambda_n^2}{R^2}\right) =$$
$$= \varepsilon(z,\omega)\left(ik\mu(z,\omega)\frac{q}{\pi c}e^{-\frac{i\omega z}{v}} + \frac{q}{\pi v}\cdot\frac{\partial}{\partial z}\left(\frac{e^{-\frac{i\omega z}{v}}}{\varepsilon(z,\omega)}\right)\right). \quad (5)$$

Here the component of $D_{z,\omega n}(z)$ is obtained from the expression for the electric induction

$$D_{z,\omega}(r,z) = \varepsilon(\omega,z)\cdot E_{z,\omega} = \sum_{n=1}^{\infty}\frac{2}{R^2 J_1^2(\alpha_n)} J_0\left(\lambda_n\frac{r}{R}\right) D_{z,\omega n}(z)$$

using the orthogonality condition of Bessel functions, R is the radius of the waveguide, and λ_n the n-th root of the zero-order Bessel function $J_0(\lambda_n) = 0$. In what follows we omit the ω, n indices.

The layered medium is represented by layers of two homogeneous and isotropic dielectrics alternating along the axis of the waveguide: the layer $-a \le z \le 0$ has dielectric and magnetic permeabilities ε_1, μ_1, respectively, the layer has $0 \le z \le b$ permeabilities ε_2, μ_2. Thus, in each layer, equation (5) is an equation with constant coefficients, the solutions of which in each layer will have the form:

$$\begin{cases} E_{z1} = A\cdot e^{ip_1 z} + B\cdot e^{-ip_1 z} + \dfrac{iqk}{\pi c}\cdot\left(\dfrac{\mu_1 - \dfrac{1}{\beta^2}\cdot\dfrac{1}{\varepsilon_1}}{p_1^2 - \dfrac{\omega^2}{v^2}}\right)\cdot e^{-\frac{i\omega z}{v}}, \\ E_{z2} = C\cdot e^{ip_2 z} + D\cdot e^{-ip_2 z} + \dfrac{iqk}{\pi c}\cdot\left(\dfrac{\mu_2 - \dfrac{1}{\beta^2}\cdot\dfrac{1}{\varepsilon_2}}{p_2^2 - \dfrac{\omega^2}{v^2}}\right)\cdot e^{-\frac{i\omega z}{v}}, \end{cases} \quad (6)$$

where $p_1{}^2 = \varepsilon_1\mu_1\cdot k^2 - k_\perp{}^2$, $p_2{}^2 = \varepsilon_2\mu_2\cdot k^2 - k_\perp{}^2$, $k = \dfrac{\omega}{c}$, $k_\perp = \dfrac{\lambda_n}{R}$.

From equations (1) we find expressions for the components of the electric and magnetic field strength in each of the regions:

$$\begin{cases} H_{\varphi 1} = \dfrac{i\varepsilon_1 k}{k_\perp}E_{z1} + \dfrac{q}{\pi c k_\perp}e^{-\frac{i\omega z}{v}}, \\ H_{\varphi 2} = \dfrac{i\varepsilon_2 k}{k_\perp}E_{z2} + \dfrac{q}{\pi c k_\perp}e^{-\frac{i\omega z}{v}}, \end{cases} \quad (7)$$

$$\begin{cases} E_{r1} = \dfrac{-1}{k_\perp}\left(\dfrac{d}{dz}E_{z1} - \dfrac{q}{\pi c\beta\varepsilon_1}e^{-\frac{ikz}{\beta}}\right), \\ E_{r2} = \dfrac{-1}{k_\perp}\left(\dfrac{d}{dz}E_{z2} - \dfrac{q}{\pi c\beta\varepsilon_1}e^{-\frac{ikz}{\beta}}\right). \end{cases} \quad (8)$$

From the boundary conditions on the surface of dielectrics and the conditions for the periodicity of the fields

$$\begin{cases} \left(H_\varphi(z)\big|_{z=0}\right)_1 = \left(H_\varphi(z)\big|_{z=0}\right)_2, \\ \left(E_r(z)\big|_{z=0}\right)_1 = \left(E_r(z)\big|_{z=0}\right)_2, \\ e^{-i\frac{\omega}{v}L}\left(H_\varphi(z)\big|_{z=-a}\right)_1 = \left(H_\varphi(z)\big|_{z=b}\right)_2, \\ e^{-i\frac{\omega}{v}L}\left(E_r(z)\big|_{z=-a}\right)_1 = \left(E_r(z)\big|_{z=b}\right)_2, \end{cases} \quad L = a+b, \quad (9)$$

we obtain a system of linear algebraic equations for finding the coefficients *A*, *B*, *C* and *D*:

$$\begin{cases} \varepsilon_1 A + \varepsilon_1 B - \varepsilon_2 C - \varepsilon_2 D = \mathrm{i}\eta Z_1, \\ p_1 A - p_1 B - p_2 C + p_2 D = -\mathrm{i}\eta\dfrac{k}{\beta}Z_2, \\ e^{-i\frac{k}{\beta}L}\left(\varepsilon_1 e^{-ip_1 a}A + \varepsilon_1 e^{ip_1 a}B\right) - \varepsilon_2 e^{ip_2 b}C - \varepsilon_2 e^{-ip_2 b}D = \mathrm{i}\eta e^{-i\frac{k}{\beta}b}Z_1, \\ e^{-i\frac{k}{\beta}L}\left(p_1 e^{-ip_1 a}A - p_1 e^{ip_1 a}B\right) - p_2 e^{ip_2 b}C + p_2 e^{-ip_2 b}D = -\mathrm{i}\eta\left(\dfrac{k}{\beta}\right)e^{-i\frac{k}{\beta}b}Z_2, \end{cases} \quad (10)$$

or in the matrix form $\hat{M}\cdot\vec{a} = \vec{b}$.

Here

$$\hat{M} = \begin{pmatrix} \varepsilon_1 & \varepsilon_1 & \varepsilon_2 & -\varepsilon_2 \\ p_1 & -p_1 & -p_2 & p_2 \\ \varepsilon_1 e^{-ip_1 a}e^{-i\frac{k}{\beta}L} & \varepsilon_1 e^{ip_1 a}e^{-i\frac{k}{\beta}L} & -\varepsilon_2 e^{ip_2 b} & -\varepsilon_2 e^{-ip_2 b} \\ p_1 e^{-ip_1 a}e^{-i\frac{k}{\beta}L} & -p_1 e^{ip_1 a}e^{-i\frac{k}{\beta}L} & -p_2 e^{ip_2 b} & p_2 e^{-ip_2 b} \end{pmatrix},$$

$$\vec{a} = \begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} \mathrm{i}\eta Z_1 \\ -\mathrm{i}\eta\left(\dfrac{k}{\beta}\right)Z_2 \\ \mathrm{i}\eta e^{-i\frac{k}{\beta}b}Z_1 \\ -\mathrm{i}\eta\left(\dfrac{k}{\beta}\right)e^{-i\frac{k}{\beta}b}Z_2 \end{pmatrix}, \quad \eta = q\frac{k_\perp{}^2}{\pi c k},$$

$Z_1 = \dfrac{1}{P_1^2} - \dfrac{1}{P_2{}^2}$, $Z_2 = \dfrac{1}{\varepsilon_1 P_1^2} - \dfrac{1}{\varepsilon_2 P_2{}^2}$, $P_m^2 = p_m{}^2 - \dfrac{\omega^2}{v^2}$, $m = 1, 2$.

From (10) we find the expression for the coefficients *A*, *B*, *C* and *D*:

$$A = \frac{\Delta A}{\Delta}, \quad B = \frac{\Delta B}{\Delta}, \quad C = \frac{\Delta C}{\Delta}, \quad D = \frac{\Delta D}{\Delta}, \quad (11)$$

where $\Delta = \det(\hat{M}) = 8\varepsilon_1\varepsilon_2 p_1 p_2\left(\cos\left(\dfrac{\omega}{v}L\right) - \cos\psi\right)$ is a matrix determinant $\hat{M}$,

$$\cos\psi = \cos(p_1 a)\cos(p_2 b) - \frac{1}{2}\left(\frac{p_1\varepsilon_2}{p_2\varepsilon_1} - \frac{p_2\varepsilon_1}{p_1\varepsilon_2}\right)\sin(p_1 a)\sin(p_2 b).$$

It should be noted that the field $\AA_{r,n\omega}(z)$ contains the derivative with respect to the longitudinal coordinate from the longitudinal field $dE_{z,n\omega}(z)/dz$, so that the right-hand sides of equations (10) have the terms proportional to $\omega/v = k/\beta$.

Thus, the equalities (11) allow us to determine the coefficients A, B, C, D.

All the singularities in the expressions for the coefficients A, B, C, D are determined by the conditions of [1]:

$$p_1{}^2 - \omega^2 / v^2 = 0, \quad (12)$$

$$p_2{}^2 - \omega^2 / v^2 = 0, \quad (13)$$

$$\cos(\omega L / v) - \cos(\psi) = 0. \quad (14)$$

We are interested in the radiation of a particle in a medium due to the interference of fields in a layered dielectric, which is determined by the roots of equation (14). Since the equation $\cos(\omega L / v) - \cos(\psi) = 0$ is the dispersion equation of a layered dielectric, the frequencies determined by the roots of this equation correspond to waves propagating in such a layered medium.

We note that the values of the coefficients A, B, C, D are expressed in terms of Z_1 and Z_2. Hence it follows that for small differences in the parameters of the medium for each of the regions, for example, for $|\varepsilon_1 - \varepsilon_2| << 1$, the values of the coefficients A, B, C, D will also be small. Physically it is explained by the fact that when the media parameters difference in two regions decrease we turn to the case of a homogeneous medium in which interference effects are absent. Therefore, to increase the interference efficiency of fields excited in layered media, it seems necessary to use dielectric layers with substantially different dielectric permittivities.

2. NUMERICAL SOLUTION OF EQUATIONS DESCRIBED OF PARTICLE RADIATION IN SPATIALLY PERIODIC LAYER DIELECTRIC

Since the expressions for the fields (6) - (8), and the dispersion equation (14) in the general case can not be analytically investigated, let us analyze them numerically. To do this, we choose the following values of the media parameters: $\mu_1 = \mu_2 = 1$, $\varepsilon_1 = 2.1$, $\varepsilon_2 = 3.5$, $a = b = 10^{-2}\sqrt{0.1}$ m, $R = 3\cdot 10^{-2}$ m, $\beta = v/c = 0.65,\ 0.95$.

In following calculations the particle charge was chosen equal to $q = 6\cdot 10^9$, $e = 9.613\cdot 10^{-10}C$.

Graphs of the dependence of the function $D(\omega/c) = \cos(\omega L/\beta c) - \cos(\psi)$, and its spectrum, shown in Figs. 1 and 3 show that the dependence of $D(\omega/c)$ is determined mainly by the beating of two cosines with a period $\Lambda_L = L/\beta$ equal to the characteristic length of the change $\cos(\omega L/\beta c)$ and with a period Λ_ψ equal to the characteristic length of the change $\cos(\psi)$.

In addition, as follows from the form of the normalized spectral power SpD, there is a weakly expressed branch of $\cos(\psi)$ with a small period of variation λ_ψ.

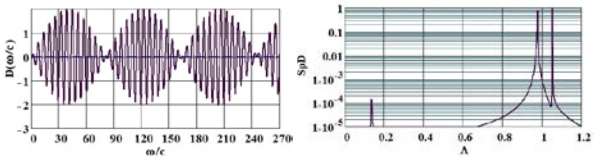


Fig. 1. Dependence $D(\omega/c) = \cos(\omega L/\beta c) - \cos(\psi)$ on ω/c and its spectrum SpD for $\beta = v/c = 0.65$. Where in $\Lambda_L = 0.973$, $\Lambda_\psi = 1.0528$, $\lambda_\psi = 0.1353$

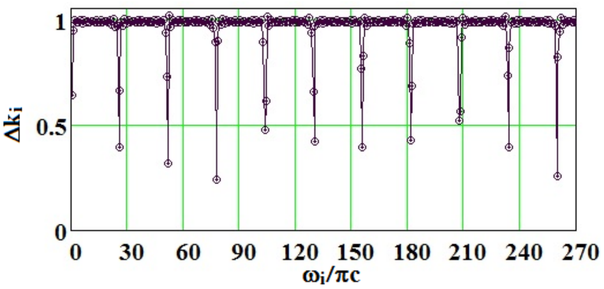


Fig. 2. Dependences of the difference $\Delta k_i = (\omega_i - \omega_{i-1})/\pi c$ of the neighboring roots of the dispersion equation $D(\omega_i/c) = 0$ of layered dielectric on the values ω/c of these roots for $\beta = v/c = 0.65$ and $k_\perp = \lambda_1/R$

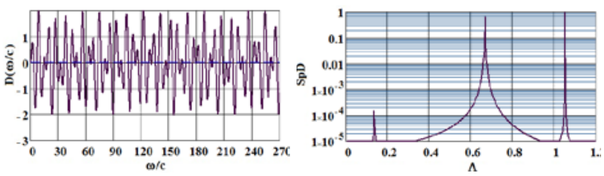


Fig. 3. Dependence $D(\omega/c) = \cos(\omega L/\beta c) - \cos(\psi)$ on ω/c and its spectrum SpD for $\beta = v/c = 0.95$, $k_\perp = \lambda_1/R$. Where in $\Lambda_L = 0.6657$, $\Lambda_\psi = 1.0528$, $\lambda_\psi = 0.1353$

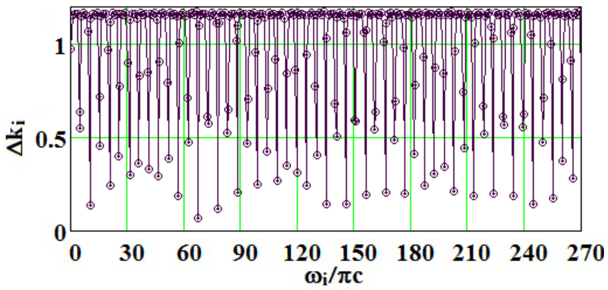


Fig. 4. Dependences of the difference $\Delta k_i = (\omega_i - \omega_{i-1})/\pi c$ of the neighboring roots of the dispersion equation $D(\omega_i/c) = 0$ of layered dielectric on the values ω/c of these roots for $\beta = v/c = 0.95$ and $k_\perp = \lambda_1/R$

On the graphs of the difference in the wavenumbers of the neighboring zeros of the dispersion equation (Figs. 2 and 4), it is clearly seen that in addition to the main two wavenumbers, which make the maximum contribution to the change in the dispersion equation (14), there is one more wavenumber indicating the roots of the dispersion equation with close values of wavenumbers. Thus, we see that there are different periods of succession of the roots of the dispersion equation (14). To determine the averages over the period of energy losses of particles, the expressions for the average field structures of the field have the form [1]:

$$\bar{E}_{z,\omega} = \frac{1}{L}\int_{-a}^{b} E_z e^{i\omega t} dz\bigg|_{z=vt}, \quad \bar{E}_{r,\omega} = \frac{1}{L}\int_{-a}^{b} E_r e^{i\omega t} dz\bigg|_{z=vt},$$

$$\bar{H}_{\varphi,\omega} = \frac{1}{L}\int_{-a}^{b} H_\varphi e^{i\omega t} dz\bigg|_{z=vt}. \qquad (15)$$

Summarizing the obtained fields for various transverse wavenumbers with allowance for their radial distribution, we find the dependences of the mean fields $\bar{E}_z, \bar{E}_r, \bar{H}_\varphi$ of the parametric Cherenkov radiation on the radius (shown in the figures below), as well as the dependence of the energy losses averages on the period of the structure on the radius:

$$-\frac{\partial W}{\partial z} = q\bar{E}_z = q\sum_{n=1}^{Nbess}\int_{-\infty}^{\infty} \bar{E}_z(\omega, r)d\omega =$$

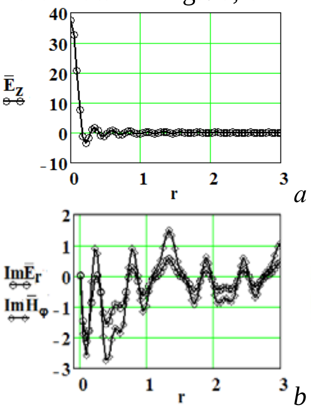
$$2\pi i q \sum_{n=1}^{Nbess} \frac{2J_0(rk_{\perp n})}{R^2 J_1^2(\lambda_n)} \sum_{j=1}^{N_{res}} (\bar{E}_z(\omega_{res_j}, r) + \bar{E}_z(-\omega_{res_j}, r)), \qquad (16)$$

where $k_{\perp n} = \lambda_n/R$, $Nbess$ is the number of radial harmonics, ω_{res_j} are the roots of the dispersion equation (14) for each $k_{\perp n}$, $Nres$ is equal to the number of roots of the dispersion equation (14) in a given frequency interval. The values for the fields $\bar{E}_z(\omega_{res_j})$ are determined from equations (6) in each region.

It follows from (16) that the particle's average energy loss is determined by the average field on the period of the structure $\bar{E}_z$.

Let us find the dependence of the fields averages on the structure period $\bar{E}_z, \bar{E}_r, \bar{H}_\varphi$ on the radius for the case when the thicknesses of the dielectric layers are the same. To represent the radial dependence, the number of steps along the radius is chosen equal to 70, $Nbess = 24$.

The graphs of the average fields dependence $\bar{E}_z, \bar{E}_r, \bar{H}_\varphi$ on radius averaged on the structure period are shown in Figs. 5, 6



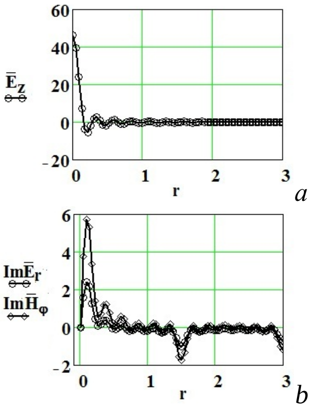
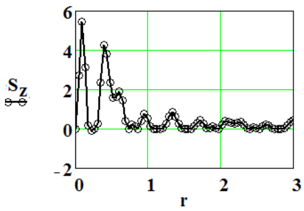


Fig. 5. Dependence of the fields on the radius averaged on the structure period: a) $\bar{E}_z$*; b)* $\bar{E}_r, \bar{H}_\varphi$ *for the parameters* $\beta = v/c = 0.65$

Fig. 6. Dependence of the fields on the radius averaged on the structure period: a) $\bar{E}_z$*; b)* $\bar{E}_r, \bar{H}_\varphi$ *for the parameters* $\beta = v/c = 0.95$

Analysis of (15) shows that the mean values of the field $\bar{E}_z$ are real, and the mean values of the fields $\bar{E}_r$ and $\bar{H}_\varphi$ are, as expected, purely imaginary. This indicates on the transfer of radiation energy along the axis of the waveguide.

The energy flux for the average over the period structure of the fields excited by the particle is determined by the Umov-Poynting vector: $\vec{S} = \mathrm{Re}\left[\vec{E}_{mid} \times \vec{H}^*_{mid}\right]$. Hence, it is not difficult to determine the values of the energy fluxes from the projections on the coordinate axes: $S_r = -\mathrm{Re}\left(\bar{E}_z \bar{H}^*_\varphi\right)$, $S_\varphi = 0$, $S_z = \mathrm{Re}\left(\bar{E}_r \bar{H}^*_\varphi\right)$.



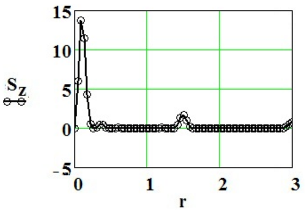


Fig. 7. The average energy fluxes over a structure period S_z *for* $\beta = v/c = 0.65$

Fig. 8. The average energy fluxes over a structure period S_z *for* $\beta = v/c = 0.95$

CONCLUSIONS

Thus, as a result of the carried out investigation of the radiation of a charged particle moving along the axis of an ideally conducting metal waveguide filled with a spatially periodic layered dielectric, the following conclusions can be drawn.

1. The problem of radiation of charged particle in an ideally conducting metal waveguide filles with a spatially periodic layered dielectric is solved without a transition to an equivalent anisotropic dielectric.

2. The dependencies of electric and magnetic radiation fields averaged on the structure period on the waveguide's radius are determined numerically under conditions when the period of the structure is of the same order as the wavelength of the radiation and the width of the dielectrics is the same.

3. It is shown that for equal thicknesses of the dielectrics the mean values of the field $\bar{E}_z$ are real, and the mean values of the fields $\bar{E}_r, \bar{H}_\varphi$ are purely imaginary.

4. The average over the structure period radiation flux for equal thicknesses of dielectrics is positive, directed along the waveguide axis, has a maximum at small distances from the waveguide axis, and decreases with approach to the waveguide wall.

5. The carried out investigation makes it possible to determine both the average fields generated by the charged particle and the energy fluxes of these fields for arbitrary values of the thicknesses and dielectric permittivities of the layers, the velocity of the charged particle, and the waveguide's radius.

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ИЗЛУЧЕНИЕ ЗАРЯЖЕННОЙ ЧАСТИЦЫ В ИДЕАЛЬНО ПРОВОДЯЩЕМ МЕТАЛЛИЧЕСКОМ ВОЛНОВОДЕ, ЗАПОЛНЕННОМ ПРОСТРАНСТВЕННО ПЕРИОДИЧЕСКИМ СЛОИСТЫМ ДИЭЛЕКТРИКОМ

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Аналитически и численно проведено исследование параметрического черенковского излучения равномерно движущейся частицы в идеально проводящем металлическом волноводе, заполненном пространственно периодическим слоистым диэлектриком для случая длин волн, сравнимых с периодом неоднородности. Описаны поля и спектры параметрического черенковского излучения. Найдены средние по периоду структуры потери энергии частицы и определены потоки энергии полей.

ВИПРОМІНЮВАННЯ ЗАРЯДЖЕНОЇ ЧАСТИНКИ У ІДЕАЛЬНО ПРОВІДНОМУ МЕТАЛЕВОМУ ХВИЛЕВОДІ, ЗАПОВНЕНОМУ ПРОСТОРОВО ПЕРІОДИЧНИМ ШАРУВАТИМ ДІЕЛЕКТРИКОМ

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Аналітично та чисельно проведено дослідження параметричного черенковського випромінювання частинки, що рівномірно рухається в ідеально провідному металевому хвилеводі, заповненому просторово періодичним шаруватим діелектриком для випадку довжин хвиль, які можна порівняти з періодом неоднорідності. Описано поля і спектри параметричного черенковського випромінювання. Знайдено середні по періоду структури втрати енергії частинки і визначені потоки енергії полів.