NUMERICAL APPROACH FOR SIMULATION OF PALMER COOLING

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In this paper the time domain algorithm is described and applied for the Palmer Cooling method. The possibility of using the Palmer system in such case for simultaneous longitudinal and transverse cooling by a suitable choice of the pickup to kicker distance was described by Hereward [1, 2]. Using his method the special computer code has been developed to calculate beam cooling in time domain approach.

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INTRODUCTION

For improving beam quality in storage rings is used Stochastic cooling (SC). It is also used for accumulation of high intensity ion beams. In our previous papers, the main principles of stochastic cooling were described. Many theoretical results for the stochastic cooling system, which is planned to be used in the Collector Ring (CR) of the FAIR complex (Darmstadt, Germany), were presented [3 - 7].

The momentum cooling and antiproton accumulation are usually simulated by numerically solving the Fokker-Planck (FP) equation [1]. This approach however can not be easily extended to account for many real-life complications, particularly the two-way transverse-longitudinal coupling due to the finite betatron size at the pickup.

An alternative approach to stochastic cooling simulation can be described as the macroparticle simulation using the discrete particles and the time- domain response functions of the pickup (PU) – kicker (KK) circuits. The beam dynamic under influence of the stochastic cooling forces can be studied by a particle by particle and turn by turn in the time-domain treatment. This treatment escapes the involvement of complicated, uncertain and changing frequency spectra, which anyhow are likely to be incomplete by considering Fokker-Planck equation and its solution. To keep the computation times within reasonable limits, the scaling law that cooling times are proportional to the number of particles (for zero preamplifier noise and all other parameters remaining unchanged, except the gain) has been applied throughout. A typical simulation super-particle number is about $(1\ldots10)\cdot10^4$.

1. THE NUMERICAL CODE FOR CALCULATION OF COOLING BY PALMER METHOD

The Collector Ring (CR) at FAIR [3] is going to be used for fast cooling of hot ions coming from separators. This ring will be equipped with stochastic cooling systems, which can allow to cool beam by different methods: TOF (time of flight), notch filter and Palmer. The Palmer cooling system will be used as a pre-cooling of radioactive ion beams (RIBs), since this system allows to avoid the Schottky band overlap of ions, for which the η – accelerator slip factor is rather large 0.178. The Palmer cooling will be useful in the first stage of stochastic cooling process at the CR. It serves to detect signals in all 3 phase space planes. After the rms Δp/p (root mean square of momentum spread) decreases below 0.1%, it is possible to switch off the signals from the Palmer Pick up and turn to Notch filter cooling.

The initial normalized particle coordinates $x_{c,i}$ and $x'_{c,i}$ are generated and transformed to the normalized coordinated $X_{n,i}$ and $X'_{n,I}$

$$X_n = \frac{x_c}{\sqrt{\beta}_{PU}}; \quad X'_n = \sqrt{\beta_{PU}}\, x'_c + \frac{\alpha_{PU}}{\sqrt{\beta_{PU}}}\, x_c . \qquad (1)$$

Here β_{PU} and α_{PU} are Twiss parameters of a ring at Pick up (beta and alfa functions). The normalized coordinates are used because of the simple modeling of betatron oscillation in the ring. The particle amplitude is defined

$$A^2 = X_n^2 + X_n'^2 .$$

The maximum radius of A^2 is interpreted as emittance ε. The particle coordinate is generated with a Gaussian distribution with the $\sigma = \varepsilon_{rms}$, where $\varepsilon_{rms} = A_{rms}^2$.

1. The initial momentum deviation $\Delta p/p_i$ is assigned to each particle. The initial particle ensemble has the Gaussian momentum distribution with $\sigma = \Delta p/p_{rms}=\delta_{rms}$.

2. The time length t_s of one sample is defined using the bandwidth characteristic of cooling system W

$$t_s = \frac{1}{2W} .$$

For a certain number of samples n_s the total time length of beam is calculated: $T_b=n_s t_s$. For each particle the initial time t_i is generated within the time domain from 0 up to T_b with a homogenous probability. Now one can say that each particle belongs to one of the samples (from 1 to n_s). This is important issue, which characterizes the particle ensemble of each sample. In the code each sample is analyzed turn by turn. Now the cooling process together with beam dynamic in the ring is calculated. It should be noted that Palmer cooling is characterized by a fact that a test particle has a horizontal orbit displacement $x_i = D_{PU}\frac{\Delta p}{p}_i$ proportional to its momentum error $\Delta p/p$, where D_{PU} is the value of the "orbit dispersion function" at the pickup as determined by the focusing properties of the storage ring. This displacement is detected by horizontal position pickup. It is assumed that the particles momentum contribution dominates at the pickup.

3. The single particle displacement at the PU is calculated by $X_{ni} = X_{ni} + D_{n,PU}\frac{\Delta p}{p}_i$, where $D_{n,PU}$ is the normalized dispersion function at PU.

4. At the PU the accessory of a particle to the certain sample *s* is defined and saved. Belonging to the certain sample depends on its time value t_i. The average value of $<X_n>_s$ is calculated for each sample *s* by $\langle X_n \rangle_s = \frac{1}{N_s}\sum_{i=1}^{N_s} X_{ni}$, where N_s is a number of particles in sample *s*.

5. On the way from PU to KK the particles migrate from sample to sample, which means the time of each particle at KK has new value and calculated by $t_i = t_i + \Delta t_i$, $\Delta t_i = T_{PK}|\eta_{PK}|\frac{\Delta p}{p}_i$, Δt_i is a time change of the particle. For each sample the creation of new ensemble with a new time characteristic is called as "mixing".

6. The particle coordinates at the KK have also mixed and new values due to betatron oscillation recalculated by

$$\begin{pmatrix} X_{n,K} \\ X'_{n,K} \end{pmatrix} = \begin{bmatrix} C_{PK} & S_{PK} \\ -S_{PK} & C_{PK} \end{bmatrix} \begin{pmatrix} X_{n,P} \\ X'_{n,P} \end{pmatrix}. \qquad (2)$$

$C_{PK} = \cos(\Delta\mu_{PK})$; $S_{PK} = \sin(\Delta\mu_{PK})$; $\Delta\mu_{PK}$ is a phase advance fro PU to KK. One can see that the betatron oscillation is considered as simple rotation in normalized coordinates with radius *A*.

7. At the KK the accessory of particle to certain sample *s* is defined and saved. Belonging to a certain sample *s* depends on its time value t_i. Depending on the sample number *s* the single particle correction is calculated by

$$\frac{\Delta p}{p}_i = \frac{\Delta p}{p}_i - \frac{g}{D_{PU}} \cdot \langle X_n \rangle_s \cdot \alpha_p \cdot s(\Delta t), \qquad (3)$$

$$X'_n = X'_n + g \cdot \langle X_n \rangle_s \cdot \alpha_t \cdot s(\Delta t). \qquad (4)$$

The *g* is a normalized gain, $\alpha_{p,t}$ is damping factor, which reduce the gain efficiency due to the noise. In the code this factor is calculated by formulae given in chapter 3. The *s*(Δ*t*) is a time profile of the signal, which is calculated by the formula given in chapter 2. The Eqs. (3) and (4) describe the cooling effect in the time domain approximation. One can see that for the Palmer method the momentum error of particles is corrected proportionally to the center gravity of sample, which characterized by average value of coordinate X_n.

8. On the way from KK to PU the particles migrate again from sample to sample, which means the time of each particle at PU is changed and calculated by

$$t_i = t_i + \Delta t_i, \quad \Delta t_i = T_{KP}|\eta_{KP}|\frac{\Delta p}{p}_i.$$

9. The particle coordinates at the PU are rearranged and recalculated by

$$\begin{pmatrix} X_{n,P} \\ X'_{n,P} \end{pmatrix} = \begin{bmatrix} C_{KP} & S_{KP} \\ -S_{KP} & C_{KP} \end{bmatrix} \begin{pmatrix} X_{n,K} \\ X'_{n,K} \end{pmatrix}.$$

$C_{KP} = \cos(\Delta\mu_{kP})$ $S_{KP} = \sin(\Delta\mu_{KP})$, $\Delta\mu_{KP}$ is a phase advance from KK to PU.

10. At the position of PU the analyses of the rms values of $\Delta p/p_{rms}$ and emittance ε_{rms} are performed and saved.

11. The next turn is calculated. Go to step 4.

2. THEORETICAL DESCRIPTION OF THE MAIN PARAMETERS OF STOCHASTIC COOLING

2.1. GAIN FACTOR

For momentum cooling the gain factor *g* can be expressed [1] as

$$g_p = \frac{e^2 f_{rev}^2 \sum_{n1}^{n2} R_n}{\delta E}\frac{N}{W} = \frac{e\, f_{rev} R_{av} N}{(E/e)\frac{\delta E}{E}} \qquad (5)$$

Here the sum goes over all harmonics in the passband, i.e. from $f_{min}=n_1 f_{rev}$ to $f_{max}=n_2 f_{rev}$ and average impedance can be approximately calculated by

$$R_{av} = \frac{1}{(n_2 - n_1)}\sum R_n .$$

For the transverse cooling the gain factor *g* can be expressed as [4]

$$g_t = \frac{4\pi N Z_i^2 e^2 c}{m_c A_i E_0 \gamma \beta^2}\sqrt{n_p n_k Z_{oPU} Z_{oK}}\left(\frac{\partial S}{\partial x}\right)^2 G_a \sqrt{\beta_p \beta_k}. \qquad (6)$$

Z_{0PU} and Z_{0K} transverse impedance of Pick-up and Kicker; R_{PU} and R_K input impedance of network (shunt impedance); n_p and n_k are number of Pick-up and Kicker units; β_p and β_k are betatron functions at Pick-up and Kicker; f_{gPU} and f_{gKU} are geometry factors of electrodes; G_a is an electrical gain; Z_i and A_i are the ion charge and mass; f_0 is a revolution frequency; e is the electron charge; $\beta = v/c$; γ is a relativistic factor; *N* is the number of particles; m_c is the middle harmonic.

2.2. NOISE

The gain damping factor α_p can be calculated by $\alpha_p = 1 - \frac{g}{2}(1 + U_p)$ for momentum cooling and $\alpha_x = 1 - \frac{g}{2}(1 + U_x)$ for betatron cooling. Here U_p and U_x are the noise-to-signal ratio. For Palmer cooling this value is defined by [10]

$$U_p = \frac{\varepsilon_{rms}^2}{D_{PU}^2 \delta_{rms}^2} + \frac{\delta_n^2}{\delta_{rms}^2} \text{ and } U_x = \frac{D_{PU}^2 \delta_{rms}^2}{A_{rms}^2} + \frac{\varepsilon_n^2}{A_{rms}^2}.$$

In this equation the noise-to-signal ratio $\delta_n^2 / \delta_{rms}^2$ can be calculated according the formula

$$\frac{\delta_n^2}{\delta_{rms}^2} = \frac{10^{\frac{\nu}{10}} k T_s}{2 N Z_i^2 e^2 f_0 n_p R \lambda}.$$

The ν is noise of the amplifier (usually 1.5…3.0 dB); *R* is input resistance of the amplifier; λ is a sensitivity factor; *N* – number of particles in the beam; *e* – electron charge; f_0 – revolution frequency; n_p – number of pick-ups; *k* – Boltzmann constant; T_e – temperature (K). The possible increase of U_p as the beam shrinks, and many construction details, are hidden in λ. As first approximation, in the code it is assumed that the sensitivity factor calculated by $\lambda_p(\delta_{rms}) = \lambda_0 \frac{\delta_{rms}^2}{\delta_0^2}$ and $\lambda_x(\varepsilon_{rms}) = \lambda_0 \frac{\varepsilon_{rms}}{\varepsilon_0}$.

The parameter λ_0 is geometrical factor, which characterizes the PU. In simulation it is assumed λ_0=0.5. $\delta_0 = (\Delta p/p_{rms})_0$ is initial rms momentum spread of beam. $\varepsilon_0 = (\varepsilon_{rms})_0$ is the initial rms emittance. These equations reveal that the noise limits the cooling rate.

2.3. KICKER ACTION

Synchronism between particles and their correcting pulse on their way from pick-up to kicker must be properly calculated. In the time domain approach the incoherent heating effect is calculated in a simpler way compare to that is done by Focker-Planck Equation. It is assumed that the action at the kicker produces the time-pulse curves. These curves can be calculated by inverse Laplace transformation of a single passage of sample for a certain bandwidth cooling system. As a result of such transformation the signal shape similar to that shown in Fig. 1, can be obtained.

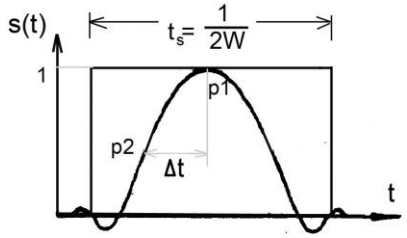

Fig. 1. Approximated correcting pulse in the kicker

In Fig. 1 Δt is the time error of the particle p2 with respect to the p1, which is located in the middle of the sample and synchronized with an ideal test particle p1. The particle p2, which arrives at KK with a time delay Δt, gets a partial kick. T_c is the useful width of the correction pulse and usually equal to the sample length t_s for low pass system. But T_c is shorter than t_s for a high frequency band-pass system. *(this is subject is still under study).*

For simplification, in this work a parabolic response model of the form

$$s(t) = 1 - \left(\frac{\Delta t}{T_c}\right)^2, \qquad (7)$$

for Palmer and TOF signal approximation is used. In Fig. 2 (left) the signal shape calculated by Eq. (7) is shown. Here is assumed that $T_s=t_s/2.3$, t_s is the time length of sample.

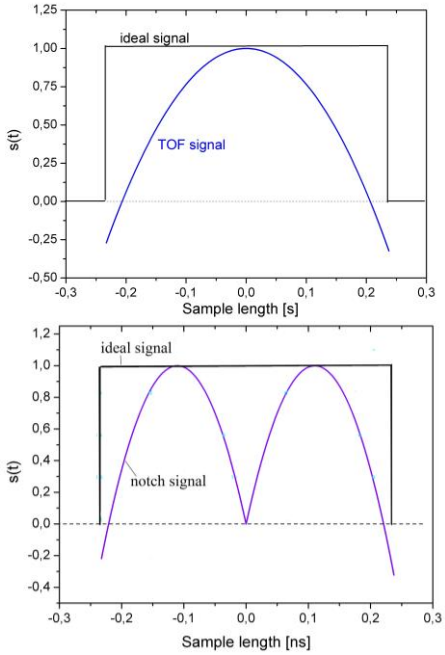

Fig. 2. Test signals s(t) reproducing a signal shape (left) for Palmer and TOF (right) for the Notch filter method

For the notch filter the signal shape is approximated by

$$s(t) = 1 - \left(\frac{|\Delta t| - T_c}{T_c}\right)^2, \qquad (8)$$

where $T_c=t_s/4.3$. The signal shape calculated by Eq. (8) is shown in Fig. 2 (right).

3. ANALYTICAL APPROACHES FOR STOCHASTIC COOLING

In this chapter the analytical formulae are presented in order to cross check the results of simulation obtained with numerical method given in this work. Using these formulae the beam evolution is calculated and results are compared with time domain approach described above. A simple, but, useful, calculation of the stochastic cooling by analytical formulae can be done using cooling rates $1/\tau$. The set of equation are solved to calculated rms emittance and rms momentum spread a each time step i with a time step of Δt:

$$\varepsilon_{rms}^{i+1} = \varepsilon_{rms}^{i} \exp\left(-\frac{\Delta t}{\tau_h}\right), \quad \frac{\Delta p}{p}^{i+1}_{rms} = \frac{\Delta p}{p}^{i}_{rms} \exp\left(-\frac{\Delta t}{\tau_p}\right). \qquad (9)$$

Here the cooling rates $1/\tau$ are calculated by formulae [5]

$$\frac{1}{\tau_p} = -\frac{W}{N}\left[2g_p B(t) - g_p^2\left(M(t) + U_p(t)\right)\right] \text{ and}$$

$$\frac{1}{\tau_h} = -\frac{W}{N}\left[-2g_h B(t)\cos(\Delta\mu_{PK}) - g_h^2\left(M(t) + U_h(t)\right)\right].$$

The parameters B, M, U are time dependent functions. The average mixing factor $M(t)$ for the Gaussian distribution is expressed as

$$M = \frac{1}{2\sqrt{2\pi}|\eta|\delta p(t)_{rms}\Delta n}\ln\left(\frac{n2}{n1}\right).$$

The δp (t) is a rms momentum spread, which depends on time. The noise-to-signal rations U_p and U_h for Palmer method are calculated by

$$U_p = \frac{\varepsilon_{rms}^i \beta_{PU} + x_{n,PU}^2}{D_{PU}^2\left(\frac{\Delta p}{p}^{i}_{rms}\right)^2}, \quad U_h = \frac{2kT_s Z_l}{Ne^2 f_0 n_p \beta_{PU} Z_{pt}^2 \varepsilon_{rms}^i},$$

$$x_{n,PU} = 10^{\frac{\delta v}{10}} kT_s, \ \delta v = 1.5 \text{ dB} - \text{noise}. \qquad (10)$$

For Palmer or TOF method $B(t) \approx \cos\left(2\pi|\eta_{PK}|n_2\alpha_{pk}\delta p(t)_{rms}\right)$. For Notch filter cooling $B(t) \approx \cos\left(0.5\pi\eta_{eff} n_2 \delta p(t)_{rms}\right)$. Z_l is characteristic impedance of the electrode; N – number of particles in the beam; e – electron charge; f_0 – revolution frequency; n_p – number of pick-ups, k – Boltzmann constant; T_e – temperature (K); β_p – beta function at the pickup. $\Delta\mu_{PK}$ – phase advance between pick-up and kicker.

The optimal value of the g_t can be calculated [4] as

$$g_{t,opt} = \frac{B(t)\cos(\Delta\mu_{PK})}{M(t) + U(t)}.$$

Some useful analytical formulae formula, which can be used for test simulations.

The noise-to-signal ration can be expressed through the noise electron current I_n and Schottky currect at PU I_{PU}.

$$U = \frac{I_n^2}{I_{PU}^2}, \quad I_{PU} = \frac{\lambda D_{rms}}{h}, \quad D_{rms} = \frac{x_{rms} I_{SC}}{2}.$$

The λ is sensitivity factor; h – half of gap of electrodes; I_n – noise current due to temperature of electronic $I_n = \sqrt{\frac{10^{\frac{v}{10}} kT_e W}{R}}$ and $I_{SC} = \sqrt{2e^2 N f_0 W}$ – Schottky current.

4. NUMERICAL SIMULATIONS

4.1. TEST CALCULATION: COMPARISON NUMERICAL WITH ANALYTICAL CALCULATIONS

The special tracking code, where the Palmer cooling is modelled in the time domain approach, has been written in FORTRAN language. As an example a set of parameters for a beam and CR cooling system [5] is used. The beam cooling with the Palmer method has been performed over the time of 5 s. In the Fig. 3 one can observe the Probability Density Function (PDF) evolution of dp/p and emittance during cooling time at each 0.4 s. In Fig. 4 the evolution of rms values of dp/p and emittance calculated by tracking code and by the analytical formulae Eq. (9, 10) are shown. Here the gain factor g is 0.5.

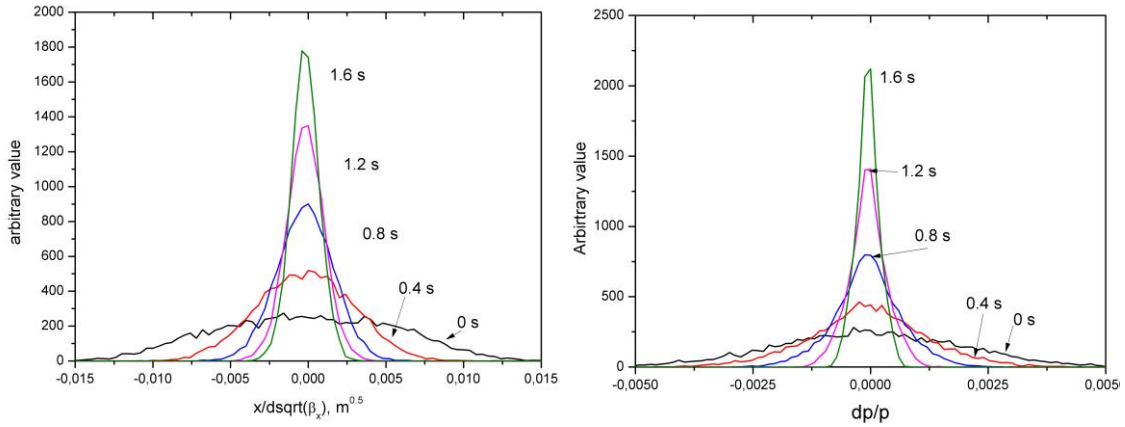

Fig. 3. Probability Density Functions (PDF) calculated by tracking code in time domain approach. Cooling of an Ur beam: horizontal beam profiles (left) and momentum spread distribution recorded every 0.4 s

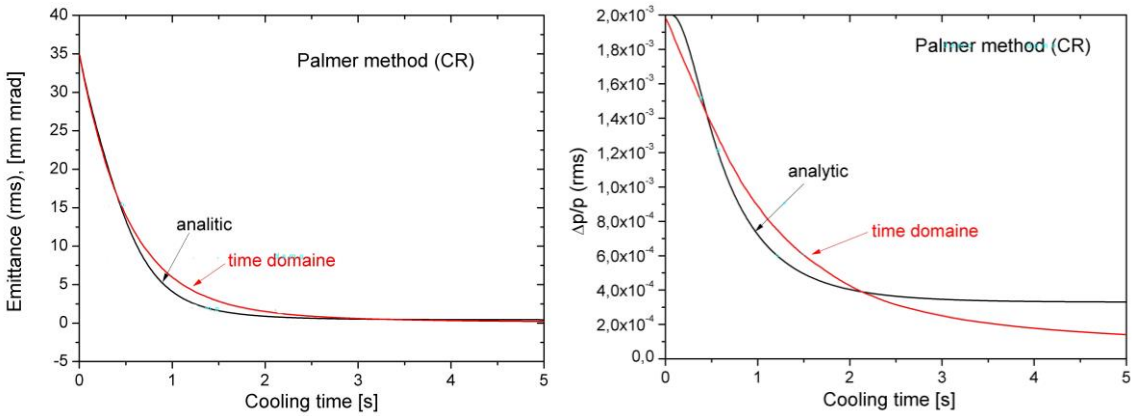

Fig. 4. Evolution of the emittance (left) and momentum spread (right) calculated with the Palmer method in the time domain approach and by the analytical formulae for the CR machine. Parameters used in calculation are given in [5]. The gain factor g = 0.5

4.2. PALMER COOLING AT THE CR

In this chapter the preliminary simulations of beam cooling with the Palmer method are presented and compared with an analytical model. In the code the Palmer cooling is modeled by following steps.

Fig. 5 shows the results of simulation for uranium beam cooling at the CR for the different gain factors. The parameters of the CR ring and beam are given in [5]. One can see that the optimal fast cooling in both planes can be performed if the gain g is 0.4…0.5 that corresponds to the amplification factor of 150 dB. Here the ideal signal is assumed at the Kicker (see chapter 2). The realistic signal shape must be obtained from inverse Laplace transformation and included in simulations. Under given above conditions the rms $\Delta p/p$ below 0.1% can be achieved in 0.5 s, while the rms emittance is cooled down to about 8 mm mrad as shown in Fig. 5. Simulations obtained with analytical formulae Eqs. (9), (10) show approximately the similar results, which are shown in Fig. 6. Here the one can see that the optimal gain factor is 0.3…0.4. Analytical model predicts that the rms emittance after 0.5 s is above 10 mm·mrad, while the rms $\Delta p/p$ is 0.1%.

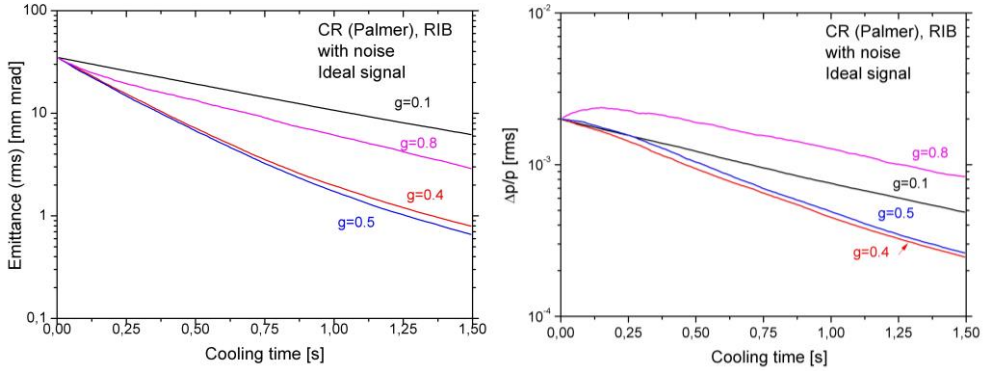

Fig. 5. Evolution of the rms emittance and the momentum spread of the U beam for different gains and obtained in numerical calculation by a time domain method

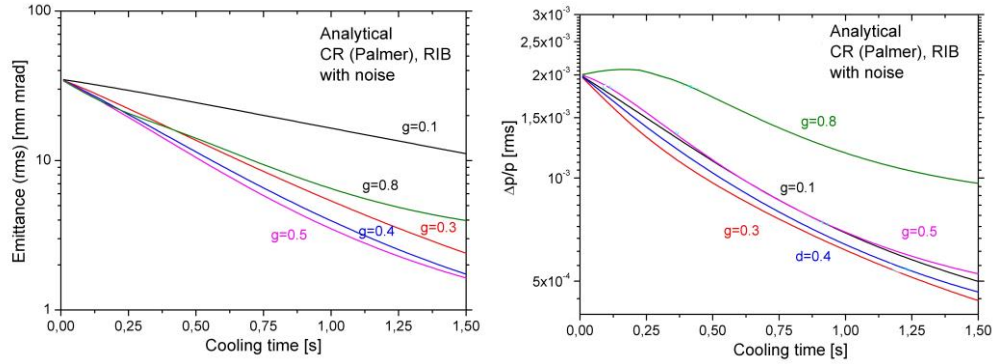

Fig. 6. Evolution of the rms emittance and the momentum spread of the U beam during cooling for different gains and obtained in calculation by analytical formula

CONCLUSIONS

The precise method for the higher dimensions Palmer cooling calculations has been presented. The special code has been developed to investigate the beam dynamic in storage ring, where the stochastic cooling process is used. Presented method can be used for TOF and Notch filter cooling systems.

Presented method can be used for the wide range of tasks and will be applied to the realistic parameters for studying the dynamics of particles in storage rings.

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ЧИСЛЕННЫЙ ПОДХОД ДЛЯ СИМУЛЯЦИИ ОХЛАЖДЕНИЯ МЕТОДОМ PALMER

М.Е. Долинская, Н.Л. Дорошко

Описывается алгоритм «timedomain», который применяется для охлаждения Palmer-методом. Возможности применения Palmer-системы для продольного и поперечного охлаждений путем подбора расстояния между pick-upи кикер-магнитами были описаны в работах Херевальда [1, 2]. Используя данный подход, была разработана компьютерная программа для расчетов параметров охлаждения с применением «timedomain»-методики.

ЧИСЕЛЬНИЙ ПІДХІД ДЛЯ СИМУЛЯЦІЇ ОХОЛОДЖЕННЯ МЕТОДОМ PALMER

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Описується алгоритм «timedomain», який застосовується для охолодження Palmer-методом. Можливості застосування Palmer-системи для поздовжнього і поперечного охолоджень шляхом підбору відстані між pick-up і кікер-магнітами були описані в роботах Херевальда [1, 2]. Використовуючи даний підхід, була розроблена комп'ютерна програма для розрахунків параметрів охолодження із застосуванням «timedomain»-методики.