A NOVEL APPROACH TO THE SYNTHESIS OF THE ELECTROMAGNETIC FIELD DISTRIBUTION IN A CHAIN OF COUPLED RESONATORS

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A novel approach to the synthesis of the electromagnetic field distribution in a chain of coupled resonators has been developed. This approach is based on the new matrix form of the solutions of the second-order difference equations. If a chain of coupled resonators can be described by the second-order difference equation for amplitudes of expansion of the electromagnetic field, two linearly independent solutions can be constructed on the basis of the solutions of nonlinear Riccati equation. Setting the structure of one solution, from the Riccati equation we can find the electrodynamical characteristics of resonators and coupling holes, at which the desired distribution of amplitudes is realized. On the base of this approach we considered the problem of separation of the electromagnetic field into “forward” and “backward” components in the inhomogeneous chain of resonators. It was shown that in the frame of considered model such separation is not defined uniquely.

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INTRODUCTION

There are three main fields of using the coupled resonator chains – accelerators [1], RF-sources, mainly travelling wave tubes (TWT) [2] and RF filters [3]. If for the first two applications it is necessary to create the special field distribution for the given frequency (accelerators) or some frequency range (TWT) along of the chain, then for the RF filters requirements are imposed on the amplitude-frequency and phase-frequency characteristics at the chain output.

Coupled-resonator circuits are of importance for design of RF/microwave filters, in particular, the narrow-band bandpass filters that play a significant role in many applications. There is a general technique for designing coupled-resonator filters in the sense that it can be applied to any type of resonator despite its physical structure [4, 5].

In coupled-cavity TWTs several tens of coupled cavities are used as the slow wave structure. The efficiency of a TWT is limited by peculiarity of the bunching process and the bunch transfer from decelerating phase into the accelerating phase of the RF field. The usual technique suggested for increasing the efficiency involves tapering of the wave phase velocity so that the decelerated bunches remain within the decelerating phase of the wave. There were proposed several methods for synthesis of the optimum phase velocity distribution along the slow wave structure (see, for example, [6 - 13]).

The widest use the the cavity chains have found in the accelerator technique. At the very beginning of its development, the RF accelerators have the RF resonators as the main element of its construction. Disk-loaded waveguides [14 - 19], different side-coupling standing wave structures [20, 21], hybrid (combined) accelerating structures (the initial part of the structure is a standing wave buncher, and its main part is a disk-loaded waveguide) [22, 23] – this is a short enumeration of the different coupled resonator chains that are used in accelerators. There is enormous number of publications that describe the calculation and design the accelerating structures.

The calculation of parameters and the design play an important role in the process of developing an accelerating structure. No less important role is played by the process of tuning cells after section brazing.

In order to provide synchronism with the beam and electromagnetic field in the accelerating structure, the phase advance of each cell needs to be adjusted to its nominal value. This can be done after brazing by correcting machining deviations, assembly and brazing mismatching. This adjustment process is called tuning (post-tuning).

The tuning methods based on the non-resonant perturbation field distribution measurement [24 - 30] have been widely used for tuning travelling-wave structures, especially in tuning the constant-gradient ones [31 - 44].

There are several approaches for post-tuning. The most widespread tuning method became one, in which the field distribution was considered to be a linear superposition of forward and backward waves in each cell [31]. The internal reflection of each cell was obtained by calculating the difference of the amplitudes of the backward waves seen before and after that cell. But forward and backward waves were not strictly determined. Their amplitudes were introduced phenomenologically.

Development of the Coupling Cavity Model (CCM) [45 - 48] gives possibility to look into this method more deeply [49, 50]. However, the problem of expanding the electromagnetic field into the forward and backward waves in each cell of the inhomogeneous chain has not been cleared up yet.

In this article a novel approach to analysis of the electromagnetic field distribution in a chain of coupled resonators is presented. This approach is based on the new matrix form of the solutions of the second-order difference equations [51].

1. SECOND-ORDER LINEAR DIFFERENCE EQUATION FOR THE CHAIN OF THE FINITE NUMBER OF RESONATORS

In the frame of the CCM electromagnetic field in each cavity of the chain of resonators are represented as the expansion with the short-circuit resonant cavity modes [17, 18, 52 - 55]

$$\vec{E}^{(k)} = \sum_q e_q^{(k)} \vec{E}_q^{(k)}(\vec{r}), \qquad (1)$$

where $q = \{0, m, n\}$ and such coupling equations for $e_{010}^{(n)}$ can be obtained [45, 47, 48]

$$Z_k e_{010}^{(k)} = \sum_{j=-\infty, j\neq n}^{\infty} e_{010}^{(j)} \alpha_{010}^{(k,j)}. \qquad (2)$$

Here $e_{010}^{(k)}$ – amplitudes of E_{010} modes, $Z_k = 1 - \frac{\omega^2}{\omega_{010}^{(k)2}} - \alpha_{010}^{(k,k)}$, $\omega_{010}^{(k)}$ – eigen frequencies of these modes, $\alpha_{010}^{(k,j)}$ – real coefficients that depend on both the frequency ω and geometrical sizes of all volumes. Sums in the right side can be truncated

$$Z_k^{(N)} e_{010}^{(N,k)} = \sum_{j=k-N, j\neq k}^{k+N} e_{010}^{(N,j)} \alpha_{010}^{(k,j)}. \qquad (3)$$

In the case of $N = 1$, the system of coupled equations (3) is very similar to the one that can be constructed on the basis of equivalent circuits approach (see, for example, [20, 56 - 58]). But in the frame of the CCM the coefficients $\alpha_{0mn}^{(k,j)}$ are electrodynamically strictly defined for arbitrary N and can be calculated with necessary accuracy. In the theory of RF filters the coupling matrix circuit model is used intensively (see, for example, [59] and cited there literature). The main problem is how to calculate the matrix elements.

Amplitudes of other modes ($(m,n) \neq (1,0)$) can be found by summing the relevant series

$$e_{0mn}^{(k)} = \frac{\omega_{0mn}^{(k)2}}{\omega_{0mn}^{(k)2} - \omega^2} \sum_{j=k-N}^{k+N} e_{010}^{(j)} \alpha_{0mn}^{(k,j)}. \qquad (4)$$

For the chain of cylindrical resonators longitudinal component of electric field at $r = 0$ (on the system longitudinal axis) is:

$$E_z^{(k)} = \sum_{m,n} e_{0mn}^{(j)} \cos\left(\frac{\pi}{d} nz\right). \qquad (5)$$

If we can ignore "long coupling" interaction, the set of coupling equations (3) takes the form[1]

$$Z_k e_{010}^{(k)} = e_{010}^{(k-1)} \alpha_{010}^{(k,k-1)} + e_{010}^{(k+1)} \alpha_{010}^{(k,k+1)}, \qquad (6)$$

where $Z_k = \left(1 - \frac{\omega^2}{\omega_{010}^{(k)2}} - \alpha_{010}^{(k,k)} - i\frac{\omega}{\omega_{010}^{(k)} Q_k}\right)$.

The set of coupling equations (6) can be considered as the second-order difference equation. This difference equation, which defines the amplitudes of the basic modes $e_{010}^{(k)}$, is the main equation of the CCM. It is reasonable to note that the amplitudes of the basic modes $e_{010}^{(k)}$ are non-measured values. Indeed, we can measure the components of electric field in any point, for example, by the nonresonant perturbation method, but we cannot measure $e_{0mn}^{(k)}$ and have to use numerical methods for finding these amplitudes by using the expansion (1). This circumference create difficulties in studding the properties of the real slow-wave waveguides, including their tuning [31, 49]. The similar situation arises also in other electrodynamic models. For example, the space harmonics in homogeneous periodic waveguides are non-measured values, too.

We will consider the chain with the finite number of resonators (Fig. 1). The first and the last resonators are connected to the transmission lines[2] and the equations (6) for $k = 1$ and $k = N$ have to be changed [58]

$$\left[-\frac{\omega^2}{\omega_{010}^{(1)2}} - i\frac{\omega(1+\beta_1)}{\omega_{010}^{(1)} Q_1} + (1 - \alpha_{010}^{(1,1)})\right] e_{010}^{(1)} = \\ = \alpha_{010}^{(1,2)} e_{010}^{(1)} + \frac{2i\omega}{\omega_{010}^{(1)} Q_1} \sqrt{\frac{\beta_1 R_1}{Z}} \frac{1}{d_1} U, \qquad (7)$$

$$\left[-\frac{\omega^2}{\omega_{010}^{(N)2}} - i\frac{\omega(1+\beta_N)}{\omega_{010}^{(N)} Q_N} + (1 - \alpha_{010}^{(N,N)})\right] e_{010}^{(N)} = \\ = \alpha_{010}^{(N,N-1)} e_{010}^{(N-1)}, \qquad (8)$$

where β_1, β_N – coupling factors of the first and the last resonators with transmission lines; Z – impedance of the input transmission line; R_1 – shunt impedance of the first resonator, $U = \sqrt{PZ/2}$; P – power of the external RF source; d_k – length of the k-th resonator.

Amplitude of the reflected wave in the input transmission line is

$$U_R = -\sqrt{\frac{\beta_1 Z}{R_1}} d_1 e_{010}^{(1)} - U. \qquad (9)$$

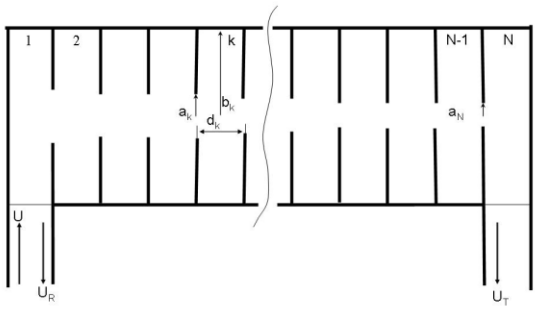

Fig. 1

According to the results of the work [51], we will seek a solution of difference equations (6) - (8) as

$$e_{010}^{(k)} = y_k^{(1)} + y_k^{(2)}, \; 1 \le k \le N; \\ e_{010}^{(k+1)} = \rho_k^{(1)} y_k^{(1)} + \rho_k^{(2)} y_k^{(2)}, \; 1 \le k \le N-1, \; \rho_k^{(1)} \neq \rho_k^{(2)}. \qquad (10)$$

Using this representation, the equations (7), (8) can be rewritten as

$$\left(Z_1 - \alpha_{010}^{(1,2)} \rho_1^{(1)}\right) y_1^{(1)} + \left(Z_1 - \alpha_{010}^{(1,2)} \rho_1^{(2)}\right) y_1^{(2)} = \\ = \frac{2i\omega}{Q_1 \omega_{010}^{(1)}} \sqrt{\frac{\beta_1 R_1}{Z}} \frac{1}{d_1} U, \qquad (11)$$

$$\left(Z_N \rho_{N-1}^{(1)} - \alpha_{010}^{(N,N-1)}\right) y_{N-1}^{(1)} + \left(Z_N \rho_{N-1}^{(2)} - \alpha_{010}^{(N,N-1)}\right) y_{N-1}^{(2)} = 0, \qquad (12)$$

and there are such matrix difference equation for new unknowns [51]

$$\begin{pmatrix} y_{k+1}^{(1)} \\ y_{k+1}^{(2)} \end{pmatrix} = T_k \begin{pmatrix} y_k^{(1)} \\ y_k^{(2)} \end{pmatrix}, \; 1 \le k \le N-2, \qquad (13)$$

[1] There is a problem of taking into account absorption of RF energy in walls as there are difficulties in obtaining appropriate eigen functions for cylindrical regions. All developed procedures in the frame of the CCM do not include this phenomenon. We used the simplest approach for including absorption into consideration. We supposed that the coupling coefficient do not depend on absorption and include the quality factor into the resonant term in the equations for e_{010} amplitudes.

[2] We will consider the chains with the transmission lines connected to the first and end resonators. Other connections can be considered similarly.

where

$$T_{k,11} = -\frac{\left\{\alpha_{010}^{(k+1,k)} - \left(Z_{k+1} - \alpha_{010}^{(k+1,k+2)}\rho_{k+1}^{(2)}\right)\rho_k^{(1)}\right\}}{\alpha_{010}^{(k+1,k+2)}\left(\rho_{k+1}^{(1)} - \rho_{k+1}^{(2)}\right)},$$
$$T_{k,12} = -\frac{\left\{\alpha_{010}^{(k+1,k)} - \left(Z_{k+1} - \alpha_{010}^{(k+1,k+2)}\rho_{k+1}^{(2)}\right)\rho_k^{(2)}\right\}}{\alpha_{010}^{(k+1,k+2)}\left(\rho_{k+1}^{(1)} - \rho_{k+1}^{(2)}\right)},$$
$$T_{k,21} = \frac{\left\{\alpha_{010}^{(k+1,k)} - \left(Z_{k+1} - \alpha_{010}^{(k+1,k+2)}\rho_{k+1}^{(1)}\right)\rho_k^{(1)}\right\}}{\alpha_{010}^{(k+1,k+2)}\left(\rho_{k+1}^{(1)} - \rho_{k+1}^{(2)}\right)},$$
$$T_{k,22} = \frac{\left\{\alpha_{010}^{(k+1,k)} - \left(Z_{k+1} - \alpha_{010}^{(k+1,k+2)}\rho_{k+1}^{(1)}\right)\rho_k^{(2)}\right\}}{\alpha_{010}^{(k+1,k+2)}\left(\rho_{k+1}^{(1)} - \rho_{k+1}^{(2)}\right)}. \quad (14)$$

Values of the grid vectors in the first and the ($N-1$)-th cells are connected by a linear relation

$$y_{N-1}^{(1)} = T_{11}^{\Sigma} y_1^{(1)} + T_{12}^{\Sigma} y_1^{(2)},$$
$$y_{N-1}^{(2)} = T_{21}^{\Sigma} y_1^{(1)} + T_{22}^{\Sigma} y_1^{(2)}. \quad (15)$$

Using these relations, the equations (7) and (8) can be rewritten in the form

$$\left(Z_1 - \alpha_{010}^{(1,2)}\rho_1^{(1)}\right) y_1^{(1)} + \left(Z_1 - \alpha_{010}^{(1,2)}\rho_1^{(2)}\right) y_1^{(2)} = = \frac{2i\omega}{Q_1\,\omega_{010}^{(1)}}\sqrt{\frac{\beta_1 R_1}{Z}}\frac{1}{d_1}U, \quad (16)$$

$$\left[\left(Z_N\rho_{N-1}^{(1)} - \alpha_{010}^{(N,N-1)}\right)T_{11}^{\Sigma} + \left(Z_N\rho_{N-1}^{(2)} - \alpha_{010}^{(N,N-1)}\right)T_{21}^{\Sigma}\right] y_1^{(1)} + +\left[\left(Z_N\rho_{N-1}^{(1)} - \alpha_{010}^{(N,N-1)}\right)T_{12}^{\Sigma} + \left(Z_N\rho_{N-1}^{(2)} - \alpha_{010}^{(N,N-1)}\right)T_{22}^{\Sigma}\right] y_1^{(2)} = 0. \quad (17)$$

We can choose the sequences $\rho_k^{(1)}$ and $\rho_k^{(2)}$ in such way that the matrix T_k will be the diagonal one [51]. From (14) it follows that $T_{k,12} = T_{k,21} = 0$ for $\rho_k^{(1)}, \rho_k^{(2)}$ which fulfilled Riccati type difference equation (the second-order rational difference equation) [60, 61] with different initial values of $\rho_1^{(1)}$ and $\rho_1^{(2)}$ ($\rho_1^{(1)} \neq \rho_1^{(2)}$)

$$\alpha_{010}^{(k+1,k)} - \left(Z_{k+1} - \alpha_{010}^{(k+1,k+2)}\rho_{k+1}\right)\rho_k = 0,\ 1 \le k \le N-2\,. \quad (18)$$

Solution of the matrix difference equation (13) with the diagonal matrix T_k is

$$y_k^{(1)} = \prod_{s=1}^{k-1}\rho_s^{(1)} y_1^{(1)},\ 2 \le k \le N,$$
$$y_k^{(2)} = \prod_{s=2}^{k-1}\rho_s^{(2)} y_1^{(2)}. \quad (19)$$

We will call $\rho_k^{(1)}$ and $\rho_k^{(2)}$ as characteristic multipliers.

In this case, the equation (17) transforms into

$$\left(Z_N\rho_{N-1}^{(1)} - \alpha_{010}^{(N,N-1)}\right)T_{11}^{\Sigma} y_1^{(1)} + +\left(Z_N\rho_{N-1}^{(2)} - \alpha_{010}^{(N,N-1)}\right)T_{22}^{\Sigma} y_1^{(2)} = 0, \quad (20)$$

where

$$T_{11}^{\Sigma} = \prod_{s=1}^{N-2} T_{s,11} = \prod_{s=1}^{N-2}\rho_s^{(1)}\,, \quad (21)$$

$$T_{22}^{\Sigma} = \prod_{s=1}^{N-2} T_{s,22} = \prod_{s=1}^{N-2}\rho_s^{(2)}\,. \quad (22)$$

Solving the equations (16) and (20), we obtain

$$y_1^{(1)} = \frac{2i\omega T_{22}^{\Sigma}}{gQ_1\,\omega_{010}^{(1)}}\sqrt{\frac{\beta_1 R_1}{Z}}\frac{1}{d_1}U\left(Z_N\rho_{N-1}^{(2)} - \alpha_{010}^{(N,N-1)}\right), \quad (23)$$

$$y_1^{(2)} = -\frac{2i\omega T_{11}^{\Sigma}}{gQ_1\,\omega_{010}^{(1)}}\sqrt{\frac{\beta_1 R_1}{Z}}\frac{1}{d_1}U\left(Z_N\rho_{N-1}^{(1)} - \alpha_{010}^{(N,N-1)}\right), \quad (24)$$

where

$$g = \left(Z_1 - \alpha_{010}^{(1,2)}\rho_1^{(1)}\right)\left(Z_N\rho_{N-1}^{(2)} - \alpha_{010}^{(N,N-1)}\right)T_{22}^{\Sigma} - -\left(Z_1 - \alpha_{010}^{(1,2)}\rho_1^{(2)}\right)\left(Z_N\rho_{N-1}^{(1)} - \alpha_{010}^{(N,N-1)}\right)T_{11}^{\Sigma}\ldots \quad (25)$$

We introduced the two linearly independent grid functions $y_k^{(1)}, y_k^{(2)}$ which are the product of multipliers $\rho_k^{(1)}$ and $\rho_k^{(2)}$ (see (19)). These multipliers are the solutions of the nonlinear difference equation (18) with different initial values of $\rho_1^{(1)}$ and $\rho_1^{(2)}$. These initial values of $\rho_1^{(1)}$ and $\rho_1^{(2)}$ can be chosen arbitrarily. Therefore, we have a continuous set of the two linearly independent grid functions $y_k^{(1)}, y_k^{(2)}$, sum of which gives the same grid function $e_{010}^{(k)} = y_k^{(1)} + y_k^{(2)}$ for the given structure of the chain. In the process of synthesis we can change the structure of the chain in such way that $\rho_k^{(1)}, \rho_k^{(2)}$ and $y_1^{(1)}, y_1^{(2)}$ will take the required values and the desired electromagnetic field distribution ($e_{010}^{(k)}$) in a chain of coupled resonators will be realized.

It is a usual requirement to insure no reflected signal in steady-state, which corresponds to the matching the input transmission line to the considered chain

$$U_R = -\sqrt{\frac{\beta_1 Z}{R_1}}d_1\left(y_1^{(1)} + y_1^{(2)}\right) - U = 0\,. \quad (26)$$

Substituting (23) and (24) into (26), we obtain

$$\beta_1 = \frac{Q_1\,\omega_{010}^{(1)}}{2i\omega}\left[\frac{\omega^2}{\omega_{010}^{(1)2}} + i\frac{\omega(1+\beta_1)}{\omega_{010}^{(1)}Q_1} - (1-\alpha_{010}^{(1,1)}) + \alpha_{010}^{(1,2)}G\right], \quad (27)$$

where

$$G = \frac{\left(\rho_1^{(1)} - \rho_1^{(2)}G_N\right)}{\left(1 - G_N\right)}, \quad (28)$$

$$G_N = \frac{\left(Z_N\rho_{N-1}^{(1)} - \alpha_{010}^{(N,N-1)}\right)T_{11}^{\Sigma}}{\left(Z_N\rho_{N-1}^{(2)} - \alpha_{010}^{(N,N-1)}\right)T_{22}^{\Sigma}}\,. \quad (29)$$

From (27) it follows that the critical value of the coupling factor β_1 is

$$\beta_1 = Q_1\,\alpha_{010}^{(1,2)}\frac{\omega_{010}^{(1)}}{\omega}\operatorname{Im}G + 1\,, \quad (30)$$

and an additional condition is to be fulfilled

$$1 + \alpha_{010}^{(1,1)} - \frac{\omega^2}{\omega_{010}^{(1)2}} - \alpha_{010}^{(1,2)}\operatorname{Re}G = 0\,. \quad (31)$$

As β_1 is a real positive value, then $\operatorname{Im}G$ has a minimal value

$$\operatorname{Im}G > -\frac{\omega}{\omega_{010}^{(1)}Q_1\,\alpha_{010}^{(1,1)}}\,. \quad (32)$$

If the chain has a single input (standing wave structure), we can create the desired field distribution by choosing the values of $\rho_k^{(1)}$, $\rho_k^{(2)}$ and finding the geometrical parameters of resonators and coupling openings from the Riccati difference equation (18). Characteristics of the first resonator are determined by equations (30) and (31).

If the chain has two ports (traveling wave structure), there is additional possibilities for manipulating with field distribution. We can create the field distribution based on the one solution $y_k^{(1)}$ ($y_k^{(2)}$ =0). In this case the value of amplitude $e_{010}^{(k)}$ equals the value of amplitude $e_{010}^{(k-1)}$ multiplied by the factor $\rho_k^{(1)}$ (quasiperiodic structure):

$$e_{010}^{(k+1)} = \rho_k^{(1)} e_{010}^{(k)} = \prod_{s=1}^{k} \rho_s^{(1)} e_{010}^{(1)} . \qquad (33)$$

Such electromagnetic field distribution can be realized if the initial value of the second solution equals to zero

$$y_1^{(2)} = 0 . \qquad (34)$$

From (24) it follows that such condition must be fulfilled

$$Z_N \rho_{N-1}^{(1)} - \alpha_{010}^{(N,N-1)} = 0 . \qquad (35)$$

This equation determines the characteristics of the last resonator and the value of coupling with the output transmission line

$$1 + \alpha_{010}^{(N,N)} - \frac{\omega^2}{\omega_{010}^{(N)2}} - \frac{\alpha_{010}^{(N,N-1)} \operatorname{Re} \rho_{N-1}^{(1)*}}{\left|\rho_{N-1}^{(1)}\right|^2} = 0 , \qquad (36)$$

$$\beta_N = -Q_N \alpha_{010}^{(N,N-1)} \frac{\omega_{010}^{(N)}}{\omega} \frac{\operatorname{Im} \rho_{N-1}^{(1)*}}{\left|\rho_{N-1}^{(1)}\right|^2} - 1 , \qquad (37)$$

$$\operatorname{Im} \rho_{N-1}^{(1)*} < -\frac{\omega \left|\rho_{N-1}^{(1)}\right|^2}{Q_N \omega_{010}^{(N)} \alpha_{010}^{(N,N-1)}} . \qquad (38)$$

From (28) and (29) it follows that $G = \rho_1^{(1)}$ and the matching condition (27) takes the form

$$\left[Z_1 - \alpha_{010}^{(1,2)} \rho_1^{(1)} \right] = -\frac{2i\omega}{Q_1 \omega_{010}^{(1)}} \beta_1 . \qquad (39)$$

The equations (30) and (31) are also simplified

$$\beta_1 = Q_1 \alpha_{010}^{(1,2)} \frac{\omega_{010}^{(1)}}{\omega} \operatorname{Im} \rho_1^{(1)} + 1 , \qquad (40)$$

$$1 + \alpha_{010}^{(1,1)} - \frac{\omega^2}{\omega_{010}^{(1)2}} - \alpha_{010}^{(1,2)} \operatorname{Re} \rho_1^{(1)} = 0 , \qquad (41)$$

$$\operatorname{Im} \rho_1^{(1)} > -\frac{\omega}{\omega_{010}^{(1)} Q_1 \alpha_{010}^{(1,2)}} . \qquad (42)$$

The initial value $y_1^{(1)}$ (23) do not depend on the characteristic multipliers $\rho_k^{(2)}$

$$y_1^{(1)} = -\sqrt{\frac{R_1}{\beta_1 Z}} \frac{1}{d_1} U . \qquad (43)$$

It is important to note that from (37) and (40) it follows that the coupler is not a symmetric element. Only at $Q \to \infty$ the coupler do not reflect from two sides.

2. SOLUTIONS OF THE DIFFERENCE EQUATIONS FOR THE HOMOGENEOUS CHAIN

Characteristic multipliers $\rho_k^{(1)}$ and $\rho_k^{(2)}$ are the solutions of the nonlinear difference equation (18) with the initial values $\rho_1^{(1)} \neq \rho_1^{(2)}$. In the general case, these initial values can be chosen arbitrary. Input transmission line matching requirement imposes some restrictions (see (32), (38), (42)) on these values.

For the homogeneous chain, the equation (18) takes the form

$$\rho_{k+1}\rho_k - \rho_k \frac{Z}{\alpha_{010}} + 1 = 0 . \qquad (44)$$

This equation has two stationary points

$$\rho_k = \chi_{1,2} = \frac{Z}{2\alpha_{010}} \pm i\sqrt{1 - \left(\frac{Z}{2\alpha_{010}}\right)^2} = \exp\left(\pm i\varphi \mp \gamma\right) . (45)$$

The first stationary point is unstable ($\left|\chi_1\right|^{-2} > 1$), as the other one is attractive ($\left|\chi_2\right|^{-2} < 1$).

The solutions of the equation (44) is [61]

$$\rho_k = \frac{\left(\rho_1 - \chi_2\right)\chi_1^k - \left(\rho_1 - \chi_1\right)\chi_2^k}{\left(\rho_1 - \chi_2\right)\chi_1^{k-1} - \left(\rho_1 - \chi_1\right)\chi_2^{k-1}} . \quad (46)$$

If $\rho_1 = \chi_1$, then $\rho_k = \chi_1$, if $\rho_1 = \chi_2$, then $\rho_k = \chi_2$.

If we choose $\rho_1^{(1)} = \chi_1, \ \rho_1^{(2)} = \chi_2$, the grid function $y_k^{(1)}$ will correspond to a “forward traveling wave” and $y_k^{(2)}$ to a “backward one”.

$$\begin{aligned} y_k^{(1)} &= \chi_1^{k-1} y_1^{(1)}, \ 2 \le k \le N, \\ y_k^{(2)} &= \chi_2^{k-1} y_1^{(2)}, \ 2 \le k \le N. \end{aligned} \qquad (47)$$

If we choose $\rho_1^{(1)} \neq \chi_1$ and $\rho_1^{(2)} \neq \chi_2$, the grid functions $y_k^{(1)}$ and $y_k^{(2)}$ will correspond to some combinations of the “traveling waves”.

$$\begin{aligned} y_k^{(1)} &= \frac{\left(\rho_1^{(1)} - \chi_2\right)\chi_1^{k-1} - \left(\rho_1^{(1)} - \chi_1\right)\chi_2^{k-1}}{\chi_1 - \chi_2} y_1^{(1)}, \ 2 \le k \le N; \\ y_k^{(2)} &= \frac{\left(\rho_1^{(2)} - \chi_2\right)\chi_1^{k-1} - \left(\rho_1^{(2)} - \chi_1\right)\chi_2^{k-1}}{\chi_1 - \chi_2} y_1^{(2)}, \ 2 \le k \le N, \end{aligned} \qquad (48)$$

where $\rho_1^{(1)} \neq \rho_1^{(2)}$.

The sum of these grid functions ($y_k^{(1)} + y_k^{(2)}$) is a grid function that do not depend on $\rho_1^{(1)}$ and $\rho_1^{(2)}$.

3. SYNTHESIS OF THE COUPLED RESONATOR CHAIN WITH DESIRED ELECTROMAGNETIC FIELD DISTRIBUTION

In the CCM electromagnetic field distribution is defined by the amplitudes of the basic oscillations [45 - 48]. For description the lowest passband we have to choose the amplitudes of E_{010} mode as basic oscillations.

In the considered above approach the distribution of amplitude $e_{010}^{(k)}$ is defined by the characteristic multipliers $\rho_k^{(1)}$, $\rho_k^{(2)}$ and initial values of grid functions $y_1^{(1)}$, $y_1^{(2)}$. So, during the synthesis process, we must choose a chain structure (parameters of resonators and coupling elements) such that the coefficients $\rho_k^{(1)}$, $\rho_k^{(2)}$ and $y_1^{(1)}$, $y_1^{(2)}$ will take on the required values and the desired value of amplitudes $e_{010}^{(k)}$ in a chain of coupled resonators will be realized.

Below we will consider the chain of cylindrical resonators that are connected via circular central openings in the walls – the disk loaded waveguides (DLW)[3]. It was shown that the DLWs, that are usually used in linacs, with disk spacing large enough ($d \ge \lambda/3$) can be describe with sufficient accuracy by the difference equation (6) [62]. Appropriate values of the coupling coefficients $\alpha_{010}^{(k,k)}, \alpha_{010}^{(k,k+1)}$ at fixed frequency can be approximated by some functions of geometrical sizes. Calculations on the base of the CCM show that for the most often used in linacs DLWs such approximations can be used

$$\alpha_{010}^{(k,k)} = -\alpha \frac{u_k \bar{p}_k + u_{k+1} \bar{p}_{k+1}}{\tilde{b}_k^2 \tilde{d}_k},$$

$$\alpha_{010}^{(k,k-1)} = \alpha \frac{u_k}{\tilde{b}_k^2 \tilde{d}_k}, \tag{49}$$

$$\alpha_{010}^{(k,k+1)} = \alpha \frac{u_{k+1}}{\tilde{b}_k^2 \tilde{d}_k},$$

where $u_k = \dfrac{\alpha a_k^3}{b_*^2 d_*} p_k^{(c)}$, $\tilde{b}_k = \dfrac{b_k}{b_*}$, $\tilde{d}_k = \dfrac{d_k}{d_*}$; a_k – the hole radius between $k-1$ and k resonators; b_k – the radius of k cylindrical resonator; d_k – the resonator length, $b_* = c\dfrac{\lambda_{01}}{\omega}$, d_* – normalizing parameters, $\omega_{010}^{(k)} = c\dfrac{\lambda_{01}}{b_k}$, $J_0(\lambda_{01}) = 0$, $\alpha = \dfrac{2}{3\pi J_1^2(\lambda_{01})}$, $\bar{p}_k = \dfrac{p_k^{(s)}}{p_k^{(c)}}$.

Analysis shows that we can consider parameters $p_k^{(s)}, p_k^{(c)}$ as the functions of the geometric sizes of the diaphragms only (the opening radius a_k, the thickness t_k of the diaphragm between $k-1$ and k resonators and the radius of the rounding of the disk hole edges).

For t_k = 0.4 cm, d_k = 3.0989 cm, parameters $p_k^{(s)}, p_k^{(c)}$ can be represented[4] as

$$\begin{aligned} p_k^{(s)} &= 0.0142 a_k^2 - 0.1329 a_k + 0.9133, \\ p_k^{(c)} &= -0.0928 a_k^2 + 0.4491 a_k - 0.0444. \end{aligned} \tag{50}$$

The parameter $p_k^{(c)}$ determines the deviation of the dependence of the coupling coefficient $\alpha_{010}^{(k,k-1)}$ on a_k from the law a_k^3, $p_k^{(s)}$ – the deviation of the dependence of the resonator frequency shift due the hole in the k – disk on a_k from the law a_k^3 (see (49)).

The equation (18) after separation of the real and imaginary parts and making some transformations takes the form

$$\begin{aligned} &\bar{b}_k^4 - \bar{b}_k^2 - \frac{\bar{b}_k^3 \left[\left|\rho_k^{(1)}\right| \cos(\varphi_k) - \bar{p}_{k+1} \right]}{\left|\rho_k^{(1)}\right| \sin(\varphi_k) Q_k} + \\ &+ u_k \frac{\left[\left|\rho_k^{(1)}\right| \left(\sin(\varphi_k + \varphi_{k-1}) - \sin(\varphi_k)\bar{p}_k \left|\rho_{k-1}^{(1)}\right| \right) - \sin(\varphi_{k-1})\bar{p}_{k+1} \right]}{\sin(\varphi_k) \left|\rho_k^{(1)}\right| \left|\rho_{k-1}^{(1)}\right| \bar{d}_k} = 0, \end{aligned} \tag{51}$$

$$\begin{aligned} u_{k+1} &= u_k \frac{\sin(\varphi_{k-1})}{\left|\rho_k^{(1)}\right| \left|\rho_{k-1}^{(1)}\right| \sin(\varphi_k)} - \\ &- \frac{\bar{b}_k^3 \bar{d}}{\left|\rho_k^{(1)}\right| \sin(\varphi_k) Q_k}, \; 2 \le k \le N-1. \end{aligned} \tag{52}$$

Parameters of the first and last resonators can be found from equations

$$\bar{b}_1^4 - \bar{b}_1^2 - \frac{u_2}{\bar{d}_1}\bar{p}_2 + \frac{u_2}{\bar{d}_1}\left|\rho_1^{(1)}\right| \cos(\varphi_1^{(1)}) = 0, \tag{53}$$

$$\beta_1 = 1 + \frac{u_2}{\bar{b}_1^3 \bar{d}_1} Q_1 \left|\rho_1^{(1)}\right| \sin(\varphi_1^{(1)}), \tag{54}$$

$$\bar{b}_N^4 - \bar{b}_N^2 - \frac{u_N}{\bar{d}_N \left|\rho_{N-1}^{(1)}\right|} \left[\bar{p}_N \left|\rho_{N-1}^{(1)}\right| - \cos(\varphi_{N-1}^{(1)}) \right] = 0, \tag{55}$$

$$\beta_N = \frac{u_N}{\bar{d}_N \bar{b}_N^3 \left|\rho_{N-1}^{(1)}\right|} Q_N \sin(\varphi_{N-1}^{(1)}) - 1. \tag{56}$$

By specifying the values of the multipliers $\rho_k^{(1)}$ and a certain set of resonator parameters, from equations (51) - (56) we can find the missing set of parameters. Amplitudes $e_{010}^{(k)}$ in the chain with this full set of resonator parameters will distribute along structure in accordance with the formula (33).

Proposed approach can be used for developing of different inhomogeneous DLWs.

Among the slow wave waveguides, the most complex structure have the ones with phase velocities that change along the longitudinal coordinate (an injector in linacs [1, 63 - 65], TWT [2, 6 - 13]). They must ensure not only the acceleration (deceleration) of particles, but also their grouping into small bunches. Injector sections for linacs are usually designed with a constant phase shift between cells, but with a variable length of resonators. The proposed above approach gives possibility to design the structures with the inhomogeneous phase shifts.

As example, we considered the possibility of creating smooth transition between the DLW with $\varphi_1 = 14\pi/15$ and the DLW with $\varphi_2 = 2\pi/3$ ($Q = \infty$). For f = 2856 MHz, $d = 3.0989$ cm, $t = 0.4$ cm the phase velocity changes from $0.71c$ to c.

We chose two sequences for $\rho_k^{(1)}$. The first one (the sequence N1) is

$$\rho_k^{(1)} = \begin{cases} \exp(i\varphi_1), & k < s \\ \exp(i13\pi/15), & k = s \\ \exp(i12\pi/15), & k = s+1 \\ \exp(i11\pi/15), & k = s+2 \\ \exp(i\varphi_2), & k >= s+3 \end{cases}. \tag{57}$$

The second one (the sequence N2) is

$$\rho_k^{(1)} = \begin{cases} \exp(i\varphi_1), & k < s \\ 0.949\exp(i13\pi/15), & k = s \\ 0.949\exp(i12\pi/15), & k = s+1 \\ 0.949\exp(i11\pi/15), & k = s+2 \\ \exp(i\varphi_2), & k >= s+3 \end{cases}. \tag{58}$$

Geometry calculated on the basis of equations (51) - (52) are presented in Table. Geometry used for calcula-

[3]DLW structures are the most often used in linacs and represent the chain of cavities in which the phase varies smoothly from cell to cell in such way, that an accelerated particle constantly locates in accelerating field.

[4]For simplicity, we will consider the case without of the rounding of the disk hole edges. For taking into account the rounding of the disk hole edges.

tion on the base the CCM differs in the homogeneous parts less than 2 μm.

	N1		N2	
	a_k	b_k	a_k	b_k
k=s-2	1.4	4.19633	1.4	4.19633
k=s-1	1.4	4.19633	1.4	4.19633
k=s	1.4	4.15983	1.4	4.16117
k=s+1	1.16863	4.10769	1.18496	4.11500
k=s+2	1.06051	4.08584	1.10506	4.09765
k=s+3	0.99754	4.07353	1.06792	4.08760
k=s+4	0.95872	4.06882	1.04010	4.08261
k=s+5	0.95872	4.06882	1.04010	4.08261
	R=7.29E-003		R=7.64E-003	

Calculation results of the longitudinal electric field distribution in the resonator centres obtained on the basis of the CCM are presented in Figs. 2 and 3 (s =11). We see that the longitudinal electric field has nearly the same phase distribution as the chosen one for the $e_{010}^{(k)}$ amplitudes. We can also see that for the same phase distribution we can create different amplitude distributions[5] which are desirable for different types of injectors – the first distribution with the increasing amplitude [65] and the second one with the constant amplitude [64].

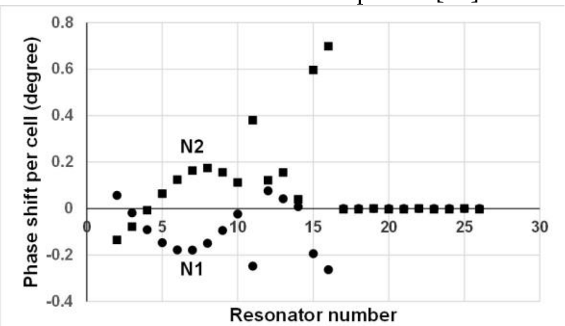

Fig. 2

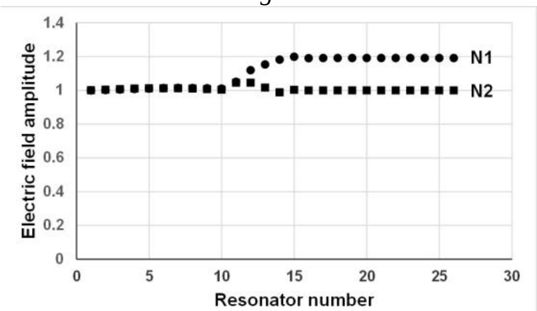

Fig. 3

Difference between the specified phase shifts per cells and calculated on the basis of the CCM.

Amplitude distributions calculated on the basis of the CCM.

For high current linacs it is needed to develop accelerating sections with constant phase shifts between the cells ($\varphi_k = const$) and the amplitudes of the electric field increasing along the structure ($\left|e_{010}^{(k)}\right| \neq const$) (see, for example, [62, 66]). Setting the law of amplitude variation along the structure $\left|e_{010}^{(k)}\right|$, we can find the full set of resonator parameters from equations (51) - (52) with such characteristic multipliers

$$\rho_k^{(1)} = \frac{\left|e_{010}^{(k+1)}\right|}{\left|e_{010}^{(k)}\right|}\exp(i\varphi). \qquad (59)$$

4. FORWARD AND BACKWARD FIELDS

In light of work on the new matrix form of second-order linear difference equations [51], we can look at the problem of expanding the electromagnetic field into the forward and backward waves in each cell of the inhomogeneous chain of resonators from the new point of view.

We have shown that in the chain that is described by the second-order difference equation (6) we can realize any reasonable ($a_n \exp(i\varphi_n)$) amplitude-phase distribution that is the product of the characteristic multipliers

$$e_{010}^{(k)} = y_k^{(1)} = y_1^{(1)}\prod_{s=1}^{k-1}\rho_s^{(1)},\ 2 \le k \le N. \qquad (60)$$

For that we have to choose the resonator and opening sizes that are fulfilled the relations (51) - (52) and the parameters of couplers (53) - (56). At such geometrical sizes the second independent solution of the equation (6) equals to zero. As there is no reflection from the input coupler, we can consider that RF power transmits through the structure without reflection. This electromagnetic field we can consider as the "forward" one.

Let's suppose that the output coupler is detuned ($Z_N\rho_{N-1}^{(1)} - \alpha_{010}^{(N,N-1)} \neq 0$). What changes will occur in the distribution of the amplitudes $e_{010}^{(k)}$?

From (10), (19) it follows that the new field $\tilde{y}_k^{(2)}$ will appear in addition to the "forward" field

$$\begin{gathered}\tilde{e}_{010}^{(k)} = \tilde{y}_k^{(1)} + \tilde{y}_k^{(2)} = \\ = \tilde{y}_1^{(1)}\prod_{s=1}^{k-1}\rho_s^{(1)} + \tilde{y}_k^{(2)}\prod_{s=1}^{k-1}\rho_s^{(2)},\ 2 \le k \le N.\end{gathered} \qquad (61)$$

The amplitude of the "forward" field $\tilde{y}_1^{(1)}$ differs from the unperturbed one $y_1^{(1)}$ and depends on the initial value of the characteristic multiplier $\rho_1^{(2)}$ and the value of the output coupler detuning (see (23), (25)). The characteristic multipliers $\rho_s^{(2)}$ are the solution of the difference equation (18) with defined coefficients and with the initial value $\rho_1^{(2)}$ which we can choose arbitrary.

In the limit $Q = \infty$, when Z_k is the real value and the couplers become the symmetrical elements, there is a reasonable background to consider that the amplitude of the "forward" field $\tilde{y}_1^{(1)}$ do not depends on the tuning of the output coupler. Then from (23) and (25) we obtain the initial value $\rho_1^{(2)}$

$$Z_1 - \alpha_{010}^{(1,2)}\rho_1^{(2)} = 0. \qquad (62)$$

As β_1 / Q_1 has a finite value (see (39), (40)), we have

$$Z_1 - \alpha_{010}^{(1,2)}\rho_1^{(1)} = -2i\alpha_{010}^{(1,2)}\,\mathrm{Im}\,\rho_1^{(1)}. \qquad (63)$$

From (62) it follows that

$$\rho_1^{(2)} = \mathrm{Re}\,\rho_1^{(1)} - i\,\mathrm{Im}\,\rho_1^{(1)} = \rho_1^{(1)*} \qquad (64)$$

[5] Many possible structures realize the variable phase velocity. As the power flow must be constant ($Q = \infty$), then needed distribution of electric field amplitudes determine the law of change of the aperture sizes. We can realize the increase of the phase velocity at the constant (or increasing) apertures, but the amplitudes have to increase strongly (see, for example, [8, 9]).

and, as Z_k is the real value, from (18) we obtain

$$\rho_k^{(2)} = \rho_k^{(1)*}. \quad (65)$$

Therefore, the additional field that arises due to reflection from the output coupler becomes the conventional backward field.

The problem becomes more difficult at $Q \neq \infty$. The characteristic multipliers $\rho_k^{(2)}$ that define the structure of additional field are the solution of the difference equation (18). This equation we can rewrite as

$$\rho_{k+1}^{(2)} = -\frac{\alpha_{010}^{(k+1,k)}}{\alpha_{010}^{(k+1,k+2)}\rho_k^{(2)}} + \frac{\alpha_{010}^{(k+1,k)}}{\alpha_{010}^{(k+1,k+2)}\rho_k^{(1)}} + \rho_{k+1}^{(1)}, 1 \le k \le N-2, \quad (66)$$

where $\rho_1^{(2)} \neq \rho_1^{(1)}$ is a free parameter. If $\alpha_{010}^{(k+1,k)} = \alpha_{010}^{(k+1,k+2)}$, $1 \le k \le N-2$ (the homogeneous chain) and $\rho_k^{(1)} = \exp(i\varphi - \gamma)$ ($1 \le k \le N-1$) we have the solution of the equation (66) in the analytical form

$$\rho_k^{(2)} = \frac{1}{\rho_k^{(1)}} = \exp(-i\varphi + \gamma), \ 1 \le k \le N-1. \quad (67)$$

In the general case, the equation (66) has no simple solution. As $\rho_1^{(2)}$ is a free parameter and there is not reasonable background for its choice, then the structure of additional field and its amplitude $\tilde{y}_1^{(2)}$ are not define uniquely. Moreover, the amplitude of the "forward" field $\tilde{y}_1^{(1)}$ which depend on $\rho_1^{(2)}$ and $\rho_{N-1}^{(2)}$ (see (23)) is not define uniquely, too.

Therefore, in the frame of considered model the separation of the electromagnetic field into "forward" and "backward" components in the inhomogeneous chain of resonators is not define uniquely. It is needed to apply some additional criteria for defining the properties of "reflected" fields.

CONCLUSIONS

We presented the novel approach to the synthesis of the electromagnetic field distribution in a chain of coupled resonators that can be described by the second-order difference equation for amplitudes of expansion of the electromagnetic field. This approach is based on the new matrix form of the solutions of the second-order difference equations that give possibility to construct the two linearly independent solutions. Setting the structure of one solution, from the Riccati equation we can find the electrodynamical characteristics of resonators and coupling holes, at which the desired distribution of amplitudes is realized. Several examples show that proposed approach can be useful in solving different physical problems. On the base of this approach we also considered the problem of separation of the electromagnetic field into "forward" and "backward" components in the inhomogeneous chain of resonators. It was shown that in the frame of considered model such separation is not defined uniquely.

The problem of creating a special field distribution is attracting attention of different researchers. This problem arises at the construction and design of new materials including nano-materials with so called cloaking properties (see, for example, [67 - 70]). The proposed approach can be used as a numerical tool to design 1-D devices and materials that manipulate waves in a specified manner.

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НОВЫЙ МЕТОД СИНТЕЗА РАСПРЕДЕЛЕНИЯ ЭЛЕКТРОМАГНИТНОГО ПОЛЯ В ЦЕПОЧКЕ СВЯЗАННЫХ РЕЗОНАТОРОВ

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Разработан новый метод синтеза распределения электромагнитного поля в цепочке связанных резонаторов. Этот метод базируется на новой матричной форме решений разностного уравнения второго порядка. Для случая, когда цепочку связанных резонаторов можно описать разностным уравнением второго порядка для амплитуд разложения электромагнитного поля, два независимых решения могут быть построены на основе решений нелинейного уравнения Риккати. Задавая структуру одного решения, из уравнения Риккати можно найти электродинамические характеристики резонаторов и отверстий связи, при которых реализуется необходимое распределение амплитуд. На основе этого подхода рассмотрена проблема разделения электромагнитного поля на «прямые» и «обратные» компоненты в неоднородной цепочке резонаторов. Было показано, что в рамках рассматриваемой модели такое разделение не определяется однозначно.

НОВИЙ МЕТОД СИНТЕЗУ РОЗПОДІЛУ ЕЛЕКТРОМАГНІТНОГО ПОЛЯ В ЛАНЦЮЖКУ ЗВ'ЯЗАНИХ РЕЗОНАТОРІВ

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Розроблено новий метод синтезу розподілу електромагнітного поля в ланцюжку пов'язаних резонаторів. Цей метод базується на новій матричній формі рішень різницевого рівняння другого порядку. Для випадку, коли ланцюжок пов'язаних резонаторів можна описати різницевим рівнянням другого порядку для амплітуд розкладання електромагнітного поля, два незалежних рішення можуть бути побудовані на основі рішень нелінійного рівняння Ріккаті. Ставлячи структуру одного рішення, з рівняння Ріккаті можна знайти електродинамічні характеристики резонаторів і отворів зв'язку, при яких реалізується необхідний розподіл амплітуд. На підставі такого підходу розглянуто проблему розділення електромагнітного поля на «прямі» та «зворотні» компоненти в неоднорідному ланцюзі резонаторів. Показано, що в рамках розглянутої моделі таке розділення не визначено однозначно.