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Two-Point Functions on Deformed Spacetime*

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Abstract. We present a review of the one-loop photon (Π) and neutrino (Σ) two-point functions in a covariant and deformed U(1) gauge-theory on the 4-dimensional noncommutative spaces, determined by a constant antisymmetric tensor $\theta^{\mu\nu}$, and by a parameter-space (κ_f, κ_g), respectively. For the general fermion-photon $S_f(\kappa_f)$ and photon self-interaction $S_g(\kappa_g)$ the closed form results reveal two-point functions with all kind of pathological terms: the UV divergence, the quadratic UV/IR mixing terms as well as a logarithmic IR divergent term of the type $\ln(\mu^2(\theta p)^2)$. In addition, the photon-loop produces new tensor structures satisfying transversality condition by themselves. We show that the photon two-point function in the 4-dimensional Euclidean spacetime can be reduced to two finite terms by imposing a specific full rank of $\theta^{\mu\nu}$ and setting deformation parameters $(\kappa_f, \kappa_g) = (0, 3)$. In this case the neutrino two-point function vanishes. Thus for a specific point $(0, 3)$ in the parameter-space (κ_f, κ_g), a covariant θ-exact approach is able to produce a divergence-free result for the one-loop quantum corrections, having also both well-defined commutative limit and point-like limit of an extended object.

Key words: non-commutative geometry; photon and neutrino physics; non-perturbative effects

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1 Introduction

Prior to the late 1990s, the possibility of experimentally testing the nature of quantum gravity was not seriously contemplated because of the immensity of the Planck scale ($E = 1.2 \times 10^{19}$ GeV). Now this view has been significantly alternated: A possibility to have a string scale significantly below the Planck scale in a braneworld scenario [8] become the core of current experimental protocols searching for quantum-gravity phenomena (notably production of black holes) at the Large Hadron Collider at CERN. Almost simultaneously, another possibility arose, where it was pointed out that distant astrophysical objects with rapid time variations could provide the most sensitive opportunities to probe *very high energy* scales, i.e., almost the near-Planck scale physics [7].

Another route to search for quantum-gravity effects involves a *spontaneous breaking of Lorentz symmetry* in string theory, when a tensor field acquires a vacuum expectation value (vev). Unlike the case of scalars, these tensor vevs do carry spacetime indices, causing the interaction represented by the Standard Model (SM) fields coupled to these vevs to depend on the direction or velocity of the said fields. Stated differently, these background vevs bring about the breakdown of Lorentz symmetry. This entails a distinctive fact of most of Lorentz violating (LV) theories on the existence of *preferred reference frames*, where the equations of motion take on the simplest

*This paper is a contribution to the Special Issue on Deformations of Space-Time and its Symmetries. The full collection is available at http://www.emis.de/journals/SIGMA/space-time.html form. In contrast to the notion of the *motionless aether* from the end of the 19th century, we have a rather unique example of such a frame in modern cosmology today: the frame in which the Cosmic Microwave Background Radiation (CMBR) looks isotropic. From the determination of the detailed spectrum of the CMBR dipole (generally interpreted as a Doppler shift due to the Earth's motion), our velocity with respect to that frame, of order of 10^{-3} c, can be inferred.

An eligible way to infer the preferred reference frame predicted by generic quantum gravity frameworks, is to study dispersion relations for propagating particles. Instead of propagating (in a vacuum) with the speed of light, in Lorentz violating theories one expects an energy-dependent velocity $\mathbf{v}(E)$ for massless particles. This is a consequence of the loss of Lorentz covariance in the dispersion relations for propagating particles, with the implication that a specific form $\mathbf{v}(E)$ can be at best valid only in one specific reference frame. Thus, a preferred reference frame in which the equation of motions possess the simplest form is singled out. This opens up a unique possibility to study constraints on violations of Lorentz invariance. The modification of the photon velocity of the form $\mathbf{v}(E)$ would induce time lag for photons of different energies, which could be subsequently detected if such particles can propagate at cosmological distances. Such an alternation of the photon velocities has already been obtained in *Loop quantum gravity* (being another popular approach to quantum gravity) [63, 64] as well as in heuristic models of space-time foam inspired by string theory [7].

One of the most striking observation regarding spontaneous Lorentz breaking via tensor vevs in the string theory framework is that it can be formulated as deformed field theories. Specifically, a low-energy limit is identified where the entire boson-string dynamics in a Neveu-Schwartz condensate is described by a minimally coupled supersymmetric gauge theory on noncommutative (NC) space [67] such that the mathematical framework of noncommutative geometry/field theory [25, 26, 49, 70] does apply. In such a scenario, noncommutative Dirac–Born–Infeld (DBI) action is realized as a special limit of open strings in a background $B^{\mu\nu}$ field, in which closed string (i.e. gravitational) modes are decoupled, leaving only open string interactions. Since in string theory $B^{\mu\nu}$ field is a rather mild background, the antisymmetric tensor $\theta^{\mu\nu}$ governing spacetime noncommutative deformations is not specified, and therefore the scale of noncommutativity could, in principle, lie anywhere between the weak and the Planck scale [15, 26, 70, 71]. It is thus of crucial importance to set a bound on this scale from experiments [71].

It is important to stress the invariance of the theory under coordinate changes, i.e., the invariance under an observer transformation (where the coordinates of the observer are boosted or rotated). This transformation is not related to the concept of Lorentz violation since in this transformation the properties of the background fields transform to a new set of coordinates as well. On the other hand, an invariance under *active* or *particle* transformation, where both fields and states are being transformed, is now broken by the background fields themselves, leading to the concept of Lorentz violation.

In a simple model of the noncommutative spacetime we consider coordinates x^μ as the Hermitian operators $\hat{x}^\mu$ [45],

$$[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu}, \qquad |\theta^{\mu\nu}| \sim \Lambda_{\rm NC}^{-2}, \tag{1.1}$$

where $\theta^{\mu\nu}$ is *a constant* real antisymmetric matrix of dimension *length*^{2}, and $\Lambda_{\rm NC}$ being the scale of noncomutativity. It is straightforward to formulate field theories on such noncommutative spaces as a deformation of the ordinary field theories [15, 26, 70, 71]. The noncommutative deformation is implemented by replacing the usual pointwise product of a pair of fields $\phi(x)$ and $\psi(x)$ by the star($\star$)-product in any action:

$$\phi(x)\psi(x) \longrightarrow (\phi \star \psi)(x) = \phi(x)\psi(x) + \mathcal{O}(\theta, \partial\phi, \partial\psi).$$

The specific Moyal–Weyl $\star$-product is relevant for the case of a constant antisymmetric noncommutative deformation tensor $\theta^{\mu\nu}$ and is defined as follows:

$$(\phi\star\psi)(x) = e^{\frac{i}{2}\theta^{\mu\nu}\partial^{\eta}_{\mu}\partial^{\xi}_{\nu}}\phi(x+\eta)\psi(y+\xi)\big|_{\eta,\xi\to 0} \equiv \phi(x)e^{\frac{i}{2}\overleftarrow{\partial_{\mu}}\theta^{\mu\nu}\overrightarrow{\partial_{\nu}}}\psi(x). \tag{1.2}$$

The $\star$-product has also an alternative integral formulation, making its non-local character more transparent. The coordinate-operator commutation relation (1.1) is then realized by the star($\star$)-commutator of the usual coordinates

$$[\hat{x}^{\mu},\hat{x}^{\nu}] = [x^{\mu} \overset{\star}{,} x^{\nu}] = i\theta^{\mu\nu},$$

implying the following spacetime uncertainty relations

$$\Delta x^{\mu}\Delta x^{\nu} \geq \frac{1}{2}|\theta^{\mu\nu}|.$$

The above procedure introduces in general the field operators ordering ambiguities and breaks ordinary gauge invariance.

Since commutative local gauge transformations for the D-brane effective action do not commute with $\star$-products, it is important to note that the introduction of $\star$-products induces field operator ordering ambiguities and also breaks ordinary gauge invariance in the naive sense. However both the commutative gauge symmetry and the deformed noncommutative gauge symmetry describe the same physical system, therefore they are expected to be equivalent. This disagreement is remedied by a set of nonlocal and highly nonlinear parameter redefinitions called Seiberg–Witten (SW) map [67]. This map promotes not only the noncommutative fields and composite operators of the commutative fields, but also the noncommutative gauge transformations as the composite operators of the commutative gauge fields and gauge transformations. Through this procedure, deformed gauge field theories can be defined for arbitrary gauge groups/representations. Consequently, building semi-realistic deformed particle physics models are made much easier.

It is reasonable to expect that the new underlying mathematical structures in the NC gauge field theories (NCGFT) could lead to profound observable consequences for the low energy physics. This is realized by the perturbative loop computation first proposed by Filk [28]. There are also famous examples of running of the coupling constant in the U(1) NCGFT in the $\star$-product formalism [53], and the exhibition of fascinating dynamics due to the celebrated ultraviolet/infrared (UV/IR) phenomenon, without [56, 60], and with the Seiberg–Witten map [38, 40, 41, 57, 66] included. Precisely, in [15, 60, 56] it was shown for the first time how UV short distance effects, considered to be irrelevant, could alter the IR dynamics, thus becoming known as the UV/IR mixing. Some significant progress on UV/IR mixing and related issues has been achieved [16, 17, 32, 50, 58] while a proper understanding of loop corrections is still sought for.

More serious efforts on formulating NCQFT models with potential phenomenological influence have started for about a decade ago. Strong boost came from the Seiberg–Witten map [67] based enveloping algebra approach, which enables a direct deformation of comprehensive phenomenological models like the standard model or GUTs [9, 12, 24]. It appeared then relevant to study ordinary gauge theories with the additional couplings inspired by the SW map/deformation included [9, 12, 24].

To include a reasonably relevant part of all SW map inspired couplings, one usually calls for an expansion and cut-off procedure, that is, an expansion of the action in powers of $\theta^{\mu\nu}$ [9, 12, 24, 36, 52, 74]. Next follows theoretical studies of one loop quantum properties [10, 13, 14, 19, 21, 22, 23, 30, 48, 51, 54, 55], as well as studies of some new physical phenomena, like breaking of Landau–Yang theorem, [3, 4, 5, 20, 59, 61, 65], etc. It was also observed that allowing a deformation-freedom via varying the ratio between individual gauge invariant terms could improve the renormalizability at one loop level [23, 48].

The studies on phenomenology (possible experimental signal/bounds on noncommutative background) started parallel to the pure theoretical developments of NCQFTs. The majority of the accelerator processes had been surveyed up to the second power in $\theta^{\mu\nu}$ [3, 4, 5, 61]. The processes involving photons in noncommutative U(1) gauge theory now involve corrections to the known processes, since the new couplings, of which the most distinctive being the various photon self couplings, now emerge in the noncommutative background even at tree level. Such couplings might give rise to novel processes (normally forbidden in the standard theory) or to provide new channels in the already known processes.

The formulation of the SW-mapped actions has recently been made exact with respect to the noncommutative parameter $\theta^{\mu\nu}$ [57, 66, 75], offering thus an opportunity to compute various processes across full energy scale [42, 43]. Accordingly, one should no longer rely on the expansion in powers in θ, which could be especially beneficial in case when the quantum gravity scale is not so tantalizing close to the Planck scale. Thus, in this and several prior work(s) we formulate the θ-exact model action employing formal powers of fields [11, 31, 37, 42, 43, 47, 57, 62, 75], aiming at the same time to keep the nonlocal nature of the modified theory. Introduction of a nonstandard momentum dependent quantity of the type $\sin^2(p\theta k/2)/(p\theta k/2)^2$ into the loop integrals makes these theories drastically different from their θ-expanded cousins, being thus interesting for pure field theoretical reasons. The deformation-freedom parameters (ratios of weight-parameters of each gauge invariant terms in the actions) are found to be compatible with the θ-exact action therefore included to study their possible effects on divergence cancelation(s).

In this review we present closed forms for fermion-loop and photon-loop corrections to the photon and the neutrino two-point functions using dimensional regularization technique and we combine parameterizations of Schwinger, Feynman, and modified heavy quark effective theory parameterization (HQET) [33]. Both two-point functions were obtained as a function of unspecified number of the integration dimensions D. Next we specify gauge field theory dimension 4, and discuss the limits $D \to 4$.

The review is structured as follows: In the following section we describe generalized deformation freedom induced actions, and we give the relevant Feynman rules. Sections 3 and 4 are devoted to the computation/presentation/discussions of photon and neutrino two-point functions containing the fermion and the photon loop. Section 5 is devoted to discussions and conclusions.

2 The model construction

The main principle that we are implementing in the construction of our θ-exact noncommutative model is that electrically neutral matter fields will be promoted via hybrid SW map deformations [39] to the neutral noncommutative fields that couple to photons and transform in the adjoint representation of $\mathrm{U}_\star(1)$. We consider a U(1) gauge theory with a neutral fermion which decouples from the gauge boson in the commutative limit. We specify the action and deformation as a *minimal* θ-exact completion of the prior first order in θ models [12, 23, 24, 59, 65], i.e. the new (inter-)action has the prior tri-particle vertices as the leading order.

In the tree-level neutrino-photon coupling processes only vertices of the form $\bar\psi a\psi$ contribute, therefore an expansion to lowest nontrivial order in a_μ (but all orders in θ) is enough. There are at least three known methods for θ-exact computations: The closed formula derived using deformation quantization based on Kontsevich formality maps [47], the relationship between open Wilson lines in the commutative and noncommutative picture [57, 62], and direct recursive computations using consistency conditions. For the lowest nontrivial order a direct deduction from the recursion and consistency relations

$$\begin{aligned}
&\delta_\Lambda A_\mu = \partial_\mu \Lambda - i[A_\mu \stackrel{\star}{,} \Lambda] \equiv A_\mu[a_\mu + \delta_\lambda a_\mu] - A_\mu[a_\mu], \\
&\delta_\Lambda \Psi = i[\Lambda \stackrel{\star}{,} \Psi] \equiv \Psi[a_\mu + \delta_\lambda a_\mu, \psi + \delta_\lambda \psi] - \Psi[a_\mu, \psi], \\
&\Lambda[[\lambda_1, \lambda_2], a_\mu] = [\Lambda[\lambda_1, a_\mu] \stackrel{\star}{,} \Lambda[\lambda_2, a_\mu]] + i\delta_{\lambda_1}\Lambda[\lambda_2, a_\mu] - i\delta_{\lambda_2}\Lambda[\lambda_1, a_\mu],
\end{aligned}$$

with the ansatz

$$\begin{aligned}
&\Lambda = \hat{\Lambda}[a_\mu]\lambda = \big(1 + \hat{\Lambda}^1[a_\mu] + \hat{\Lambda}^2[a_\mu] + \mathcal{O}\big(a^3\big)\big)\lambda, \\
&\Psi = \hat{\Psi}[a_\mu]\psi = \big(1 + \hat{\Psi}^1[a_\mu] + \hat{\Psi}^2[a_\mu] + \mathcal{O}\big(a^3\big)\big)\psi,
\end{aligned}$$

is already sufficient. Capital letters denote noncommutative objects, small letters denote commutative objects, hatted capital letters denote differential operator maps from the latter to the former. In particular, here $\hat{\Psi}[a_\mu]$ and $\hat{\Lambda}[a_\mu]$ are gauge-field dependent differential operators that we shall now determine: Starting with the fermion field Ψ, at lowest order we have

$$i[\lambda \stackrel{\star}{,} \psi] = \hat{\Psi}[\partial\lambda]\psi.$$

Writing the $\star$-commutator explicitly as

$$\begin{aligned}
[\phi \stackrel{\star}{,} \psi] &= \phi(x)\left(e^{i\frac{\partial_x\theta\partial_y}{2}} - e^{-i\frac{\partial_x\theta\partial_y}{2}}\right)\psi(y)\Big|_{x=y} = 2i\phi(x)\sin\left(\frac{\partial_x\theta\partial_y}{2}\right)\psi(y)\Big|_{x=y} \\
&= i\theta^{ij}\left(\frac{\partial\phi(x)}{\partial x^i}\right)\frac{\sin\left(\frac{\partial_x\theta\partial_y}{2}\right)}{\frac{\partial_x\theta\partial_y}{2}}\left(\frac{\partial\psi(y)}{\partial y^j}\right)\Big|_{x=y},
\end{aligned}$$

we observe that

$$\hat{\Psi}[a_\mu] = -\theta^{ij} a_i \star_2 \partial_j,$$

will fulfill the consistency relation. The generalized star-product $\star_2$, appearing in the above, is defined, respectively, as [38, 40, 57, 66]:

$$\phi(x) \star_2 \psi(x) = \frac{\sin\frac{\partial_1\theta\partial_2}{2}}{\frac{\partial_1\theta\partial_2}{2}}\phi(x_1)\psi(x_2)\Big|_{x_1=x_2=x}.$$

Here $\star$-product (1.2) is associative but noncommutative, while $\star_2$ is commutative but nonassociative. The $\star$-commutator can then be rewritten as the following $\star_2$-products

$$[\phi \stackrel{\star}{,} \psi] = i\theta^{ij}\partial_i\phi \star_2 \partial_j\psi. \tag{2.1}$$

The gauge transformation Λ can be worked out similarly, namely

$$0 = [\lambda_1 \stackrel{\star}{,} \lambda_2] + i\hat{\Lambda}[\partial\lambda_1]\lambda_2 - i\hat{\Lambda}[\partial\lambda_2]\lambda_1 = \frac{1}{2}\big([\lambda_1 \stackrel{\star}{,} \lambda_2] - [\lambda_2 \stackrel{\star}{,} \lambda_1]\big) + i\hat{\Lambda}[\partial\lambda_1]\lambda_2 - i\hat{\Lambda}[\partial\lambda_2]\lambda_1,$$

and hence

$$\hat{\Lambda}^1 = -\frac{1}{2}\theta^{ij} a_i \star_2 \partial_j.$$

The gauge field a_μ requires slightly more work. The lowest order terms in its consistency relation are

$$-\partial_\mu\left(\frac{1}{2}\theta^{ij} a_i \star_2 \partial_j\lambda\right) - i[\lambda \stackrel{\star}{,} a_\mu] = A^2_\mu[a_\mu + \partial_\mu\lambda] - A^2_\mu[a_\mu],$$

where A^2 is the a^2 order term in the expansion of A as power series of a. Using the relation (2.1), the left hand side can be rewritten as $-\frac{1}{2}\theta^{ij}\partial_\mu a_i\star_2\partial_j\lambda-\frac{1}{2}\theta^{ij}a_i\star_2\partial_\mu\partial_j\lambda-\theta^{ij}\partial_i\lambda\star_2\partial_j a_\mu$, where the first term comes from $-\frac{1}{2}\theta^{ij}\partial_\mu a_i\star_2 a_j$, while the third one comes from $-\theta^{ij}a_i\star_2\partial_j a_\mu$. After a gauge transformation, the sum of the first and third terms equals the second term. Ultimately, we obtain SW map solutions up to the $\mathcal{O}(a^2)$ order:

$$
\begin{aligned}
&A_\mu = a_\mu - \frac{1}{2}\theta^{\nu\rho}a_\nu\star_2(\partial_\rho a_\mu + f_{\rho\mu}) + \mathcal{O}(a^3),\\
&\Psi = \psi - \theta^{\mu\nu}a_\mu\star_2\partial_\nu\psi + \mathcal{O}(a^2)\psi, \qquad \Lambda = \lambda - \frac{1}{2}\theta^{\mu\nu}a_\mu\star_2\partial_\nu\lambda + \mathcal{O}(a^2)\lambda,
\end{aligned}
\tag{2.2}
$$

with $f_{\mu\nu}$ being the commutative Abelian field strength $f_{\mu\nu}=\partial_\mu a_\nu-\partial_\nu a_\mu$.

The resulting expansion defines in the next section the one-photon-two-fermion and the three-photon vertices, θ-exactly.

2.1 Actions

We start with the *minimal* NC model of a SW type $\mathrm{U}_\star(1)$ gauge theory on Euclidean spacetime. Here in the starting action, the *minimal* refers on the number of fields/gauge field strengths/covariant derivatives: two gauge fields, two gauge field strengths, one covariant derivative and three fields for gauge-fermion interactions. Thus we have

$$
S^{\min} = \int -\frac{1}{2}F^{\mu\nu}\star F_{\mu\nu} + i\bar\Psi\star \not{D}\Psi,
\tag{2.3}
$$

with the coupling constant to be set as $e=1$, and with the following definitions of the non-Abelian NC covariant derivative and the field strength, respectively:

$$
D_\mu\Psi = \partial_\mu\Psi - i[A_\mu \stackrel{\star}{,} \Psi] \qquad \text{and} \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu \stackrel{\star}{,} A_\nu].
$$

All the fields in this action are images under (hybrid) Seiberg–Witten maps of the corresponding commutative fields a_μ and ψ. In the original work of Seiberg and Witten and in virtually all subsequent applications, these maps are understood as (formal) series in powers of the noncommutativity parameter $\theta^{\mu\nu}$. Physically, this corresponds to an expansion in momenta and is valid only for low energy phenomena. Here we shall not subscribe to this point of view and instead interpret the noncommutative fields as valued in the enveloping algebra of the underlying gauge group. This naturally corresponds to an expansion in powers of the gauge field a_μ and hence in powers of the coupling constant e. At each order in a_μ we shall determine θ-exact expressions. In the following we discuss the model construction for the photon and the massless fermion case. Since we have set $e=1$, to restore the coupling constant one simply substitutes a_μ by ea_μ and then divides the gauge-field term in the Lagrangian by e^2. Coupling constant e, carries (mass) dimension $(4-d)/2$ in the d-dimensional field theory.

The expansion in powers of the commutative gauge field content is motivated by the obvious fact that in perturbative quantum field theory one can sort the interaction vertices by the number of external legs and this is equivalent to the number of field operators in the corresponding interacting terms. For any specific process and loop order there exists an upper limit on the number of external legs. So if one expands the noncommutative fields with respect to the formal power of the commutative fields which are the primary fields in the theory up to an appropriate order, the relevant vertices in a specific diagram will automatically be exact to all orders of θ.

The *minimal* gauge invariant nonlocal interaction (2.3) includes the gauge boson self-coupling as well as the fermion-gauge boson coupling, denoted here as S_g and S_f, respectively:

$$
S^{\min} = S_{\mathrm{U}(1)} + S_\mathrm{g} + S_\mathrm{f}.
$$

In the next step we expand the action (2.3) in terms of the commutative gauge parameter λ and fields a_μ and ψ using the U(1) SW map solutions (2.2). This way, the photon self-interaction up to the lowest nontrivial order is obtained [38, 41]:

$$\begin{aligned}S_g &= \int if^{\mu\nu}\star[a_\mu \stackrel{\star}{,} a_\nu] + \partial_\mu\big(\theta^{\rho\sigma}a_\rho\star_2(\partial_\sigma a_\nu + f_{\sigma\nu})\big)\star f^{\mu\nu} + \mathcal{O}\big(a^4\big)\\ &= \int \theta^{\rho\tau}f^{\mu\nu}\left(\frac{1}{4}f_{\rho\tau}\star_2 f_{\mu\nu} - f_{\mu\rho}\star_2 f_{\nu\tau}\right) + \mathcal{O}\big(a^4\big).\end{aligned} \tag{2.4}$$

The the lowest order photon-fermion interaction (first three terms of equation (2.7) from [38]) reads as follows

$$\begin{aligned}S_f &= \int \bar\psi\gamma^\mu[a_\mu \stackrel{\star}{,} \psi] + i(\theta^{ij}\partial_i\bar\psi\star_2 a_j)\partial\!\!\!/\psi - i\bar\psi\star\partial\!\!\!/(\theta^{ij}a_i\star_2\partial_j\psi) + \bar\psi\mathcal{O}\big(a^2\big)\psi\\ &= -\int i\theta^{\rho\tau}\bar\psi\gamma^\mu\left(\frac{1}{2}f_{\rho\tau}\star_2\partial_\mu\psi - f_{\mu\rho}\star_2\partial_\tau\psi\right) + \bar\psi\mathcal{O}\big(a^2\big)\psi.\end{aligned} \tag{2.5}$$

Note that actions for the gauge and the matter fields obtained above, (2.4) and (2.5) respectively, are nonlocal objects due to the presence of the (generalized) star products.

2.2 General deformed actions: $S_{\rm f}$ and $S_{\rm g}$

It is easy to see that each of the interactions (2.4) and (2.5) contains two U(1) gauge invariant terms, therefore one could vary the ratio between them without spoiling the gauge invariance. Prior studies have also indicated that varying these ratios can improve the one-loop behavior of the model [23, 38, 41, 42, 48]. For this propose we introduce further two-dimensional deformation-parameter-space (κ_f, κ_g).

The deformation parameter κ_f in the photon-gauge boson interaction can be so chosen that it realizes the linear superposition of two possible nontrivial noncommutative deformations of a free neutral fermion action proposed in [38, 41, 42]. Its existence was already hinted in the θ-expanded expressions in [65] but not fully exploited in the corresponding loop computation before.

The pure gauge action $S_{\rm g}$ deformation κ_g was first presented in the non-Abelian gauge sector action of the NCSM and NC SU(N) at first order in θ, $S^\theta_{\rm g}$ [23, 48]. This could be realized by generalizing the standard SW map expression for linear in θ gauge field strength into the following form [73]:

$$F^\theta_{\mu\nu}(\kappa_g) = f_{\mu\nu} + \theta^{\rho\tau}\big(\kappa_g^{-1}f_{\mu\rho}f_{\nu\tau} - a_\rho\partial_\tau f_{\mu\nu}\big) + \mathcal{O}\big(\theta^2\big).$$

The gauge transformation for the noncommutative field strength $\delta_\lambda F_{\mu\nu} = i[\Lambda \stackrel{\star}{,} F_{\mu\nu}]$ will still be satisfied at its leading order in θ. We have observed in prior studies [37, 41, 73] that the above deformation can be made θ-exact and adopted it here

$$F_{\mu\nu}(\kappa_g) = f_{\mu\nu} + \theta^{\rho\tau}\big(\kappa_g^{-1}f_{\mu\rho}\star_2 f_{\nu\tau} - a_\rho\star_2\partial_\tau f_{\mu\nu}\big) + \mathcal{O}\big(a^3\big). \tag{2.6}$$

Using the relation (2.1) we can see that $\delta_\lambda F_{\mu\nu}(\kappa_g) = i[\lambda \stackrel{\star}{,} f_{\mu\nu}] + \mathcal{O}\big(a^2\big)\lambda$, which represents again the desired field strength consistency at the corresponding order. Thus, starting with (2.3), (2.6) and (2.5), followed by an appropriate field strength redefinition $f_{\mu\nu}\to\kappa_g f_{\mu\nu}$, and finally after an overall rescaling κ_g^{-2}, we can write the generalized manifestly gauge invariant actions with *minimal* number of fields[1]:

$$S_{\rm U(1)} = \int -\frac{1}{2}f_{\mu\nu}f^{\mu\nu} + i\bar\psi\partial\!\!\!/\psi, \tag{2.7}$$

[1]It could be simpler if we have associated κ_g with $f_{\mu\rho}$ in (2.8) as the κ_f in (2.9), we choose the other way around to unify our result with the prior works [23, 37, 41, 48, 73].

$$S_{\rm g}(\kappa_g)=\int\theta^{\rho\tau}f^{\mu\nu}\left(\frac{\kappa_g}{4}f_{\rho\tau}\star_2 f_{\mu\nu}-f_{\mu\rho}\star_2 f_{\nu\tau}\right), \tag{2.8}$$

$$S_{\rm f}(\kappa_f)=-\int i\theta^{\rho\tau}\bar\psi\gamma^\mu\left(\frac{1}{2}f_{\rho\tau}\star_2\partial_\mu\psi-\kappa_f f_{\mu\rho}\star_2\partial_\tau\psi\right). \tag{2.9}$$

Since $S_{\rm g}(\kappa_g)$ and $S_{\rm f}(\kappa_f)$ are both gauge invariant by themselves, one can incorporate either one or both of them into the full Lagrangian. The above actions were obtained by a θ-exact gauge-invariant truncation of a $\mathrm{U}_\star(1)$ model up to tri-leg vertices. Such an operation is achievable because the U(1) gauge transformation after deformation preserves the number of fields within each term.

Motivation to introduce deformation parameters κ_g and κ_f was, besides the general gauge invariance of the action, to help eliminating one-loop pathologies due to the UV and/or IR divergences in both sectors. The parameter-space (κ_f,κ_g) represents a measure of the deformation-freedom in the matter $S_{\rm f}(\kappa_f)$ and the gauge $S_{\rm g}(\kappa_g)$ sectors, respectively. We should clarify, that we are interested in the general gauge invariant interactions induced by the $\theta^{\mu\nu}$ background instead of the strictly Moyal–Weyl star-product deformation of the commutative gauge theories and its Seiberg–Witten map extension. We relax the constraint that a deformation should be Moyal–Weyl type for the hope that such variation could provide certain additional control on the novel pathologies due to the noncommutativity, which had indeed occurred in the θ-expanded models studied before, and as we will discuss later, in our θ-exact model as well. We still constrained our model building by requiring that *each* of the gauge invariant interaction terms arises within a Seiberg–Witten map type deformation, only their linear combination ratios κ_g and κ_f are allowed to vary. This is all explained in full details in [38, 41, 42]. Each parameter bears the origin from the corresponding θ-expanded theory [23, 48, 65].

By straightforward reading-out procedure from $S_{\rm g}$ (2.8) we obtain the following Feynman rule for the triple-photon vertex in momentum space:

$$\Gamma^{\mu\nu\rho}_{\kappa_g}(p,k,q)=F(k,q)V^{\mu\nu\rho}_{\kappa_g}(p,k,q),\qquad F(k,q)=\frac{\sin\frac{k\theta q}{2}}{\frac{k\theta q}{2}}, \tag{2.10}$$

where momenta p, k, q are taken to be incoming satisfying the momentum conservation $p+k+q=0$ [41]. The deformation freedom ambiguity κ_g is included in the vertex function:

$$\begin{aligned}V^{\mu\nu\rho}_{\kappa_g}(p,k,q)=&-(p\theta k)[(p-k)^\rho g^{\mu\nu}+(k-q)^\mu g^{\nu\rho}+(q-p)^\nu g^{\mu\rho}]\\&-\theta^{\mu\nu}[p^\rho(kq)-k^\rho(pq)]-\theta^{\nu\rho}[k^\mu(pq)-q^\mu(pk)]-\theta^{\rho\mu}[q^\nu(pk)-p^\nu(kq)]\\&+(\theta p)^\nu[g^{\mu\rho}q^2-q^\nu q^\rho]+(\theta p)^\rho[g^{\mu\nu}k^2-k^\mu k^\nu]+(\theta k)^\mu[g^{\nu\rho}q^2-q^\nu q^\rho]\\&+(\theta k)^\rho[g^{\mu\nu}p^2-p^\mu p^\nu]+(\theta q)^\nu[g^{\mu\rho}p^2-p^\mu p^\rho]+(\theta q)^\mu[g^{\nu\rho}k^2-k^\nu k^\rho]\\&+(\kappa_g-1)\big((\theta p)^\mu[g^{\nu\rho}(kq)-q^\nu k^\rho]+(\theta k)^\nu[g^{\mu\rho}(qp)-q^\mu p^\rho]\\&+(\theta q)^\rho[g^{\mu\nu}(kp)-k^\mu p^\nu]\big).\end{aligned} \tag{2.11}$$

The above vertex function (2.11) is in accord with corresponding Feynman rule for triple neutral gauge-boson coupling in [20].

From $S_{\rm f}$ (2.9) the fermion-photon vertex reads as follows

$$\Gamma^\mu_{\kappa_f}(k,q)=F(k,q)V^\mu_{\kappa_f}(k,q)=F(k,q)\big[\kappa_f\big(\not k(\theta q)^\mu-\gamma^\mu(k\theta q)\big)-(\theta k)^\mu\not q\big], \tag{2.12}$$

where k is the photon incoming momentum, and the fermion momentum q flows through the vertex, as it should [41].

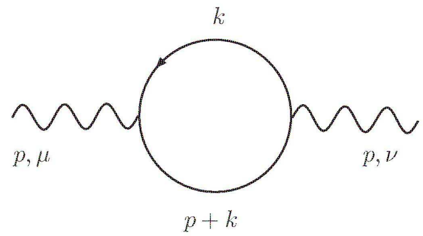

Figure 1. Fermion-loop contribution to the photon two-point function.

3 Photon two-point function

3.1 Computing photon two-point function using dimensional regularization

Employing the parametrization given in this section we illustrate the way we have performed the computation of the integrals which differ from regular ones by the existence of a non-quadratic $k\theta p$ denominators. The key point was to introduce the HQET parametrization [33], represented as follows

$$\frac{1}{a_1^{n_1}a_2^{n_2}}=\frac{\Gamma(n_1+n_2)}{\Gamma(n_1)\Gamma(n_2)}\int_0^\infty\frac{i^{n_1}y^{n_1-1}dy}{(ia_1y+a_2)^{n_1+n_2}}.$$

To perform computations of our integrals, we first use the Feynman parametrization on the quadratic denominators, then the HQET parametrization help us to combine the quadratic and linear denominators. For example

$$\frac{1}{k^2(p+k)^2}\frac{1}{k\theta p}=2i\int_0^1dx\int_0^\infty dy\big[\big(k^2+i\epsilon\big)(1-x)+\big((p+k)^2+i\epsilon\big)x+iy(k\theta p)\big]^{-3}.$$

After employing the Schwinger parametrization, the phase factors from (2.12) can be absorbed by redefining the y integral. This way we obtain

$$\frac{2-e^{ik\theta p}-e^{-ik\theta p}}{k^2(p+k)^2(k\theta p)}\cdot\{\text{numerator}\}$$
$$=2i\int_0^1dx\int_0^{\frac{1}{\lambda}}dy\int_0^\infty d\lambda\lambda^2e^{-\lambda\left(l^2+x(1-x)p^2+\frac{y^2}{4}(\theta p)^2\right)}\cdot\{y\text{ odd terms of the numerator}\},$$

with loop-momenta being $l=k+xp+\frac{i}{2}y(\theta p)$. By this means the y-integral limits take the places of planar/nonplanar parts of the loop integral. For higher negative power(s) of $k\theta p$, the parametrization follows the same way except the appearance of the additional y-integrals which lead to *finite* hypergeometric functions[2]. Following [39], we are enabled to follow the general procedure of dimensional regularization in computing one-loop two point functions. Thus we start the computations with respect to general integration dimension D, next we set the $D\to4$ limits and perform the full analysis of the one-loop two point functions behavior.

Photon two-point function: fermion-loop. The fermion-loop contribution is read out from Fig. 1. Dimensional regularization than gives photon polarization tensor full expression

$$\Pi^{\mu\nu}_{\kappa_f}(p)_D=-\operatorname{tr}\mu^{4-D}\int\frac{d^Dk}{(2\pi)^D}\Gamma^\mu_{\kappa_f}(-p,p+k)\frac{i(\not p+\not k)}{(p+k)^2}\Gamma^\nu_{\kappa_f}(p,k)\frac{i\not k}{k^2},$$

where the momentum structure and dependence on the parameter κ_f is encoded. Performing a large amount of computations we have found the following structure:

$$\Pi^{\mu\nu}_{\kappa_f}(p)_D=\frac{1}{(4\pi)^2}\big[\big(g^{\mu\nu}p^2-p^\mu p^\nu\big)F_1^{\kappa_f}(p)+(\theta p)^\mu(\theta p)^\nu F_2^{\kappa_f}(p)\big],\qquad(3.1)$$

[2]See http://functions.wolfram.com/07.32.06.0031.01.

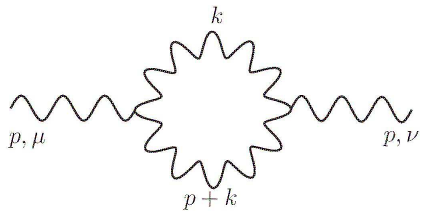

Figure 2. Photon-loop contribution to the photon two-point function.

while the full details of the loop-coefficients $F^{\kappa_f}_{1,2}(p)$ computations are given in [41]. It is straightforward to see that each term in the tensor structure (3.1) does satisfy the Ward identity by itself, therefore $p_\mu\Pi^{\mu\nu}_{\kappa_f}(p)_D = p_\nu\Pi^{\mu\nu}_{\kappa_f}(p)_D = 0$, $\forall\,(D,\kappa_f)$.

In the limit $D\to 4-\epsilon$, the loop-coefficients can be expressed in the following closed forms:

$$
\begin{aligned}
F^{\kappa_f}_1(p) &= -\kappa_f^2\frac{8}{3}\bigg[\frac{2}{\epsilon}+\ln\pi e^{\gamma_{\rm E}}+\ln\big(\mu^2(\theta p)^2\big)\bigg]+4\kappa_f^2p^2(\theta p)^2\sum_{k=0}^{\infty}\frac{(k+2)(p^2(\theta p)^2)^k}{4^k\Gamma[2k+6]}\\
&\quad\times\big[(k+2)\big(\ln\big(p^2(\theta p)^2\big)-\psi(2k+6)-\ln 4\big)+2\big], \qquad (3.2)\\
F^{\kappa_f}_2(p) &= \kappa_f\frac{8}{3}\frac{p^2}{(\theta p)^2}\bigg[\kappa_f-8\big(\kappa_f+2\big)\frac{1}{p^2(\theta p)^2}\bigg]-4\kappa_f^2p^4\sum_{k=0}^{\infty}\frac{(p^2(\theta p)^2)^k}{4^k\Gamma[2k+6]}\\
&\quad\times\big[(k+1)(k+2)\big(\ln\big(p^2(\theta p)^2\big)-2\psi(2k+6)-\ln 4\big)+2k+3\big], \qquad (3.3)
\end{aligned}
$$

with $\gamma_{\rm E}\simeq 0.577216$ being Euler's constant. The above expressions for $F^{\kappa_f}_{1,2}(p)$ contain both contributions, from the planar as well as from the non-planar graphs. All of the *divergences* arising from the fermion-loop (Fig. 1) could be removed by the unique choice $\kappa_f=0$, as in that case the whole general amplitude (3.1) vanishes for any integration dimensions D [41].

Evaluation of the four-dimensional θ-exact fermion-part contribution to the photon polarization tensor, i.e. the fermion-loop photon two-point function (3.1) yields two already known tensor structures [18, 34, 35]. The loop-coefficients $F^{\kappa_f}_{1,2}(p)$, on the other hand, exhibit nontrivial κ_f dependence. Namely, in the limit $\kappa_f\to 0 \Longrightarrow F^{\kappa_f}_1(p) = F^{\kappa_f}_2(p) = 0$, thus the photon polarization tensor (3.1) vanishes, while $\kappa_f = 1$ appears to be identical to the non SW-map model. Fermion-loop contains UV and logarithmic divergence in $F^{\kappa_f}_1(p)$ for $\kappa_f\neq 0$, while the quadratic UV/IR mixing could be removed by setting $\kappa_f = 0,-2$ in $F^{\kappa_f}_2(p)$.

Finally, it is important to stress that there is an additional fermion-loop tadpole diagram contribution to the photon 2-point function, arising from 2-photon-2-fermion ($\bar\psi a^2\psi$) interaction vertices [38, 42]. However, it was shown in [66], that this tadpole diagram vanishes due to the internal Lorentz structure.

Photon two-point function: photon-loop. The photon-loop computation involves a single photon-loop integral contribution to the photon polarization tensor from Fig. 2. Using dimensional regularization, in [41] we have computed the integral,

$$
\Pi^{\mu\nu}_{\kappa_g}(p)_D=\frac{1}{2}\mu^{4-D}\int\frac{d^Dk}{(2\pi)^D}\Gamma^{\mu\rho\sigma}_{\kappa_g}(-p;-k,p+k)\frac{-ig_{\rho\rho'}}{k^2}\Gamma^{\nu\rho'\sigma'}_{\kappa_g}(p;k,-k-p)\frac{-ig_{\sigma\sigma'}}{(p+k)^2},
$$

as a function of deformation freedom κ_g ambiguity. We obtained the following compact form of the photon-loop contribution to the photon two-point function in D-dimensions,

$$
\begin{aligned}
\Pi^{\mu\nu}_{\kappa_g}(p)_D=\frac{1}{(4\pi)^2}\Big\{&\big[g^{\mu\nu}p^2-p^\mu p^\nu\big]B^{\kappa_g}_1(p)+(\theta p)^\mu(\theta p)^\nu B^{\kappa_g}_2(p)\\
&+\big[g^{\mu\nu}(\theta p)^2-(\theta\theta)^{\mu\nu}p^2+p^{\{\mu}(\theta\theta p)^{\nu\}}\big]B^{\kappa_g}_3(p)
\end{aligned}
$$

$$+\Big[(\theta\theta)^{\mu\nu}(\theta p)^2+(\theta\theta p)^{\mu}(\theta\theta p)^{\nu}\Big]B_4^{\kappa_g}(p)+(\theta p)^{\{\mu}(\theta\theta\theta p)^{\nu\}}B_5^{\kappa_g}(p)\Big\}. \tag{3.4}$$

Each term of the above tensor structure satisfies Ward identities by itself, i.e. $p_\mu\Pi^{\mu\nu}_{\kappa_g}(p)_D = p_\nu\Pi^{\mu\nu}_{\kappa_g}(p)_D = 0, \forall(D,\kappa_g)$.

The structure of the photon-loop contribution to the photon polarization tensor contains various previously unknown new momentum structures. It is much reacher with respect to earlier non SW map θ-exact results based on $\star$-product only [18, 34]. These higher order in θ ($\theta\theta\theta\theta$ types) terms suggest certain connection to the open/closed string correspondence [47, 67] (in an inverted way). We consider such connection plausible given the connection between noncommutative field theory and quantum gravity/string theory.

All coefficients $B_i^{\kappa_g}(p)$ can be expressed as sum over integrals over modified Bessel and generalized hypergeometric functions. A complete list of coefficients $B_i^{\kappa_g}(p)$ including computations, as a functions of dimension D, is given in full details in [41]. Next we concentrate on the *divergent* parts in the limit $D\to 4-\epsilon$, and in the IR regime only

$$\begin{aligned}
B_1^{\kappa_g}(p) \sim{}& \bigg(\frac{2}{3}(\kappa_g-3)^2+\frac{2}{3}(\kappa_g+2)^2\frac{p^2(\operatorname{tr}\theta\theta)}{(\theta p)^2}+\frac{4}{3}\big(\kappa_g^2+4\kappa_g+1\big)\frac{p^2(\theta\theta p)^2}{(\theta p)^4}\bigg)\\
&\times\bigg[\frac{2}{\epsilon}+\ln(\mu^2(\theta p)^2)\bigg]-\frac{16}{3}(\kappa_g-1)^2\frac{1}{(\theta p)^6}\big((\operatorname{tr}\theta\theta)(\theta p)^2+4(\theta\theta p)^2\big),
\end{aligned}\tag{3.5}$$

$$\begin{aligned}
B_2^{\kappa_g}(p) \sim{}& \bigg(\frac{8}{3}(\kappa_g-1)^2\frac{p^4(\theta\theta p)^2}{(\theta p)^6}+\frac{2}{3}\big(\kappa_g^2-2\kappa_g-5\big)\frac{p^4(\operatorname{tr}\theta\theta)}{(\theta p)^4}+\frac{2}{3}\big(25\kappa_g^2-86\kappa_g+73\big)\\
&\times\frac{p^2}{(\theta p)^2}\bigg)\bigg[\frac{2}{\epsilon}+\ln\big(\mu^2(\theta p)^2\big)\bigg]-\frac{16}{3}(\kappa_g-3)(3\kappa_g-1)\frac{1}{(\theta p)^4}\\
&+\frac{32}{3}(\kappa_g-1)^2\frac{1}{(\theta p)^8}\big((\operatorname{tr}\theta\theta)(\theta p)^2+6(\theta\theta p)^2\big),
\end{aligned}\tag{3.6}$$

$$B_3^{\kappa_g}(p) \sim -\frac{1}{3}\big(\kappa_g^2-2\kappa_g-11\big)\frac{p^2}{(\theta p)^2}\bigg[\frac{2}{\epsilon}+\ln\big(\mu^2(\theta p)^2\big)\bigg]-\frac{8}{3(\theta p)^4}\big(\kappa_g^2-10\kappa_g+17\big), \tag{3.7}$$

$$B_4^{\kappa_g}(p) \sim -2(\kappa_g+1)^2\frac{p^4}{(\theta p)^4}\bigg[\frac{2}{\epsilon}+\ln\big(\mu^2(\theta p)^2\big)\bigg]-\frac{32p^2}{3(\theta p)^6}\big(\kappa_g^2-6\kappa_g+7\big), \tag{3.8}$$

$$B_5^{\kappa_g}(p) \sim \frac{4}{3}\big(\kappa_g^2+\kappa_g+4\big)\frac{p^4}{(\theta p)^4}\bigg[\frac{2}{\epsilon}+\ln\big(\mu^2(\theta p)^2\big)\bigg]+\frac{64p^2}{3(\theta p)^6}(\kappa_g-1)(\kappa_g-2). \tag{3.9}$$

Note that all $B_i^{\kappa_g}(p)$ coefficients are computed for abitrary κ_g and the notation $\sim$ means that in the above equations we have neglected all finite terms. We observe here the presence of the UV divergences as well as a quadratic UV/IR mixing in all $B_i^{\kappa_g}$'s. Up to the $1/\epsilon$ terms, the UV divergence is at most logarithmic, i.e. there is a logarithmic ultraviolet/infrared term representing a soft UV/IR mixing. The results (3.5)–(3.9) in four dimensions for arbitrary κ_g show power type UV/IR mixing, therefore diverge at both the commutative limit ($\theta\to 0$) and the size-of-the-object limit ($|\theta p|\to 0$). Inspecting (3.5) to (3.9) together with general structure (3.4) we found decouplings of UV and logarithmic IR divergences from the power UV/IR mixing terms. The latter exists in all $B_i^{\kappa_g}(p)$'s. The logarithmic IR divergences from planar and nonplanar sources appear to have identical coefficient and combine into a single $\ln\mu^2(\theta p)^2$ term. Finally it is important to stress that no single κ_g value is capable of removing all novel divergences.

3.2 Photon-loop with a special $\theta^{\mu\nu}$ in four dimensions

In our prior analysis we have found that in the $D\to 4-\epsilon$ limit the general off-shell contribution of photon self-interaction loop to the photon two-point function contains complicated non-vanishing UV and IR divergent terms with existing and new momentum structures, regardless the κ_g values we take. To see whether there exists certain remedy to this situation we explore two conditions which have emerged in the prior studies. First we tested the zero mass-shell condition/limit ($p^2 \to 0$) used in θ-expanded models [55]. Inspection of equations (3.2)–(3.3) and (3.5)–(3.9) show some simplification but not the full cancelation of the pathological divergences. Such condition clearly appears to be unsatisfactory.

Next we have turned into the other one, namely the special full rank $\theta^{\mu\nu}$ choice

$$\theta^{\mu\nu} \equiv \theta^{\mu\nu}_{\sigma_2} = \frac{1}{\Lambda^2_{\rm NC}}\begin{pmatrix}0 & -1 & 0 & 0\\ 1 & 0 & 0 & 0\\ 0 & 0 & 0 & -1\\ 0 & 0 & 1 & 0\end{pmatrix} = \frac{1}{\Lambda^2_{\rm NC}}\begin{pmatrix}i\sigma_2 & 0\\ 0 & i\sigma_2\end{pmatrix} \equiv \frac{1}{\Lambda^2_{\rm NC}} i\sigma_2 \otimes I_2, \tag{3.10}$$

with σ_2 being famous Pauli matrix. This constraints was used in the renormalizability studies of 4d NCGFT without SW map [16, 17]. Note also that this $\theta^{\mu\nu}_{\sigma_2}$ is full rank and thus breaks in general the unitarity if one performs Wick rotation to the Minkowski spacetime [29]. This choice, in 4d Euclidean spacetime, induces a relation $(\theta\theta)^{\mu\nu} = -\frac{1}{\Lambda^4_{\rm NC}} g^{\mu\nu}$. The tensor structures (3.4), with restored coupling constant e included then reduces into two parts and we obtain the same Lorentz structure as we did from the fermion-loop (3.1):

$$\Pi^{\mu\nu}_{\kappa_g}(p)_4\Big|^{\theta_{\sigma_2}} = \frac{e^2}{(4\pi)^2}\big\{\big[g^{\mu\nu}p^2 - p^\mu p^\nu\big]B^{\kappa_g}_{\rm a}(p) + (\theta p)^\mu(\theta p)^\nu B^{\kappa_g}_{\rm b}(p)\big\}.$$

Neglecting the IR safe terms we have found that the $B^{\kappa_g}_{\rm a}(p)$ and $B^{\kappa_g}_{\rm b}(p)$ exhibits *divergent* structures [41]:

$$\begin{aligned}
B^{\kappa_g}_{\rm a}(p) &\sim \frac{4(\kappa_g-3)^2}{3}\left(\frac{2}{\epsilon} + \ln\big(\mu^2(\theta p)^2\big)\right) + \frac{16}{3}\frac{(\kappa_g-3)(\kappa_g+1)}{p^2(\theta p)^2},\\
B^{\kappa_g}_{\rm b}(p) &\sim 2p^2\frac{(\kappa_g-3)(7\kappa_g-9)}{(\theta p)^2}\left(\frac{2}{\epsilon} + \ln\big(\mu^2(\theta p)^2\big)\right) - \frac{16}{3}\frac{(\kappa_g-3)(7\kappa_g-5)}{(\theta p)^4},
\end{aligned}$$

which can all be eliminated by choosing the deformation freedom point $\kappa_g = 3$. A careful evaluation of the full photon-loop at this point exhibits a simple structures

$$B^{\kappa_g=3}_{\rm a}(p) = 2\left[\frac{56}{3} + I\right], \qquad B^{\kappa_g=3}_{\rm b}(p) = -\frac{p^2}{(\theta p)^2}9[8-I],$$

where in [41] we have shown that

$$I = 0.$$

Thus, for special choice (3.10) in the $D \to 4-\epsilon$ limit, and at $\kappa_g = 3$ point, we have found

$$B^{\kappa_g=3}_{\rm a}(p) = \frac{112}{3}, \qquad B^{\kappa_g=3}_{\rm b}(p) = -72\frac{p^2}{(\theta p)^2}.$$

Summing up the contributions to the photon polarization tensor. To simplify the tremendous divergent structures in the loop-coefficients $F^{\kappa_f}_{1,2}(p)$'s, and $B^{\kappa_g}_{1,\dots,5}(p)$'s at $D \to 4-\epsilon$, we have been forced to probe two additional constraints: One which appears to be ineffective is the zero mass-shell condition/limit $p^2 \to 0$, due to the uncertainty on its own validity when quantum corrections present. The other constraint, namely setting $\theta^{\mu\nu}$ to a special full ranked value $\theta^{\mu\nu}_{\sigma_2}$ (3.10), reduces the number of different momentum structures from five to two. Then all divergences and the IR safe contributions disappear at a deformation parameter-space unique point $(\kappa_f, \kappa_g) = (0, 3)$ leaving,

$$\Pi^{\mu\nu}_{(0,3)}(p)\Big|^{\theta_{\sigma_2}} \equiv \big[\Pi^{\mu\nu}_{\kappa_f=0}(p) + \Pi^{\mu\mu}_{\kappa_g=3}(p)\big]\Big|^{\theta_{\sigma_2}} = \frac{e^2p^2}{\pi^2}\left[\frac{7}{3}\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right) - \frac{9}{2}\frac{(\theta p)^\mu(\theta p)^\nu}{(\theta p)^2}\right],$$

as the only one-loop-finite contribution/correction to the photon two-point function.

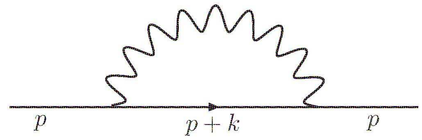

Figure 3. Bubble-graph contribution to the neutrino two-point function.

4 Neutrino two-point function

One-loop contributions as a function of κ_f receive the same spinor structure as in [38]. We now reconfirm that by using the action (2.3) together with the Feynman rule (2.12), out of four diagrams in Fig. 2 of [38], only the non-vanishing bubble-graph is considered in this manuscript.

Thus, in the present scenario a contribution from Fig. 3 reads

$$\Sigma_{\kappa_f}(p)_D = \frac{-1}{(4\pi)^2}\big[\gamma_\mu p^\mu N_1^{\kappa_f}(p) + \gamma_\mu(\theta\theta p)^\mu N_2^{\kappa_f}(p)\big]. \tag{4.1}$$

Complete loop-coefficients $N_{1,2}^{\kappa_f}(p)$, as a functions of an arbitrary dimensions D are given in [41].

For arbitrary κ_f in the limit $D \to 4-\epsilon$ we have obtained the following loop-coefficients:

$$\begin{aligned}
N_1^{\kappa_f}(p) = \kappa_f \bigg\{ & \bigg[2\kappa_f + \big(\kappa_f - 1\big)\bigg(\frac{2}{\epsilon} + \ln \pi e^{\gamma_{\mathrm{E}}} + \ln\big(\mu^2(\theta p)^2\big)\bigg)\bigg] \\
& - \frac{p^2(\theta p)^2}{4}\sum_{k=0}^{\infty}\frac{(p^2(\theta p)^2)^k}{4^k k(k+1)(2k+1)^2(2k+3)\Gamma[2k+4]} \\
& \times \big[k(k+1)(2k+1)(2k+3)\big(\kappa_f(2k+3)-1\big)\big(\ln\big(p^2(\theta p)^2\big) - 2\psi(2k) - \ln 4\big) \\
& + 3 + 28k + 46k^2 + 20k^3 - \kappa_f\big(2k+3\big)^2\big(1+8k+8k^2\big)\big] \\
& + (\mathrm{tr}\,\theta\theta)\bigg\{\frac{p^2}{(\theta p)^2}\bigg[\frac{2}{\epsilon} + 2 + \gamma_{\mathrm{E}} + \ln\pi + \ln\big(\mu^2(\theta p)^2\big) + \frac{8\big(\kappa_f - 1\big)}{3\kappa_f(\theta p)^2 p^2}\bigg] \\
& - \frac{p^4}{4}\sum_{k=0}^{\infty}\frac{(p^2(\theta p)^2)^k}{4^k k(k+1)(2k+1)^2(2k+3)}\big[k(k+1)(2k+1)(2k+3) \\
& \times \big(\ln\big(p^2(\theta p)^2\big) - 2\psi(2k) - \ln 4\big) - 2k\big(14 + k(23+10k)\big) - 3\big]\bigg\} \\
& + (\theta\theta p)^2\bigg\{2\frac{p^2}{(\theta p)^4}\bigg[\frac{2}{\epsilon} + 1 + \gamma_{\mathrm{E}} + \ln\pi + \ln\big(\mu^2(\theta p)^2\big) + \frac{16\big(\kappa_f - 1\big)}{3\kappa_f(\theta p)^2 p^2}\bigg] \\
& + \frac{p^4}{2(\theta p)^2}\sum_{k=0}^{\infty}\frac{k(p^2(\theta p)^2)^k}{(k+1)(2k+1)^2(2k+3)\Gamma[2k+4]}\big[(k+1)(2k+1)(2k+3) \\
& \times \big(\ln\big(p^2(\theta p)^2\big) - 2\psi(2k) - \ln 4\big) + 16k^2 - 34k - 17\big]\bigg\}\bigg\}, \\
N_2^{\kappa_f}(p) = -\kappa_f \frac{p^2}{(\theta p)^2} \bigg\{ & 4 + \big(\kappa_f - 1\big)\bigg[\frac{2}{\epsilon} + \ln\pi e^{\gamma_{\mathrm{E}}} + \ln\big(\mu^2(\theta p)^2\big)\bigg] - \frac{16\big(\kappa_f - 1\big)}{3(\theta p)^2 p^2} \\
& - \frac{p^2(\theta p)^2}{4}\sum_{k=0}^{\infty}\frac{(p^2(\theta p)^2)^k}{4^k k(k+1)(2k+1)^2(2k+3)\Gamma[2k+4]}\big[k(1+k)(1+2k)(3+2k) \\
& \times \big(1 + 3k_f + 2\big(k_f+1\big)k\big)\big(\ln\big(p^2(\theta p)^2\big) - 2\psi(2k) - \ln 4\big) - 3 - 9\kappa_f \\
& - 4k\big(7 + 21\kappa_f + \big(20 + 43\kappa_f + 2k\big(11 + 4k + 4\kappa_f(4+k)\big)\big)k\big)\big]\bigg\}.
\end{aligned}$$

In the expressions for $N^{\kappa_f}_{1,2}(p)$ contributions from both the planar as well as the non-planar graphs are present. For any $\kappa_f \neq 1$ our neutrino two point function receive UV, and power as well as logarithmic UV/IR mixing terms.

First we analyze the choice $\boldsymbol{\kappa_f = 1}$. For $D = 4 - \epsilon$ in the limit $\epsilon \to 0$, we obtain the final expression as

$$\Sigma_{\kappa_f=1}(p) = \frac{-e^2}{(4\pi)^2}\gamma_\mu\left[p^\mu N_1(p) + (\theta\theta p)^\mu \frac{p^2}{(\theta p)^2} N_2(p)\right], \tag{4.2}$$

with restored coupling constant e included. Here $N_{1,2}(p)$ coefficients are as follows

$$N_1(p) = p^2\left(\frac{\operatorname{tr}\theta\theta}{(\theta p)^2} + 2\frac{(\theta\theta p)^2}{(\theta p)^4}\right)A + \left[1 + p^2\left(\frac{\operatorname{tr}\theta\theta}{(\theta p)^2} + \frac{(\theta\theta p)^2}{(\theta p)^4}\right)\right]B, \tag{4.3}$$

$$A = \frac{2}{\epsilon} + \ln(\mu^2(\theta p)^2) + \ln(\pi e^{\gamma_{\rm E}}) + \sum_{k=1}^{\infty}\frac{\left(p^2(\theta p)^2/4\right)^k}{\Gamma(2k+2)}\left(\ln\frac{p^2(\theta p)^2}{4} + 2\psi_0(2k+2)\right), \tag{4.4}$$

$$\begin{aligned} B = -8\pi^2 N_2(p) &= -2 + \sum_{k=0}^{\infty}\frac{\left(p^2(\theta p)^2/4\right)^{k+1}}{(2k+1)(2k+3)\Gamma(2k+2)} \\ &\times\left(\ln\frac{p^2(\theta p)^2}{4} - 2\psi_0(2k+2) - \frac{8(k+1)}{(2k+1)(2k+3)}\right). \end{aligned} \tag{4.5}$$

It is to be noted here that the spinor structure proportional to $\gamma_\mu(\theta p)^\mu$ is missing in the final result. This conforms with the calculation of the neutral fermion self-energy in the θ-expanded SW map approach [27].

The $1/\epsilon$ UV divergence could in principle be removed by a properly chosen counterterm. However (as already mentioned) due to the specific momentum-dependent coefficient in front of it, a nonlocal form for it is required[3]. It is important to stress here that amongst other terms contained in both coefficients $N_{1,2}(p)$, there are structures proportional to

$$\left(p^2(\theta p)^2\right)^{n+1}\left(\ln\left(p^2(\theta p)^2\right)\right)^m, \qquad \forall\, n \quad \text{and} \quad m = 0, 1.$$

The numerical factors in front of the above structures are rapidly-decaying, thus series are always convergent for finite argument, as we demonstrate in [38].

Turning to the UV/IR mixing problem, we recognize a soft UV/IR mixing term represented by a logarithm,

$$\Sigma^{\rm UV/IR}_{\kappa_f=1} = \frac{-e^2}{(4\pi)^2}\not{p}\, p^2\left(\frac{\operatorname{tr}\theta\theta}{(\theta p)^2} + 2\frac{(\theta\theta p)^2}{(\theta p)^4}\right)\cdot\ln\left|\mu^2(\theta p)^2\right|.$$

Instead of dealing with nonlocal counterterms, we take a different route here to cope with various divergences besetting (4.2). Since $\theta^{0i} \neq 0$ makes a NC theory nonunitary [29], we can, without loss of generality, chose θ to lie in the (1, 2) plane

$$\theta^{\mu\nu}_{\rm spec} = \frac{1}{\Lambda^2_{\rm NC}}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{4.6}$$

yielding

$$\frac{\operatorname{tr}\theta\theta}{(\theta p)^2} + 2\frac{(\theta\theta p)^2}{(\theta p)^4} = 0, \quad \forall\, p. \tag{4.7}$$

[3]Any quadratic counterterm for the neutral fermion would be gauge invariant since the neutral fermion field is invariant under the U(1) gauge transformation.

With (4.7), $\Sigma^{\text{spec}}_{\kappa_f=1}$, in terms of Euclidean momenta, receives the following form:

$$\Sigma^{\text{spec}}_{\kappa_f=1}(p) = \frac{-e^2}{(4\pi)^2}\gamma_\mu\left[p^\mu\left(1+\frac{\operatorname{tr}\theta\theta}{2}\frac{p^2}{(\theta p)^2}\right)-2(\theta\theta p)^\mu\frac{p^2}{(\theta p)^2}\right]N_2(p). \tag{4.8}$$

By inspecting (4.5) one can be easily convinced that $N_2(p)$ is free from the $1/\epsilon$ divergence and the UV/IR mixing term, being also well-behaved in the infrared, in the $\theta\to 0$ as well as $\theta p \to 0$ limit. We see, however, that the two terms in (4.8), one being proportional to p^μ and the other proportional to $(\theta\theta p)^\mu$, are still ill-behaved in the $\theta p\to 0$ limit. If, for the choice (4.6), P denotes the momentum in the (1,2) plane, then $\theta p = \theta P$. For instance, a particle moving inside the noncommutative plane with momentum P along the one axis, has a spatial extension of size $|\theta P|$ along the other. For the choice (4.6), $\theta p\to 0$ corresponds to a zero momentum projection onto the (1,2) plane. Thus, albeit in our approach the commutative limit ($\theta\to 0$) is smooth at the quantum level, the limit when an extended object (arising due to the fuzziness of space) shrinks to zero, is not. We could surely claim that in our approach the UV/IR mixing problem is considerably softened; on the other hand, we have witnessed how the problem strikes back in an unexpected way. This is, at the same time, the first example where this two limits are not degenerate (or two limits do not commute).

Next we analyze the choice $\boldsymbol{\kappa_f = 0}$. Using the Feynman rule (2.12) for $\kappa_f = 0$ and for general θ, we find the following closed form contribution to the neutrino two point function (from diagram Σ_1 in [38, 40]):

$$\Sigma_{\kappa_f=0}(p) = \frac{e^2}{(4\pi)^2}\not{p}\left[\frac{8}{3}\frac{1}{(\theta p)^2}\left(\frac{\operatorname{tr}\theta\theta}{(\theta p)^2}+4\frac{(\theta\theta p)^2}{(\theta p)^4}\right)\right]. \tag{4.9}$$

It is important to stress that we have found that diagram $\Sigma_2 = 0$ in all κ_f-cases, while diagrams Σ_3 and Σ_4 vanish due to charge conjugation symmetry, see [38, 40]. There is no alternative dispersion relation in degenerate case (4.6), since the factor that multiplies $\not{p}$ in (4.9), does not dependent on the time-like component p_0 (energy).

Considering neutrino two point function (4.1), our results extends the prior works [38, 40] by completing the behavior for general κ_f. Here we discuss some novel behaviors associated with general κ_f. The neutrino two point function does posses power UV/IR mixing phenomenon for arbitrary values of κ_f, except $\kappa_f = 1$. In the limit $\kappa_f\to 0$ all UV, IR divergent terms as well as constant terms in $N^{\kappa_f}_{1,2}(p)$ vanish; what remains are only the power UV/IR mixing terms. The UV divergence can be localized using the special θ value [38, 40] in $N^{\kappa_f}_1(p)$ but not in $N^{\kappa_f}_2(p)$. The UV and the power IR divergence in $N^{\kappa_f}_2(p)$ can be removed by setting $\kappa_f = 1$.

Summing up, choice $\kappa_f = 1$ eliminates some of divergences, but not all of them. Imposing the special $\theta^{\mu\nu}_{\sigma_2}$ reduces the contribution to quadratic UV/IR mixing into a single term from $N^{\kappa_f}_2(p)$, which has two zero points $\kappa_f = 0, 1$. Only $\kappa_f = 0$ can induce full divergence cancelations, by removing the whole $\Sigma(p)$, i.e. we have

$$\Sigma_{\kappa_f=0}(p)\Big|^{\theta_{\sigma_2}} = 0.$$

5 Discussion and conclusion

In this review we present a θ-exact quantum one-loop contributions to the photon (Π) and neutrino (Σ) two point functions and analyze their properties. In principle the quantum corrections in NCQFTs are extremely profound, revealing a structure of pathological terms far beyond that found in ordinary field theories. For practical purposes, perturbative loop computation was the most intensively studied for the Moyal–Weyl (constant $\theta^{\mu\nu}$) type deformation [16, 17, 28, 32, 50], for its preservation of translation invariance allows a (modified) Feynman diagrammatic calculation. Much of efforts went in taming divergences related to the ultraviolet-infrared connection/mixing, see for example [15, 38, 40, 41, 56, 60]. The UV/IR mixing, built-in as a new principle in all NCQFT models, and closely related to the Black Hole Complementarity and/or Holographic principle [44, 68, 69, 72], does reverse the well-established connection (via the uncertainty principle) between energy and size. The existence of such pathological terms in NCQFTs raises a serious concern on the renormalizability/consistency of the theory. Although no satisfactory resolution for this issue had been achieved so far, very recently it has been observed that certain control over novel divergences may be obtained by certain gauge invariant variation of the SW mapped action [41]. The anomalous structures in the two point function further suggests possible modifications to obtain trouble-free and meaningful loop-results, necessary for studying the particle propagation [38]. Such effects were largely left untouched in literature so far, mostly due to the prior concern on the consistency/renormalizability from a purely theoretical viewpoint.

After having defined and explained the full noncommutative action-model origin of the deformation parameter-space (κ_f, κ_g), we obtained the relevant Feynman rules. Our method, extending the modified Feynman rule procedure [28], yields the one-loop quantum corrections for arbitrary dimensions in closed form, as function of the deformation-freedom parameters κ_f, κ_g, as well as momentum p^μ and noncommutative parameter $\theta^{\mu\nu}$. Full parameter-space freedom is kept in our evaluation here. Following the extended dimensional regularization technique we expressed the diagrams as D-dimensional loop-integrals and identify the relevant momentum structures with corresponding loop-coefficients. We have found that total contribution to photon two-point function satisfies the Ward–(Slavnov–Taylor) identity for arbitrary dimensions D and for any point in the (κ_f, κ_g) parameter-space:

$$p_\mu \Pi^{\mu\nu}_{(\kappa_f,\kappa_g)}(p)_D = p_\mu\big(\Pi^{\mu\nu}_{\kappa_f}(p)_D + \Pi^{\mu\nu}_{\kappa_g}(p)_D\big) = p_\nu\big(\Pi^{\mu\nu}_{\kappa_f}(p)_D + \Pi^{\mu\nu}_{\kappa_g}(p)_D\big) = 0.$$

The one-loop photon polarization tensor in four dimensions contains the UV divergence and UV/IR mixing terms dependent on the freedom parameters κ_f and κ_g. The introduction of the freedom parameters univocally has a potential to improve the situation regarding cancellation of divergences, since certain choices for κ_f and κ_g could make some of the terms containing singularities to vanish.

We observe the following general behavior of one-loop two-point functions in the $D \to 4-\epsilon$ limit: The total expressions for both the photon and the neutrino two point functions contain the $1/\epsilon$ ultraviolet term, the celebrated UV/IR mixing power terms as well as the logarithmic (soft) UV/IR mixing term. The $1/\epsilon$ divergence is always independent of the noncommutative scale. The logarithmic terms from the ϵ-expansion and the modified Bessel function integral sum into a common term $\ln(\mu^2(\theta p)^2)$, which is divergent in the infrared $|p| \to 0$, in the size-of-the-object $|\theta p| \to 0$ limit, as well as in the vanishing noncommutativity $\theta \to 0$ limit. Thus, the existences of UV/IR mixings for both, photons and neutrinos respectively, in 4d spaces deformed by spacetime noncommutativity at low energies, suggests that the relation of quantum corrections to observations [1, 2, 6, 44, 46] is not entirely clear. However, in the context of the UV/IR mixing it is very important to mention a complementary approach [1, 2] where NC gauge theories are realized as effective QFT's, underlain by some more fundamental theory such as string theory. It was claimed that for a large class of more general QFT's above the UV cutoff the phenomenological effects of the UV completion can be quite successfully modeled by a threshold value of the UV cutoff. So, in the presence of a finite UV cutoff no one sort of divergence will ever appear since the problematic phase factors effectively transform the highest energy scale (the UV cutoff) into the lowest one (the IR cutoff). What is more, not only the full scope of noncommutativity is experienced only in the range delimited by the two cutoffs, but for the scale of NC high enough, the whole standard model can be placed below the IR cutoff [44].

Thus, a way the UV/IR mixing problem becomes hugely less pressing, making a study of the theory at the quantum level much more reliable.

We have demonstrated how quantum effects in the θ-exact Seiberg–Witten map approach to NC gauge field theory reveal a much richer structure for the one-loop quantum correction to the photon and fermion two-point functions (and accordingly for the UV/IR mixing problem) than observed previously in approximate models restricting to low-energy phenomena. Our analysis can be considered trustworthy since we have obtained the final result in an analytic, closed-form manner. We believe that a promising avenue of research would be using the enormous freedom in the Seiberg–Witten map to look for other forms which UV/IR mixing may assume. Two alternative forms have been already found [38]. Finally, our approach to UV/IR mixing should not be confused with the one based on a theory with UV completion ($\Lambda_{\rm UV} < \infty$), where a theory becomes an effective QFT, and the UV/IR mixing manifests itself via a specific relationships between the UV and the IR cutoffs [1, 2, 6, 44, 46].

In conclusion, our main result in four-dimensional space is that we have all pathological terms under full control after the introduction of the deformation-freedom parameter-space (κ_f, κ_g) and a special choice for $\theta^{\mu\nu}$. Namely, working in the 4d Euclidean space with a special full rank of $\theta^{\mu\nu}_{\sigma_2}$ and setting the point $(\kappa_f, \kappa_g) = (0, 3)$, the fermion plus the photon-loop contribution to $\Pi^{\mu\nu}_{(\kappa_f,\kappa_g)}(p)$ contain only two finite terms, i.e. all divergent terms are eliminated. In this case the neutrino two-point function vanishes.

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References

[1] Abel S., Chu C.-S., Goodsell M., Noncommutativity from the string perspective: modification of gravity at a mm without mm sized extra dimensions, *J. High Energy Phys.* **2006** (2006), no. 11, 058, 27 pages, hep-th/0606248.

[2] Abel S.A., Jaeckel J., Khoze V.V., Ringwald A., Vacuum birefringence as a probe of Planck scale noncommutativity, *J. High Energy Phys.* **2006** (2006), no. 9, 074, 18 pages, hep-ph/0607188.

[3] Alboteanu A., Ohl T., Rückl R., Probing the noncommutative standard model at hadron colliders, *Phys. Rev. D* **74** (2006), 096004, 9 pages, hep-ph/0608155.

[4] Alboteanu A., Ohl T., Rückl R., Noncommutative standard model at $\mathcal{O}(\theta^2)$, *Phys. Rev. D* **76** (2007), 105018, 10 pages, arXiv:0707.3595.

[5] Alboteanu A., Ohl T., Rückl R., The noncommutative standard model at colliders, *Acta Phys. Polon. B* **38** (2007), 3647–3651, arXiv:0709.2359.

[6] Álvarez-Gaumé L., Vázquez-Mozo M.A., General properties of non-commutative field theories, *Nuclear Phys. B* **668** (2003), 293–321, hep-th/0305093.

[7] Amelino-Camelia G., Ellis J., Mavromatos N.E., Nanopoulos D.V., Sarkar S., Tests of quantum gravity from observations of gamma-ray bursts, *Nature* **393** (1998), 763–765, astro-ph/9712103.

[8] Arkani-Hamed N., Dimopoulos S., Dvali G., The hierarchy problem and new dimensions at a millimeter, *Phys. Lett. B* **429** (1998), 263–272, hep-ph/9803315.

[9] Aschieri P., Jurčo B., Schupp P., Wess J., Noncommutative GUTs, Standard Model and C, P, T, *Nuclear Phys. B* **651** (2003), 45–70, hep-th/0205214.

[10] Banerjee R., Ghosh S., Seiberg–Witten map and the axial anomaly in non-commutative field theory, *Phys. Lett. B* **533** (2002), 162–167, hep-th/0110177.

[4] Wolfram Research Inc., Mathematica, Version 8.0, Champaign, IL, 2010.

[5] Martin-Garcia J., xAct, http://www.xact.es/

[11] Barnich G., Brandt F., Grigoriev M., Local BRST cohomology and Seiberg–Witten maps in noncommutative Yang–Mills theory, *Nuclear Phys. B* **677** (2004), 503–534, hep-th/0308092.

[12] Behr W., Deshpande N., Duplančić G., Schupp P., Trampetić J., Wess J., The $Z \to \gamma\gamma, gg$ decays in the noncommutative standard model, *Eur. Phys. J. C Part. Fields* **29** (2003), 441–446, hep-ph/0202121.

[13] Bichl A., Grimstrup J., Popp L., Schweda M., Grosse H., Wulkenhaar R., Renormalization of the noncommutative photon self-energy to all orders via Seiberg–Witten map, *J. High Energy Phys.* **2001** (2001), no. 6, 013, 15 pages, hep-th/0104097.

[14] Bichl A.A., Grimstrup J.M., Popp L., Schweda M., Wulkenhaar R., Perturbative analysis of the Seiberg–Witten map, *Internat. J. Modern Phys. A* **17** (2002), 2219–2231, hep-th/0102044.

[15] Bigatti D., Susskind L., Magnetic fields, branes, and noncommutative geometry, *Phys. Rev. D* **62** (2000), 066004, 6 pages, hep-th/9908056.

[16] Blaschke D.N., A new approach to non-commutative $\mathrm{U}_\star(N)$ gauge fields, *Europhys. Lett.* **91** (2010), 11001, 6 pages, arXiv:1005.1578.

[17] Blaschke D.N., Grosse H., Kronberger E., Schweda M., Wohlgenannt M., Loop calculations for the non-commutative $\mathrm{U}_\star(1)$ gauge field model with oscillator term, *Eur. Phys. J. C Part. Fields* **67** (2010), 575–582, arXiv:0912.3642.

[18] Brandt F.T., Das A., Frenkel J., General structure of the photon self-energy in noncommutative QED, *Phys. Rev. D* **65** (2002), 085017, 13 pages, hep-th/0112127.

[19] Burić M., Latas D., Nikolić B., Radovanović V., The role of the Seiberg–Witten field redefinition in renormalization of noncommutative chiral electrodynamics, *Eur. Phys. J. C Part. Fields* **73** (2013), 2542, 10 pages, arXiv:1304.4451.

[20] Burić M., Latas D., Radovanović V., Trampetić J., Nonzero $Z \to \gamma\gamma$ decays in the renormalizable gauge sector of the noncommutative standard model, *Phys. Rev. D* **75** (2007), 097701, 4 pages, hep-ph/0611299.

[21] Burić M., Latas D., Radovanović V., Trampetić J., The absence of the 4ψ divergence in noncommutative chiral models, *Phys. Rev. D* **77** (2008), 045031, 7 pages, arXiv:0711.0887.

[22] Burić M., Latas D., Radovanović V., Trampetić J., Chiral fermions in noncommutative electrodynamics: renormalisability and dispersion, *Phys. Rev. D* **83** (2011), 045023, 10 pages, arXiv:1009.4603.

[23] Burić M., Radovanović V., Trampetić J., The one-loop renormalization of the gauge sector in the θ-expanded noncommutative standard model, *J. High Energy Phys.* **2007** (2007), no. 3, 030, 17 pages, hep-th/0609073.

[24] Calmet X., Jurčo B., Schupp P., Wess J., Wohlgenannt M., The standard model on non-commutative space-time, *Eur. Phys. J. C Part. Fields* **23** (2002), 363–376, hep-ph/0111115.

[25] Connes A., Noncommutative geometry, Academic Press, Inc., San Diego, CA, 1994.

[26] Douglas M.R., Nekrasov N.A., Noncommutative field theory, *Rev. Modern Phys.* **73** (2001), 977–1029, hep-th/0106048.

[27] Ettefaghi M.M., Haghighat M., Massive neutrino in noncommutative space-time, *Phys. Rev. D* **77** (2008), 056009, 8 pages, arXiv:0712.4034.

[28] Filk T., Divergencies in a field theory on quantum space, *Phys. Lett. B* **376** (1996), 53–58.

[29] Gomis J., Mehen T., Space-time noncommutative field theories and unitarity, *Nuclear Phys. B* **591** (2000), 265–276, hep-th/0005129.

[30] Grimstrup J.M., Wulkenhaar R., Quantisation of θ-expanded non-commutative QED, *Eur. Phys. J. C Part. Fields* **26** (2002), 139–151, hep-th/0205153.

[31] Grosse H., Lechner G., Noncommutative deformations of Wightman quantum field theories, *J. High Energy Phys.* **2008** (2008), no. 9, 131, 29 pages, arXiv:0808.3459.

[32] Grosse H., Wulkenhaar R., Renormalisation of ϕ^4-theory on noncommutative $\mathbb{R}^4$ in the matrix base, *Comm. Math. Phys.* **256** (2005), 305–374, hep-th/0401128.

[33] Grozin A.G., Lectures on perturbative HQET. I, hep-ph/0008300.

[34] Hayakawa M., Perturbative analysis on infrared aspects of noncommutative QED on $\mathbb{R}^4$, *Phys. Lett. B* **478** (2000), 394–400, hep-th/9912094.

[35] Hayakawa M., Perturbative analysis on infrared and ultraviolet aspects of noncommutative QED on $\mathbb{R}^4$, hep-th/9912167.

[36] Hinchliffe I., Kersting N., Ma Y.L., Review of the phenomenology of noncommutative geometry, *Internat. J. Modern Phys. A* **19** (2004), 179–204, hep-ph/0205040.

[37] Horvat R., Ilakovac A., Kekez D., Trampetić J., You J., Forbidden and invisible Z boson decays in a covariant θ-exact noncommutative standard model, *J. Phys. G: Nuclear Part. Phys.* **41** (2014), 055007, 13 pages, arXiv:1204.6201.

[38] Horvat R., Ilakovac A., Schupp P., Trampetić J., You J., Neutrino propagation in noncommutative spacetimes, *J. High Energy Phys.* **2012** (2012), no. 4, 108, 28 pages, arXiv:1111.4951.

[39] Horvat R., Ilakovac A., Schupp P., Trampetić J., You J., Yukawa couplings and seesaw neutrino masses in noncommutative gauge theory, *Phys. Lett. B* **715** (2012), 340–347, arXiv:1109.3085.

[40] Horvat R., Ilakovac A., Trampetić J., You J., On UV/IR mixing in noncommutative gauge field theories, *J. High Energy Phys.* **2011** (2011), no. 12, 081, 11 pages.

[41] Horvat R., Ilakovac A., Trampetić J., You J., Self-energies on deformed spacetimes, *J. High Energy Phys.* **2013** (2013), no. 11, 071, 27 pages, arXiv:1306.1239.

[42] Horvat R., Kekez D., Schupp P., Trampetić J., You J., Photon-neutrino interaction in θ-exact covariant noncommutative field theory, *Phys. Rev. D* **84** (2011), 045004, 10 pages, arXiv:1103.3383.

[43] Horvat R., Kekez D., Trampetić J., Spacetime noncommutativity and ultra-high energy cosmic ray experiments, *Phys. Rev. D* **83** (2011), 065013, 5 pages, arXiv:1005.3209.

[44] Horvat R., Trampetić J., Constraining noncommutative field theories with holography, *J. High Energy Phys.* **2011** (2011), no. 1, 112, 8 pages, arXiv:1009.2933.

[45] Jackiw R., Pi S.-Y., Covariant coordinate transformations on noncommutative space, *Phys. Rev. Lett.* **88** (2002), 111603, 4 pages, hep-th/0111122.

[46] Jaeckel J., Khoze V.V., Ringwald A., Telltale traces of U(1) fields in noncommutative standard model extensions, *J. High Energy Phys.* **2006** (2006), no. 2, 028, 21 pages, hep-ph/0508075.

[47] Jurčo B., Schupp P., Wess J., Nonabelian noncommutative gauge theory via noncommutative extra dimensions, *Nuclear Phys. B* **604** (2001), 148–180, hep-th/0102129.

[48] Latas D., Radovanović V., Trampetić J., Noncommutative SU(N) gauge theory and asymptotic freedom, *Phys. Rev. D* **76** (2007), 085006, 7 pages, hep-th/0703018.

[49] Madore J., An introduction to noncommutative differential geometry and its physical applications, *London Mathematical Society Lecture Note Series*, Vol. 257, 2nd ed., Cambridge University Press, Cambridge, 1999.

[50] Magnen J., Rivasseau V., Tanasa A., Commutative limit of a renormalizable noncommutative model, *Europhys. Lett.* **86** (2009), 11001, 6 pages, arXiv:0807.4093.

[51] Martín C.P., The gauge anomaly and the Seiberg–Witten map, *Nuclear Phys. B* **652** (2003), 72–92, hep-th/0211164.

[52] Martín C.P., The minimal and the new minimal supersymmetric grand unified theories on noncommutative space-time, *Classical Quantum Gravity* **30** (2013), 155019, 15 pages, arXiv:1302.3732.

[53] Martín C.P., Sánchez-Ruiz D., One-loop UV divergent structure of U(1) Yang–Mills theory on noncommutative $\mathbb{R}^4$, *Phys. Rev. Lett.* **83** (1999), 476–479, hep-th/9903077.

[54] Martín C.P., Tamarit C., Noncommutative GUT inspired theories and the UV finiteness of the fermionic four point functions, *Phys. Rev. D* **80** (2009), 065023, 6 pages, arXiv:0907.2464.

[55] Martín C.P., Tamarit C., Renormalisability of noncommutative GUT inspired field theories with anomaly safe groups, *J. High Energy Phys.* **2009** (2009), no. 12, 042, 18 pages, arXiv:0910.2677.

[56] Matusis A., Susskind L., Toumbas N., The IR/UV connection in non-commutative gauge theories, *J. High Energy Phys.* **2000** (2000), no. 12, 002, 18 pages, hep-th/0002075.

[57] Mehen T., Wise M.B., Generalized $\star$-products, Wilson lines and the solution of the Seiberg–Witten equations, *J. High Energy Phys.* **2000** (2000), no. 12, 008, 10 pages, hep-th/0010204.

[58] Meljanac S., Samsarov A., Trampetić J., Wohlgenannt M., Scalar field propagation in the ϕ^4 κ-Minkowski model, *J. High Energy Phys.* **2011** (2011), no. 12, 010, 23 pages, arXiv:1111.5553.

[59] Minkowski P., Schupp P., Trampetić J., Neutrino dipole moments and charge radii in non-commutative space-time, *Eur. Phys. J. C Part. Fields* **37** (2004), 123–128, hep-th/0302175.

[60] Minwalla S., Van Raamsdonk M., Seiberg N., Noncommutative perturbative dynamics, *J. High Energy Phys.* **2000** (2000), no. 2, 020, 31 pages, hep-th/9912072.

[61] Ohl T., Reuter J., Testing the noncommutative standard model at a future photon collider, *Phys. Rev. D* **70** (2004), 076007, 10 pages, hep-ph/0406098.

[62] Okawa Y., Ooguri H., Exact solution to the Seiberg–Witten equation of noncommutative gauge theory, *Phys. Rev. D* **64** (2001), 046009, 11 pages, hep-th/0104036.

[63] Rovelli C., Loop quantum gravity: the first 25 years, *Classical Quantum Gravity* **28** (2011), 153002, 35 pages, arXiv:1012.4707.

[64] Rovelli C., Zakopane lectures on loop gravity, *PoS Proc. Sci.* (2011), PoS(QGQGS2011), 003, 60 pages, arXiv:1102.3660.

[65] Schupp P., Trampetić J., Wess J., Raffelt G., The photon-neutrino interaction induced by non-commutativity and astrophysical bounds, *Eur. Phys. J. C Part. Fields* **36** (2004), 405–410, hep-ph/0212292.

[66] Schupp P., You J., UV/IR mixing in noncommutative QED defined by Seiberg–Witten map, *J. High Energy Phys.* **2008** (2008), no. 8, 107, 10 pages, arXiv:0807.4886.

[67] Seiberg N., Witten E., String theory and noncommutative geometry, *J. High Energy Phys.* **1999** (1999), no. 9, 032, 93 pages, hep-th/9908142.

[68] Susskind L., Lindesay J., An introduction to black holes, information and the string theory revolution. The holographic universe, World Scientific Publishing Co. Inc., Hackensack, NJ, 2005.

[69] Susskind L., Thorlacius L., Uglum J., The stretched horizon and black hole complementarity, *Phys. Rev. D* **48** (1993), 3743–3761, hep-th/9306069.

[70] Szabo R.J., Quantum field theory on noncommutative spaces, *Phys. Rep.* **378** (2003), 207–299, hep-th/0109162.

[71] Szabo R.J., Quantum gravity, field theory and signatures of noncommutative spacetime, *Gen. Relativity Gravitation* **42** (2010), 1–29, arXiv:0906.2913.

[72] 't Hooft G., On the quantum structure of a black hole, *Nuclear Phys. B* **256** (1985), 727–745.

[73] Trampetić J., Signal for space-time noncommutativity: the $Z \to \gamma\gamma$ decay in the renormalizable gauge sector of the θ-expanded NCSM, in Proceedings of 4th Summer School "Modern Mathematical Physics" (September 3–14, 2006, Belgrade, Serbia), Editors B. Dragovich, Z. Rakić, Institute of Physics, Belgrade, Serbia, 2007, 379–390, arXiv:0704.0559.

[74] Trampetić J., Renormalizability and phenomenology of θ-expanded noncommutative gauge field theory, *Fortschr. Phys.* **56** (2008), 521–531, arXiv:0802.2030.

[75] Zeiner J., Noncommutative quantumelectrodynamics from Seiberg–Witten maps to all orders in $\theta^{\mu\nu}$, Ph.D. Thesis, Julius-Maximilians-Universität Würzburg, 2007.