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BUCKLING ANALYSIS OF ANTI-SYMMETRIC CROSS-PLY LAMINATED COMPOSITE PLATES UNDER DIFFERENT BOUNDARY CONDITIONS

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Abstract. The buckling analysis of anti-symmetric cross-ply laminated composite plates under different boundary conditions is examined by using a refined higher order exponential shear deformation theory. The theory, which has strong similarity with classical plate theory in many aspects, accounts for a quadratic variation of the transverse shear strains across the thickness and satisfies the zero traction boundary conditions on the top and bottom surfaces of the plate without using shear correction factors. The number of independent unknowns in the present theory is four, as against five in other shear deformation theories. In this investigations, the equations of motion for simply supported thick laminated rectangular plates are derived and obtained through the use of Hamilton's principle. The closed-form solutions of anti-symmetric cross-ply and angle-ply laminates are obtained using Navier solution. Numerical results for critical buckling loads anti-symmetric cross-ply laminated composite plates is presented. The validity of the present study is demonstrated by comparison with other higher-order solutions reported in the literature. It can be concluded that the proposed theory is accurate and simple in solving the buckling behaviors of anti-symmetric cross-ply laminated composite plates under different boundary conditions.

Key words: Laminated composite plates, higher-order shear deformation theory, buckling, theoretical solutions, various boundary conditions.

1. Introduction.

Plate elements are commonly used in civil and mechanical structures. The consideration of buckling loads for laminated composite plates are essential to have an efficient and reliable design. In order to static and dynamic analysis of laminated composite plates structures, a number of plate theories are available based on considering the transverse shear deformation of plate. The classical plate theory in which the transverse shear deformation effects are neglected and the normal to the mid-plane remains straight and normal to the middle surface during the deformation. As a result, the classical plate theory usually underestimates deflection and overestimates the buckling loads for thick plates. Some shortcomings of the classical plate theory are modified by a number of shear deformable plate theories the simplest of which is the first-order shear deformation theory (FSDT). The FSDT yields a constant value of transverse shear strain through the thickness of the plate and requires shear correction factors to account for the deviation of the actual transverse shear strain from the constant one. These shear correction factors are sensitive to the geometric parameters of plate, boundary conditions and loading conditions. A number of shear deformation theories have been proposed to date. The first such theory for laminated isotropic plates was apparently [1]. This theory was generalized to laminated anisotropic plates in [2]. It was shown in [3, 4], the FSDT violates equilibrium conditions at the top and bottom faces of the plate, shear correction factors are required to rectify the unrealistic variation of the shear strain/stress through the thickness. To overcome the drawbacks of the FSDT, various higher-order plate theories have been proposed by assuming higher-order displacement fields. For instance, Reddy

[5, 6] developed a simple higher-order shear deformation plate theory by using third-order polynomial in the expansion of the displacement components through the thickness of the plate. In order to overcome the limitations of FSDT, higher-order shear deformation theories HSDT are used. Since which involve higher-order terms in Taylor's expansions of the displacements in the thickness coordinate, were developed by Librescu [7], Ren [8], Kant and Pandya [9], and Mohan et al. [10]. A good review of these theories for the analysis of laminated composite plates is available in Refs. [11 – 15]. Applications to the laminated composite plates for problems of buckling of thick plates have been discussed by a number of authors. Reddy and Phan [16] have used the Navier solution in order to analyze the free vibration and buckling of isotropic, orthotropic and laminated rectangular plates with simply supported edge condition according to the laminated composite plates of Reddy [5]. Hanna and Leissa [17] have developed a completely higher order shear deformation plate theory, including energy functional, equation of motion and boundary condition. They have used Rayleigh – Ritz method for free vibration solution of fully free rectangular plate. Doong [18] have used the average stress method in order to develop high order plate theory in which an arbitrary initial stress state is included. The governing equations are obtained using a perturbation technique. He has presented the Navier solution for natural frequencies and buckling loads for simply supported rectangular plate. Stability analysis have been performed for simply supported plate by Navier method. A broad literature survey on the vibration analysis of shear deformable plates also have been done by Liew et al. [19].

In this work, a refined exponential theory of plates is presented and applied to the investigation of buckling analysis of anti-symmetric cross-ply laminated composite plates under different boundary conditions. The governing equations of anti-symmetric cross-ply laminated composite plates are given based on the exponential shear deformation plate theory. The equations of motion are derived using Hamilton's principle. The fundamental frequencies are found by solving an eigenvalue equation. The results obtained by the present method are compared with solutions and results of the other higher-order theories. The influences of several parameters are discussed.

2. Refined plate theory for laminated composite plates.

2.1. Basic assumptions.

Consider a rectangular plate of total thickness h composed of n orthotropic layers with the coordinate system as shown in Figure 1. Assumptions of the refined plate's theory are as follows:

The displacements are small in comparison with the plate thickness and, therefore, strains involved are infinitesimal.

The transverse displacement w includes three components of bending w_b and shear w_s. These components are functions of coordinates x and y only.

$$w(x,y,z) = w_b(x,y) + w_s(x,y). \tag{1}$$

The transverse normal stress σ_z is negligible in comparison with in-plane stresses σ_x and σ_y.

The displacements U in x-direction and V in y-direction consist of extension, bending, and shear components :

$$U = u + u_b + u_s\,,\quad V = v + v_b + v_s\,. \tag{2}$$

The bending components u_b and v_b are assumed to be similar to the displacements given by the classical plate theory. Therefore, the expression for u_b and v_b can be given as:

$$u_b = -z\frac{\partial w_b}{\partial x}\,,\quad v_b = -z\frac{\partial w_b}{\partial y}\,. \tag{3a}$$

The shear components u_s and v_s give rise, in conjunction with w_s, to the parabolic variations of shear strains γ_{xz}, γ_{yz} and hence to shear stresses σ_{xz}, σ_{yz} through the thickness of the plate in such a way that shear stresses σ_{xz}, σ_{yz} are zero at the top and bottom faces of the plate. Consequently, the expression for u_s and v_s can be given as:

$$u_s = f(z)\frac{\partial w_s}{\partial x}\,,\quad v_s = f(z)\frac{\partial w_s}{\partial y}\,. \tag{3b}$$

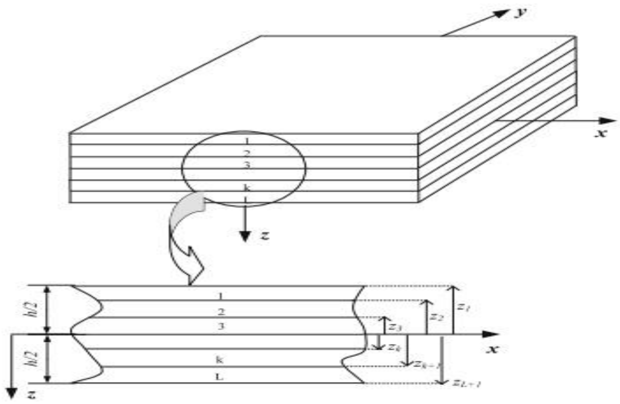

Figure 1.
Coordinate system used for a typical laminated plate.

2.2. Kinematics

Based on the assumptions made in the preceding section, the displacement field can be obtained using Eqs. (1) – (3) as:

$$u(x,y,z) = u(x,y) - z\frac{\partial w_b}{\partial x} + f(z)\frac{\partial w_s}{\partial x};$$

$$v(x,y,z) = v(x,y) - z\frac{\partial w_b}{\partial y} + f(z)\frac{\partial w_s}{\partial y};$$

$$w(x,y,z) = w_b(x,y) + w_s(x,y)\,, \tag{4a}$$

where *u* and *v* are the mid-plane displacements of the plate in the x and y direction, respectively; w_b and w_s are the bending and shear components of transverse displacement, respectively, while $f(z)$ represents shape functions determining the distribution of the transverse shear strains and stresses along the thickness and is given as the present model; the function f(z) is an exponential shape function (Exponential Shear Deformation Theory):

$$f(z) = z - ze^{\left(-2\frac{z^2}{h^2}\right)}. \tag{4b}$$

It should be noted that unlike the first-order shear deformation theory, this theory does not require shear correction factors. The strains associated with the displacements in Eq. (4) are:

$$\varepsilon_x = \varepsilon_x^0 + z\ k_x^b + f\ k_x^s;$$

$$\varepsilon_y = \varepsilon_y^0 + z\ k_y^b + f\ k_y^s;$$

$$\gamma_{xy} = \gamma_{xy}^0 + z\ k_{xy}^b + f\ k_{xy}^s;$$

$$\gamma_{yz} = g\ \gamma_{yz}^s; \tag{5}$$

$$\gamma_{xz} = g\ \gamma_{xz}^s;$$

$$\varepsilon_z = 0\,,$$

where:

$$\varepsilon_x^0 = \frac{\partial u}{\partial x}, \quad k_x^b = -\frac{\partial^2 w_b}{\partial x^2}, \quad k_x^s = -\frac{\partial^2 w_s}{\partial x^2}, \quad \varepsilon_y^0 = \frac{\partial v}{\partial y}, \quad k_y^b = -\frac{\partial^2 w_b}{\partial y^2}, \quad k_y^s = -\frac{\partial^2 w_s}{\partial y^2};$$

$$\gamma_{xy}^0 = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, \quad k_{xy}^b = -2\frac{\partial^2 w_b}{\partial x \partial y}, \quad k_{xy}^s = -2\frac{\partial^2 w_s}{\partial x \partial y}, \quad \gamma_{yz}^s = \frac{\partial w_s}{\partial y}, \quad \gamma_{xz}^s = \frac{\partial w_s}{\partial x}; \quad (6)$$

$$g(z) = 1 - f'(z) \text{ and } f'(z) = \frac{df(z)}{dz}.$$

2.3. Constitutive equations

Under the assumption that each layer possesses a plane of elastic symmetry parallel to the x-y plane, the constitutive equations for a layer can be written as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{66} & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 \\ 0 & 0 & 0 & 0 & Q_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}, \quad (7)$$

where Q_{ij} are the plane stress-reduced stiffnesses, and are known in terms of the engineering constants in the material axes of the layer:

$$Q_{11} = \frac{E_{11}}{1 - \nu_{12}\nu_{21}}, \quad Q_{22} = \frac{E_{22}}{1 - \nu_{12}\nu_{21}}, \quad Q_{12} = \frac{\nu_{12}E_{22}}{1 - \nu_{12}\nu_{21}},$$

$$Q_{66} = G_{12}, \quad Q_{44} = G_{23}, \quad Q_{55} = G_{13}. \quad (8)$$

Since the laminate is made of several orthotropic layers with their material axes oriented arbitrarily with respect to the laminate coordinates, the constitutive equations of each layer must be transformed to the laminate coordinates (*x*, *y*, *z*). The stress-strain relations in the laminate coordinates of the k^{th} layer are given as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{xz} \end{Bmatrix}^{(k)} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} & 0 & 0 \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} & 0 & 0 \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & \bar{Q}_{44} & \bar{Q}_{45} \\ 0 & 0 & 0 & \bar{Q}_{45} & \bar{Q}_{55} \end{bmatrix}^{(k)} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}^{(k)}, \quad (9)$$

where $\bar{Q}_{ij}$ are the transformed material constants given as

$$\bar{Q}_{11} = Q_{11}\cos^4\theta + 2(Q_{12} + 2Q_{66})\sin^2\theta\cos^2\theta + Q_{22}\sin^4\theta;$$

$$\bar{Q}_{12} = (Q_{11} + Q_{22} - 4Q_{66})\sin^2\theta\cos^2\theta + Q_{12}(\sin^4\theta + \cos^4\theta);$$

$$\bar{Q}_{22} = Q_{11}\sin^4\theta + 2(Q_{12} + 2Q_{66})\sin^2\theta\cos^2\theta + Q_{22}\cos^4\theta;$$

$$\bar{Q}_{16} = (Q_{11} - Q_{12} - 2Q_{66})\sin\theta\cos^3\theta + (Q_{12} - Q_{22} + 2Q_{66})\sin^3\theta\cos\theta;$$

$$\bar{Q}_{26}=(Q_{11}-Q_{12}-2Q_{66})\sin^3\theta\cos\theta+(Q_{12}-Q_{22}+2Q_{66})\sin\theta\cos^3\theta\ ; \tag{10}$$

$$\bar{Q}_{66}=(Q_{11}+Q_{22}-2Q_{12}-2Q_{66})\sin^2\theta\cos^2\theta+Q_{66}(\sin^4\theta+\cos^4\theta)\ ;$$

$$\bar{Q}_{44}=Q_{44}\cos^2\theta+Q_{55}\sin^2\theta\ ;$$

$$\bar{Q}_{45}=(Q_{55}-Q_{44})\cos\theta\sin\theta\ ;$$

$$\bar{Q}_{55}=Q_{55}\cos^2\theta+Q_{44}\sin^2\theta\ ,$$

in which θ is the angle between the global x-axis and the local x-axis of each lamina.

2.4. Governing equations

The strain energy of the plate can be written as

$$U=\frac{1}{2}\int_V\sigma_{ij}\varepsilon_{ij}dV=\frac{1}{2}\int_V(\sigma_x\varepsilon_x+\sigma_y\varepsilon_y+\sigma_{xy}\gamma_{xy}+\sigma_{yz}\gamma_{yz}+\sigma_{xz}\gamma_{xz})dV. \tag{11}$$

Substituting Eqs. (5) and (9) into Eq. (11) and integrating through the thickness of the plate, the strain energy of the plate can be rewritten as

$$\begin{aligned}U=\frac{1}{2}\int_A\Big\{&N_x\varepsilon_x^0+N_y\varepsilon_y^0+N_{xy}\gamma_{xy}^0+M_x^bk_x^b+M_y^bk_y^b+M_{xy}^bk_{xy}^b;\\&+M_x^sk_x^s+M_y^sk_y^s+M_{xy}^sk_{xy}^s+Q_{yz}^s\gamma_{yz}^s+Q_{xz}^s\gamma_{xz}^s\Big\}dxdy,\end{aligned} \tag{12}$$

where the stress resultants *N*, *M*, and *Q* are defined by

$$\begin{aligned}(N_x,N_y,N_{xy})&=\int_{-h/2}^{h/2}(\sigma_x,\sigma_y,\sigma_{xy})dz=\sum_{k=1}^{N}\int_{z_k}^{z_{k+1}}(\sigma_x,\sigma_y,\sigma_{xy})dz;\\(M_x^b,M_y^b,M_{xy}^b)&=\int_{-h/2}^{h/2}(\sigma_x,\sigma_y,\sigma_{xy})zdz=\sum_{k=1}^{N}\int_{z_k}^{z_{k+1}}(\sigma_x,\sigma_y,\sigma_{xy})zdz;\\(M_x^s,M_y^s,M_{xy}^s)&=\int_{-h/2}^{h/2}(\sigma_x,\sigma_y,\sigma_{xy})fdz=\sum_{k=1}^{N}\int_{z_k}^{z_{k+1}}(\sigma_x,\sigma_y,\sigma_{xy})fdz;\\(Q_{xz}^s,Q_{yz}^s)&=\int_{-h/2}^{h/2}(\sigma_{xz},\sigma_{yz})gdz=\sum_{k=1}^{N}\int_{z_k}^{z_{k+1}}(\sigma_{xz},\sigma_{yz})gdz.\end{aligned} \tag{13}$$

Substituting Eq. (9) into Eq. (13) and integrating through the thickness of the plate, the stress resultants are given as

$$\begin{Bmatrix}\begin{Bmatrix}N_x\\N_y\\N_{xy}\end{Bmatrix}\\\begin{Bmatrix}M_x^b\\M_y^b\\M_{xy}^b\end{Bmatrix}\\\begin{Bmatrix}M_x^s\\M_y^s\\M_{xy}^s\end{Bmatrix}\end{Bmatrix}=\begin{bmatrix}\begin{bmatrix}A_{11}&A_{12}&A_{16}\\A_{12}&A_{22}&A_{26}\\A_{16}&A_{26}&A_{66}\end{bmatrix}&\begin{bmatrix}B_{11}&B_{12}&B_{16}\\B_{12}&B_{22}&B_{26}\\B_{16}&A_{26}&A_{66}\end{bmatrix}&\begin{bmatrix}B_{11}^s&B_{12}^s&B_{16}^s\\B_{12}^s&B_{22}^s&B_{26}^s\\B_{16}^s&B_{26}^s&B_{66}^s\end{bmatrix}\\\begin{bmatrix}B_{11}&B_{12}&B_{16}\\B_{12}&B_{22}&B_{26}\\B_{16}&A_{26}&A_{66}\end{bmatrix}&\begin{bmatrix}D_{11}&D_{12}&D_{16}\\D_{12}&D_{22}&D_{26}\\D_{16}&D_{26}&D_{66}\end{bmatrix}&\begin{bmatrix}D_{11}^s&D_{12}^s&D_{16}^s\\D_{12}^s&D_{22}^s&D_{26}^s\\D_{16}^s&D_{26}^s&D_{66}^s\end{bmatrix}\\\begin{bmatrix}B_{11}^s&B_{12}^s&B_{16}^s\\B_{12}^s&B_{22}^s&B_{26}^s\\B_{16}^s&B_{26}^s&B_{66}^s\end{bmatrix}&\begin{bmatrix}D_{11}^s&D_{12}^s&D_{16}^s\\D_{12}^s&D_{22}^s&D_{26}^s\\D_{16}^s&D_{26}^s&D_{66}^s\end{bmatrix}&\begin{bmatrix}H_{11}^s&H_{12}^s&H_{16}^s\\H_{12}^s&H_{22}^s&H_{26}^s\\H_{16}^s&H_{26}^s&H_{66}^s\end{bmatrix}\end{bmatrix}\begin{Bmatrix}\begin{Bmatrix}\varepsilon_x^0\\\varepsilon_y^0\\\gamma_{xy}^0\end{Bmatrix}\\\begin{Bmatrix}k_x^b\\k_y^b\\k_{xy}^b\end{Bmatrix}\\\begin{Bmatrix}k_x^s\\k_y^s\\k_{xy}^s\end{Bmatrix}\end{Bmatrix}. \tag{14a}$$

$$\begin{Bmatrix} Q_{yz}^s \\ Q_{xz}^s \end{Bmatrix} = \begin{bmatrix} A_{44}^s & A_{45}^s \\ A_{45}^s & A_{55}^s \end{bmatrix} \begin{Bmatrix} \gamma_{yz}^s \\ \gamma_{xz}^s \end{Bmatrix}, \tag{14b}$$

where A_{ij} , B_{ij} , etc., are the plate stiffness, defined by

$$\left(A_{ij}, B_{ij}, D_{ij}, B_{ij}^s, D_{ij}^s, H_{ij}^s \right) = \int_{-h/2}^{h/2} \bar{Q}_{ij} (1, z, z^2, f(z), zf(z), f^2(z)) dz \qquad (i,j) = (1,2,6); \tag{15a}$$

$$A_{ij}^s = \int_{-h/2}^{h/2} \bar{Q}_{ij} \left[g(z) \right]^2 dz, \qquad (i,j) = (4,5). \tag{15b}$$

The work done by applied forces can be written as

$$V = \frac{1}{2} \int_A \left[N_x^0 \frac{\partial^2 (w_b + w_s)}{\partial x^2} + N_y^0 \frac{\partial^2 (w_b + w_s)}{\partial y^2} + 2 N_{xy}^0 \frac{\partial^2 (w_b + w_s)}{\partial x \partial y} \right] dxdy, \tag{16}$$

where N_x^0 , N_y^0 and N_{xy}^0 are in-plane distributed forces.

The virtual work principle is used here in order to derive the equilibrium equations appropriate to the displacement field and the constitutive equation. The principle can be stated in analytical form as

$$U + V = 0, \tag{17}$$

where δ indicates a variation with respect to *x* and *y*.

Substituting Eqs.(12) and (16) into Eq. (17) and integrating the equation by parts, collecting the coefficients of δu , δv , δw_b and δw_s , the equilibrium equations for the laminate plate are obtained as follows:

$$\delta u : \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0;$$

$$\delta v : \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0;$$

$$\delta w_b : \frac{\partial^2 M_x^b}{\partial x^2} + 2 \frac{\partial^2 M_{xy}^b}{\partial x \partial y} + \frac{\partial^2 M_y^b}{\partial y^2} + N(w) = 0; \tag{18}$$

$$\delta w_s : \frac{\partial^2 M_x^s}{\partial x^2} + 2 \frac{\partial^2 M_{xy}^s}{\partial x \partial y} + \frac{\partial^2 M_y^s}{\partial y^2} + \frac{\partial Q_{xz}^s}{\partial x} + \frac{\partial Q_{yz}^s}{\partial y} + N(w) = 0,$$

where $N(w)$ is defined by

$$N(w) = N_x^0 \frac{\partial^2 (w_b + w_s)}{\partial x^2} + N_y^0 \frac{\partial^2 (w_b + w_s)}{\partial y^2} + 2 N_{xy}^0 \frac{\partial^2 (w_b + w_s)}{\partial x \partial y}. \tag{19}$$

Eq. (18) can be expressed in terms of displacements (u , v , w_b , w_s) by substituting for the stress resultants from Eq. (14). For homogeneous laminates, the equilibrium equations (18) take the form

$$A_{11} \frac{\partial^2 u}{\partial x^2} + 2 A_{16} \frac{\partial^2 u}{\partial x \partial y} + A_{66} \frac{\partial^2 u}{\partial y^2} + A_{16} \frac{\partial^2 v}{\partial x^2} + (A_{12} + A_{66}) \frac{\partial^2 v}{\partial x \partial y} + A_{26} \frac{\partial^2 v}{\partial y^2} ;$$

$$-B_{11}\frac{\partial^3 w_b}{\partial x^3}-3B_{16}\frac{\partial^3 w_b}{\partial x^2\partial y}-(B_{12}+2B_{66})\frac{\partial^3 w_b}{\partial x\partial y^2}-B_{26}\frac{\partial^3 w_b}{\partial y^3};$$

$$-B_{11}^s\frac{\partial^3 w_s}{\partial x^3}-3B_{16}^s\frac{\partial^3 w_s}{\partial x^2\partial y}-(B_{12}^s+2B_{66}^s)\frac{\partial^3 w_s}{\partial x\partial y^2}-B_{26}^s\frac{\partial^3 w_s}{\partial y^3}=0\,; \tag{20a}$$

$$A_{16}\frac{\partial^2 u}{\partial x^2}+(A_{12}+A_{66})\frac{\partial^2 u}{\partial x\partial y}+A_{26}\frac{\partial^2 u}{\partial y^2}+A_{66}\frac{\partial^2 v}{\partial x^2}+2A_{26}\frac{\partial^2 v}{\partial x\partial y}+A_{22}\frac{\partial^2 v}{\partial y^2};$$

$$-B_{16}\frac{\partial^3 w_b}{\partial x^3}-(B_{12}+2B_{66})\frac{\partial^3 w_b}{\partial x^2\partial y}-3B_{26}\frac{\partial^3 w_b}{\partial x\partial y^2}-B_{22}\frac{\partial^3 w_b}{\partial y^3};$$

$$-B_{16}^s\frac{\partial^3 w_s}{\partial x^3}-(B_{12}^s+2B_{66}^s)\frac{\partial^3 w_s}{\partial x^2\partial y}-3B_{26}^s\frac{\partial^3 w_s}{\partial x\partial y^2}-B_{22}^s\frac{\partial^3 w_s}{\partial y^3}=0\,; \tag{20b}$$

$$B_{11}\frac{\partial^3 u}{\partial x^3}+3B_{16}\frac{\partial^3 u}{\partial x^2\partial y}+(B_{12}+2B_{66})\frac{\partial^3 u}{\partial x\partial y^2}+B_{26}\frac{\partial^3 u}{\partial y^3}+$$

$$+B_{16}\frac{\partial^3 v}{\partial x^3}+(B_{12}+2B_{66})\frac{\partial^3 v}{\partial x^2\partial y}+3B_{26}\frac{\partial^3 v}{\partial x\partial y^2}+B_{22}\frac{\partial^3 v}{\partial y^3}-$$

$$-D_{11}\frac{\partial^4 w_b}{\partial x^4}-4D_{16}\frac{\partial^4 w_b}{\partial x^3\partial y}-2(D_{12}+2D_{66})\frac{\partial^4 w_b}{\partial x^2\partial y^2}-4D_{26}\frac{\partial^4 w_b}{\partial x\partial y^3}-D_{22}\frac{\partial^4 w_b}{\partial y^4}-$$

$$-D_{11}^s\frac{\partial^4 w_s}{\partial x^4}-4D_{16}^s\frac{\partial^4 w_s}{\partial x^3\partial y}-2(D_{12}^s+2D_{66}^s)\frac{\partial^4 w_s}{\partial x^2\partial y^2}-4D_{26}^s\frac{\partial^4 w_s}{\partial x\partial y^3}-D_{22}^s\frac{\partial^4 w_s}{\partial y^4}+N(w)=0\,; \tag{20c}$$

$$B_{11}^s\frac{\partial^3 u}{\partial x^3}+3B_{16}^s\frac{\partial^3 u}{\partial x^2\partial y}+(B_{12}^s+2B_{66}^s)\frac{\partial^3 u}{\partial x\partial y^2}+B_{26}^s\frac{\partial^3 u}{\partial y^3}+$$

$$+B_{16}^s\frac{\partial^3 v}{\partial x^3}+(B_{12}^s+2B_{66}^s)\frac{\partial^3 v}{\partial x^2\partial y}+3B_{26}^s\frac{\partial^3 v}{\partial x\partial y^2}+B_{22}^s\frac{\partial^3 v}{\partial y^3}-$$

$$-D_{11}^s\frac{\partial^4 w_b}{\partial x^4}-4D_{16}^s\frac{\partial^4 w_b}{\partial x^3\partial y}-2(D_{12}^s+2D_{66}^s)\frac{\partial^4 w_b}{\partial x^2\partial y^2}-4D_{26}^s\frac{\partial^4 w_b}{\partial x\partial y^3}-D_{22}^s\frac{\partial^4 w_b}{\partial y^4}-$$

$$-H_{11}^s\frac{\partial^4 w_s}{\partial x^4}-4H_{16}^s\frac{\partial^4 w_s}{\partial x^3\partial y}-2(H_{12}^s+2H_{66}^s)\frac{\partial^4 w_s}{\partial x^2\partial y^2}-4H_{26}^s\frac{\partial^4 w_s}{\partial x\partial y^3}-H_{22}^s\frac{\partial^4 w_s}{\partial y^4}+$$

$$+A_{55}^s\frac{\partial^2 w_s}{\partial x^2}+A_{44}^s\frac{\partial^2 w_s}{\partial y^2}+2A_{45}^s\frac{\partial^2 w_s}{\partial x\partial y}+N(w)=0\,. \tag{20d}$$

3. Exact solution for antisymmetric cross-ply laminates

For anti-symmetric cross-ply laminates, the following plate stiffnesses are identically zero:

$$A_{16}=A_{26}=D_{16}=D_{26}=D_{16}^s=D_{26}^s=H_{16}^s=H_{26}^s=0\,;$$

$$B_{22}=-B_{11};\;\; B_{22}^s=-B_{11}^s; \tag{21}$$

$$B_{12}=B_{26}=B_{16}=B_{66}=B_{12}^s=B_{16}^s=B_{26}^s=B_{66}^s=A_{45}^s=0\,.$$

The exact solution of equations (20) for the anti-symmetric cross-ply laminated plate under various boundary conditions can be constructed according to [20]. The boundary conditions for an arbitrary edge with simply supported and clamped edge conditions are:

Clamped (*C*):

$$u = v = w_b = w_s = \frac{\partial w_b}{\partial x} = \frac{\partial w_b}{\partial y} = \frac{\partial w_s}{\partial x} = \frac{\partial w_s}{\partial y} = 0 \quad \text{at} \quad x = 0, a \quad \text{and} \quad y = 0, b\,; \tag{22}$$

Simply supported (*S*):

$$v = w_b = w_s = \frac{\partial w_b}{\partial y} = \frac{\partial w_s}{\partial y} = 0 \qquad \text{at} \qquad x = 0, a\,; \tag{23a}$$

$$u = w_b = w_s = \frac{\partial w_b}{\partial x} = \frac{\partial w_s}{\partial x} = 0 \qquad \text{at} \qquad y = 0, b\,. \tag{23b}$$

The boundary conditions in Eq. (22) and (23) are satisfied by the following expansions

$$\begin{aligned} u &= U_{mn} X'_m(x)\, Y_n(y); \\ v &= V_{mn}\, X_m(x)\, Y'_n(y); \\ w_b &= W_{bmn}\;\; X_m(x)\, Y_n(y); \\ w_s &= W_{smn}\;\; X_m(x)\, Y_n(y), \end{aligned} \tag{24}$$

where U_{mn}, V_{mn}, W_{bmn} *and* W_{smn} unknown parameters must be determined. The functions $X_m(x)$ and $Y_n(y)$ are suggested here to satisfy at least the geometric boundary conditions given in equations (22) and (23) and represent approximate shapes of the deflected surface of the plate. These functions, for the different cases of boundary conditions, are listed in Table 1, with $\alpha = \frac{m\pi}{a}$ and $\beta = \frac{n\pi}{b}$.

Table 1. The admissible functions $X_m(x)$ and $Y_n(y)$

	Boundary conditions		The functions $X_m(x)$ and $Y_n(y)$	
	At *x=0, a*	At *y=0, b*	$X_m(x)$	$Y_n(y)$
SSSS	$X_m(0) = X''_m(0) = 0$ $X_m(a) = X''_m(a) = 0$	$Y_n(0) = Y''_n(0) = 0$ $Y_n(b) = Y''_n(b) = 0$	$\sin(\alpha x)$	$\sin(\beta y)$
CSSS	$X_m(0) = X'_m(0) = 0$ $X_m(a) = X''_m(a) = 0$	$Y_n(0) = Y''_n(0) = 0$ $Y_n(b) = Y''_n(b) = 0$	$\sin(\alpha x)[\cos(\alpha x) - 1]$	$\sin(\beta y)$
CSCS	$X_m(0) = X'_m(0) = 0$ $X_m(a) = X''_m(a) = 0$	$Y_n(0) = Y'_n(0) = 0$ $Y_n(b) = Y''_n(b) = 0$	$\sin(\alpha x)[\cos(\alpha x) - 1]$	$\sin(\beta y)[\cos(\beta y) - 1]$
CCSS	$X_m(0) = X'_m(0) = 0$ $X_m(a) = X'_m(a) = 0$	$Y_n(0) = Y''_n(0) = 0$ $Y_n(b) = Y''_n(b) = 0$	$\sin^2(\alpha x)$	$\sin(\beta y)$
CCCC	$X_m(0) = X'_m(0) = 0$ $X_m(a) = X'_m(a) = 0$	$Y_n(0) = Y'_n(0) = 0$ $Y_n(b) = Y'_n(b) = 0$	$\sin^2(\alpha x)$	$\sin^2(\beta y)$
FFCC	$X''_m(0) = X'''_m(0) = 0$ $X''_m(a) = X'''_m(a) = 0$	$Y_n(0) = Y'_n(0) = 0$ $Y_n(b) = Y'_n(b) = 0$	$\cos^2(\alpha x)[\sin^2(\alpha x) + 1]$	$\sin^2(\beta y)$

$(\,)'$ denotes the derivative with respect to the corresponding coordinates.

Substituting Eqs. (24) and (21) into Eq. (20), the exact solution of anti-symmetric cross-ply laminates can be determined from equations

$$\begin{bmatrix} s_{11} & s_{12} & s_{13} & s_{14} \\ s_{21} & s_{22} & s_{23} & s_{24} \\ s_{31} & s_{32} & s_{33}+k & s_{34}+k \\ s_{41} & s_{42} & s_{43}+k & s_{44}+k \end{bmatrix} \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{bmn} \\ W_{smn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}, \tag{25}$$

where

$$s_{11} = \int_0^a\int_0^b \left(A_{11} X_m''' Y_n + A_{66} X_m' Y_n'' \right) X_m' Y_n dxdy ,$$

$$s_{12} = \int_0^a\int_0^b (A_{12} + A_{66}) X_m' Y_n'' X_m' Y_n dxdy ,$$

$$s_{13} = -\int_0^a\int_0^b \left[B_{11} X_m'' Y_n + \left(B_{12} + 2B_{66} \right) X_m' Y_n'' \right] X_m' Y_n dxdy ,$$

$$s_{14} = -\int_0^a\int_0^b \left[B_{11}^s X_m'' Y_n + \left(B_{12}^s + 2B_{66}^s \right) X_m' Y_n'' \right] X_m' Y_n dxdy , \; s_{21} = \int_0^a\int_0^b (A_{12} + A_{66}) X_m'' Y_n' X_m Y_n' dxdy ,$$

$$s_{22} = \int_0^a\int_0^b \left(A_{22} X_m Y_n''' + A_{66} X_m'' Y_n' \right) X_m Y_n' dxdy ,$$

$$s_{23} = -\int_0^a\int_0^b \left[B_{22} X_m Y_n''' + (B_{12} + 2B_{66}) X_m'' Y_n' \right] X_m Y_n' dxdy ,$$

$$s_{24} = -\int_0^a\int_0^b \left[B_{22}^s X_m Y_n''' + \left(B_{12}^s + 2B_{66}^s \right) X_m'' Y_n' \right] X_m Y_n' dxdy ,$$

$$s_{31} = \int_0^a\int_0^b \left[B_{11} X_m'''' Y_n + (B_{12} + 2B_{66}) X_m'' Y_n'' \right] X_m Y_n \, dxdy ,$$

$$s_{32} = \int_0^a\int_0^b \left[B_{22} X_m Y_n'''' + (B_{12} + 2B_{66}) X_m'' Y_n'' \right] X_m Y_n \, dxdy ,$$

$$s_{33} = \int_0^a\int_0^b -\left[D_{11} X_m'''' Y_n + 2(D_{12} + 2D_{66}) X_m'' Y_n'' + D_{22} X_m Y_n'''' \right] X_m Y_n \, dxdy , \tag{26}$$

$$s_{34} = \int_0^a\int_0^b -\left[D_{11}^s X_m'''' Y_n + 2\left(D_{12}^s + 2D_{66}^s \right) X_m'' Y_n'' + D_{22}^s X_m Y_n'''' \right] X_m Y_n \, dxdy ,$$

$$s_{41} = \int_0^a\int_0^b \left[B_{11}^s X_m'''' Y_n + \left(B_{12}^s + 2B_{66}^s \right) X_m'' Y_n'' \right] X_m Y_n dxdy ,$$

$$s_{42} = \int_0^a \int_0^b \left[B_{22}^s X_m Y_n'''' + \left(B_{12}^s + 2B_{66}^s \right) X_m'' Y_n'' \right] X_m Y_n \, dxdy \; ,$$

$$s_{43} = \int_0^a \int_0^b - \left[D_{11}^s X_m'''' Y_n + 2\left(D_{12}^s + 2D_{66}^s \right) X_m'' Y_n'' + D_{22}^s X_m Y_n'''' \right] X_m Y_n \, dxdy \; ,$$

$$s_{44} = \int_0^a \int_0^b - \left[H_{11}^s X_m'''' Y_n + 2\left(H_{12}^s + 2H_{66}^s \right) X_m'' Y_n'' + H_{22}^s X_m Y_n'''' - A_{55}^s X_m'' Y_n - A_{44}^s X_m Y_n'' \right] X_m Y_n \, dxdy \; ,$$

$$k = N_{cr} \int_0^a \int_0^b \left(\gamma_1 X_m'' Y_n + \gamma_2 X_m Y_n'' \right) X_m Y_n \, dxdy$$

$$\text{and } N_x^0 = \gamma_1 N_{cr} \, , \; N_y^0 = \gamma_2 N_{cr} \, , \text{ and } N_{xy}^0 = 0 \, . \qquad (27)$$

4. Numerical results

In this study, a buckling of anti-symmetrically cross-ply laminated composite plates by using the present shear deformation theory for laminated plates is suggested.

For the verification purpose, the results obtained by present model are compared with those of the CLPT, FSDT, HSDT, and exact solution of three-dimensional elasticity. In all examples, a shear correction factor of 5/6 is used for FSDT. Unless sited otherwise, and for validation purposes, the following lamina properties used according to [23] as: $E_1 = 40E_2$, $G_{12} = G_{13} = 0,6E_2$, $G_{23} = 0,5E_2$, $\nu_{12} = 0,25$, $a/h = 10$, $a/b = 1$.

For convenience, the following dimensionless formula is used in presenting the numerical results in graphical and tabular forms:

$$\bar{N} = N_{cr} \left(\frac{a^2}{E_2 h^3} \right) . \qquad (28)$$

Table 2. Dimensionless uniaxial buckling load of simply supported (SSSS) anti-symmetric cross-ply (0/90)$_n$ square laminates.

Number of layers	Theory	$\bar{N}$
$(0/90)_2$	Present Theory	22.5790
	Exact [21]	21.2796
	Reddy [5]	22.5790
	FSDT [22]	22.8060
	CLPT	30.3591
$(0/90)_3$	Present Theory	24.4596
	Exact [21]	23.6689
	Reddy [5]	24.4596
	FSDT [22]	24.5777
	CLPT	33.5817
$(0/90)_5$	Present Theory	25.4225
	Exact [21]	24.9636
	Reddy [5]	25.4225
	FSDT [22]	25.4500
	CLPT	35.2316

Table 3. Stacking sequence effect on the variation of the critical buckling load $\overline{N}$ of a simply supported (SSSS) anti-symmetric cross-ply $(0/90)_n$ square laminates.

Number of layers	a/h			
	5	20	50	100
$(0/90)_1$	8,7694	12,5770	12,8949	12,9416
$(0/90)_2$	12,8466	27,9451	29,9449	30,2545
$(0/90)_3$	13,5747	30,7097	33,0862	33,4564
$(0/90)_4$	13,8389	31,6757	34,1852	34,5770
$(0/90)_5$	13,9630	32,1227	34,6939	35,0956
$(0/90)_8$	14,0990	32,6069	35,2449	35,6575
$(0/90)_{16}$	14,1650	32,8397	35,5098	35,9276
$(0/90)_{32}$	14,1815	32,8979	35,5761	35,9951

A simply supported anti-symmetric cross-ply $(0/90)_n$ (n = 2, 3, 5) square laminate subjected to uniaxial compressive load is considered. Table 2 shows a comparison between the results obtained using the various models and the three-dimensional elasticity solutions given by Noor [21].

The results clearly indicate that the present model gives more accurate results in predicting the buckling loads when compared to Reddy [5], and indicates that Reddy's theory is closer to the present model. Compared to the three-dimensional elasticity solution, the buckling loads predicted by present model, Reddy [5], and FSDT [22] are 6 % to 7 %, respectively, for four-layer anti-symmetric cross-ply (0/90/0/90) square laminates.

The effect of side-to-thickness ratio on buckling load of simply supported four-layer (0/90/0/90) square laminates is also presented in Fig. 2.

Table 4. The effect of modulus ratio on the variation of the critical buckling load $\overline{N}$ of a simply supported (SSSS) anti-symmetric cross-ply $(0/90)_n$ square laminates.

Number of layers	E_1/E_2					
	5	10	20	30	40	50
$(0/90)_1$	5.2597	6.2721	8.1151	9.8695	11.5625	13.2019
$(0/90)_2$	6.4394	9.2315	14.2543	18.6671	22.5790	26.0724
$(0/90)_3$	6.6599	9.7762	15.3518	20.2010	24.4596	28.2310
$(0/90)_4$	6.7374	9.9672	15.7361	20.7378	25.1176	28.9866
$(0/90)_5$	6.7732	10.0557	15.9141	20.9864	25.4225	29.3368
$(0/90)_8$	6.8122	10.1517	16.1071	21.2559	25.7530	29.7167
$(0/90)_{16}$	6.8309	10.1979	16.2000	21.3856	25.9121	29.8996
$(0/90)_{32}$	6.8677	10.2270	16.2190	21.3998	25.9248	29.9131

Table 5. The stacking sequence effect on the variation of the critical buckling load $\overline{N}$ of an anti-symmetric cross-ply $(0/90)_n$ square laminates.

Number of layers	SSSS	CSSS	CSCS	CCSS	CCCC	FFCC
$(0/90)_1$	11.5625	15.6796	23.8302	21.6053	32.5819	40.4980
$(0/90)_2$	22.5790	28.0782	40.2425	36.4605	50.1704	58.7157
$(0/90)_3$	24.4596	30.1404	42.9808	38.9277	53.1913	61.9980
$(0/90)_4$	25.1176	30.8669	43.9520	39.8040	54.2816	63.2002
$(0/90)_5$	25.4225	31.2046	44.4046	40.2127	54.7933	63.7675
$(0/90)_8$	25.7530	31.5715	44.8974	40.6578	55.3531	64.3902
$(0/90)_{16}$	25.9121	31.7485	45.1353	40.8728	55.6242	64.6927
$(0/90)_{32}$	25.9248	31.8484	45.1835	40.9699	55.6993	64.7282

The critical buckling loads $\overline{N}$ have been tabulated in Tables 3 – 5 of an anti-symmetric cross-ply (0/90)*n* square laminates. These results are obtained for various boundary conditions and loading conditions, some power of laminate plate and different thickness-side and aspect ratios. we can say that, according to Tables 3; 4 and 5, it is observed that for some boundary conditions by increasing the aspect ratio, the critical buckling load remains in the constant value, whereas the numbers of half waves in the x direction of critical buckling mode shape increases.

The Numerical results are tabulated in Table 6 for the modulus ratio effect and in Table 7 for the aspect ratio effect on the variation of the critical buckling load $\overline{N}$ of an anti-symmetric cross-ply $(0/90)_4$ square laminates for different boundary conditions. We can conclude that this model is not only accurate, but also easy to predict the critical buckling loads of laminated plates. The effect of modulus ratio on dimensionless uniaxial buckling load of simply supported multilayered anti-symmetric cross-ply square laminates (Fig. 3), the critical buckling load of laminated square plates are illustrated in Figs. 3 and 4, respectively. In light of the results obtained; we can see that increasing the $E1\ /\ E2$ ratio implies an increase in the critical buckling load, and the geometrical and mechanical characteristics of the plate have a direct influence on the critical load buckling. Figures 5 and 6 show the boundary conditions effect and the aspect ratio effect on the variation of the critical buckling load $\overline{N}$ of an anti-symmetric cross-ply $(0/90)_4$ square laminates. As it is well known, the magnitudes of buckling loads are over predicted in the case of the clamped boundary condition. It can be observed that the influence of the ratio on the buckling loads for different boundary conditions. In addition, it can be seen that both the buckling loads increase with the different boundary conditions.

5. Conclusion.

In this analytical , investigations on the buckling of anti-symmetric cross-ply laminated composite plates under different boundary conditions is presented using the exponential shear deformation plate theory. The theory allows for a square-law variation in the transverse shear strains across the plate thickness and satisfies the zero-traction boundary conditions on the top and bottom surfaces of the plate without using shear correction factors. The equations of motion were derived from Hamilton's principle. The accuracy and efficiency of the present model has been demonstrated for buckling behaviors of anti-symmetric cross-ply laminated composite plates under different boundary conditions, and the results obtained by the present model are compared with other shear deformation theories being in literature. The conclusions of this theory are as follows:

The buckling load obtained using the present model with four unknowns and height order shear deformation Reddy's theory [5] with five unknowns are in good agreement.

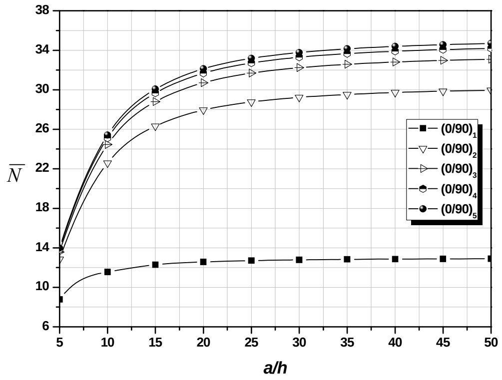

Fig. 2.
The effect of side-to-thickness ratio on dimensionless uniaxial buckling load of simply supported (SSSS) anti-symmetric cross-ply $(0/90)_n$ *square laminates.*

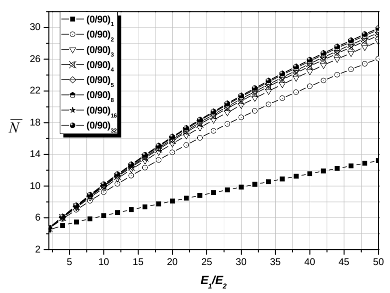

Fig. 3.
The effect of modulus ratio on dimensionless uniaxial buckling load of simply supported (SSSS) multilayered anti-symmetric cross-ply square laminates.

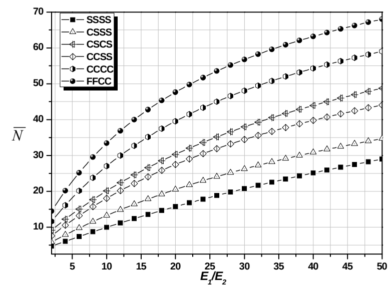

Fig. 4.
The boundary conditions effect on the variation of the critical buckling load $\overline{N}$ *of an anti-symmetric cross-ply* $(0/90)_4$ *square laminates.*

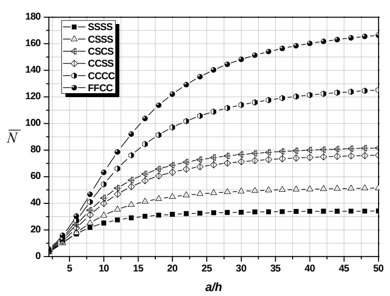

Fig. 5.
The boundary conditions effect on the variation of the critical buckling load $\overline{N}$ *of an anti-symmetric cross-ply* $(0/90)_4$ *square laminates.*

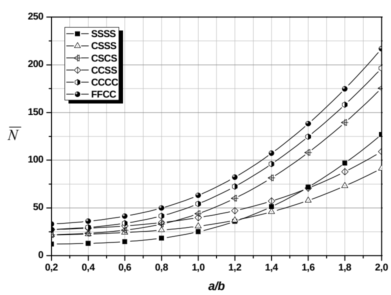

Fig. 6.
The aspect ratio effect on the variation of the critical buckling load $\overline{N}$ *of an anti-symmetric cross-ply* $(0/90)_4$ *square laminates*

Table 6. The modulus ratio effect on the variation of the critical buckling load $\overline{N}$ of an anti-symmetric cross-ply $(0/90)_4$ square laminates for different boundary conditions.

Number of layers	a/h	E_1/E_2					
		5	10	20	30	40	50
SSSS	5	5.5200	7.5135	10.3900	12.3759	13.8389	14.9687
	10	6.7374	9.9672	15.7361	20.7378	25.1176	28.9866
	20	7.1319	10.8584	18.0830	25.0163	31.6757	38.0773
	50	7.2509	11.1377	18.8729	26.5553	34.1852	41.7631
	100	7.2683	11.1787	18.9915	26.7910	34.5770	42.3495
CSSS	5	6.5195	8.6544	11.2871	12.8767	13.9638	14.7704
	10	8.8330	13.2593	20.5219	26.2400	30.8669	34.6939
	20	9.6986	15.3157	25.9077	35.7243	44.8492	53.3541
	50	9.9726	16.0128	27.9718	39.7736	51.4219	62.9201
	100	10.0130	16.1176	28.2941	40.4292	52.5237	64.5781
CSCS	5	9.4479	12.1736	15.3465	17.1897	18.4353	19.3614
	10	13.6468	20.1364	30.3249	37.9773	43.9520	48.7592
	20	15.3672	24.1305	40.3893	55.1570	68.6330	80.9820
	50	15.9307	25.5542	44.5520	63.2278	81.5908	99.6493
	100	16.0146	25.7716	45.2187	64.5804	83.8583	103.0534
CCSS	5	8.2534	10.8055	13.6761	15.3044	16.3947	17.2036
	10	11.9149	18.0424	27.4861	34.4453	39.8040	44.0709
	20	13.4140	21.7258	37.0480	50.8586	63.3748	74.7731
	50	13.9048	23.0475	41.0716	58.7635	76.1342	93.1929
	100	13.9779	23.2497	41.7198	60.1007	78.3950	96.6038
CCCC	5	11.4487	14.4464	17.5457	19.2425	20.3906	21.2677
	10	18.2147	27.0472	39.5748	48.0838	54.2816	59.0281
	20	21.413	34.7654	58.5509	79.1234	97.1033	112.9594
	50	22.5230	37.7997	67.7187	96.8375	125.1918	152.8123
	100	22.6912	38.2775	69.2715	100.0470	130.6103	160.9645
FFCC	5	13.4242	16.6078	19.7489	21.4622	22.6507	23.5883
	10	22.7620	33.4224	47.6390	56.7713	63.2002	68.0204
	20	27.6316	45.0962	75.3323	100.6289	122.1242	140.6297
	50	29.3986	50.0162	90.1632	128.9493	166.4485	202.7261
	100	29.6699	50.8092	92.7796	134.3724	175.5989	216.4655

Table 7. The aspect ratio effect on the variation of the critical buckling load $\overline{N}$ of an anti-symmetric cross-ply $(0/90)_4$ square laminates for different boundary conditions.

Number of layers	a/b	a/h				
		5	10	20	50	100
SSSS	0.5	7.9349	13.5340	16.4795	17.5527	17.7177
	1	13.8389	25.1176	31.6757	34.1852	34.5770
	2	49.3608	126.9579	216.5435	270.7636	280.8432
CSSS	0.5	10.2174	23.2749	34.7466	40.3648	41.3213
	1	13.9638	30.8669	44.8492	51.4219	52.5237
	2	35.0144	91.3392	158.7607	200.8747	208.8163
CSCS	0.5	10.9704	24.8048	36.7766	42.5786	43.5625
	1	18.4353	43.9520	68.6330	81.5908	83.8583
	2	58.9923	175.5266	396.8768	624.8799	681.2578
CCSS	0.5	11.6624	29.8388	50.5470	62.9388	65.2308
	1	16.3947	39.8040	63.3748	76.1342	78.3950
	2	40.8863	109.1107	195.9058	253.2013	264.2874
CCCC	0.5	12.2768	31.4112	53.2116	66.2575	68.6706
	1	20.3906	54.2816	97.1033	125.1918	130.6103
	2	64.9499	196.4280	502.5786	929.7097	1060.1205
FFCC	0.5	13.7037	38.2802	74.1242	101.1898	106.7882
	1	22.6507	63.2002	122.1242	166.4485	175.5989
	2	71.7481	216.8627	560.7782	1055.1549	1209.5977

Compared to the three-dimensional elasticity solution, the present model gives more accurate results of buckling load than the height order shear deformation theory.

It can be concluded that the present model proposed is accurate in solving the buckling behaviors of anti-symmetric cross-ply and angle-ply laminated composite.

Acknowledgments: The authors thank the referees for their valuable comments.

РЕЗЮМЕ. Проведено аналіз втрати стійкості антисиметрично перехресно армованої шаруватої композитної пластинки при різних граничних умовах в рамках уточненої теорії вищого порядку, що враховує зсувні деформації. Ця теорія подібна до класичної теорії пластинок у багатьох аспектах, але додатково враховує квадратичну зміну поперечних зсувних деформацій по товщині і задовольняє нульові граничні умови зчеплення на верхній та нижній сторонах пластинки без використання поправок щодо зсуву. Число невідомих у цій теорії дорівнює чотирьом як і в інших теоріях, що враховують зсувні деформації. Рівняння руху для вільно опертої товстої пластинки отримані за допомогою принципу Гамільтона. Розв‘язок у замкнутому вигляді отримано з використанням рівнянь Нав‘є для антисиметрично перехресно (під різними кутами) армованої шаруватої пластинки. Отримано числові результати щодо втрати стійкості. Обгрунтованість підходу демонструється порівнянням з розв‘язками, отриманими і опублікованими в рамках інших теорій вищого порядку. Головний висновок полягає у тому, що запропонований підхід є точним і простим.

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