UDC 519.004.942
DOI: 10.20535/SRIT.2308-8893.2016.3.03

EMPIRICAL ANALYSIS OF CHERNOBYL NUCLEAR REACTOR CORE FOR 5 SECONDS BEFORE THE EXPLOSION

YOSHIO MATSUKI, PETRO I. BIDYUK

Abstract. This study uses the methodology of empirical analysis for analyzing the transient mode of the nuclear reactor core, a few second before the explosion at the time of the Chernobyl accident. The parameters were selected from the published articles [1]. A scenario was assumed for this analysis, such as the reduction of the flow rate of the Main Circulation Pump, and regression models were constructed to examine this scenario. The results of the models application were examined, and conclusions were made regarding the reduction of the flow rate of the Main Circulation Pump and the reactivity during the last few seconds to the explosion.

Keywords: Chornobyl disaster, critical operation mode, regression analysis, void and water environment

SCOPE OF ANALYSIS

On 26 April 1986, an explosion occurred in the nuclear reactor core of Chernobyl Power Station, Unit No.4. It is known that the specific design of the reactor core was one of the main causes of the accident. This research analyzed the relations between the sudden reactor power increase and water flow in the rector core, with a methodology of empirical analysis. The result is compared with the nuclear reactor theory.

Table 1. Descriptive Statistics of Parameters (taken from Fig. 3 of [1])

Parameters	Fuel temperature, K	MCP flow rate, m^3/sec	Power (% nominal power)	Reactivity, %	Void, %
Mean	210,421	9,653	67442,4	0,533	31,819
Median	131,396	9,575	13220	0,554	34,500
Maximum	570,633	10,200	227186,7	1,000	40,050
Minimum	90,100	9,3	0	0,214	12,000
Std. Dev.	147,216	0,269	90122,460	0,250	9,202
Skewness	1,371	0,713	0,872	0,228	–0,947
Kurtosis	3,722	2,396	2,039	1,972	2,837
Observations	16	16	16	16	16

Note: Max.: maximum value. Min.: minimum value. Std. Dev.: standard deviation. Skewness: the measure of the probability distribution leaning to one side of the mean. Kurtosis: “peakedness” of probability distributions. Observation: number of observations.

This research focuses on the time period of 5 seconds before the explosion (between 01h 23 min 38 sec and 01 h 23 min 42,71 sec on 25 April 1986) with the parameters (the data) of power, MCP flow rate[1], void, reactivity and fuel temperature, which are taken from Martines, et.al 1989 [1]. Table 1–2 shows the descriptive statistics of the selected variables for this research.

Table 2. Descriptive Statistics of Parameters (taken from Table ii of [1])

Parameters	Fuel temperature, K	Fuel energy, %	Total energy, MJ	Total power,MW	Water energy, %	Water power, %
Mean	822,0312	60,0725	24623,62	89767,5	40,075	29,51
Median	604,95	69,55	5495	9600	30,73	14,08
Maximum	1524,9	94	89200	306000	100	100
Minimum	537,9	0	0	200	8,4	5,71
Std. Dev.	359,784	33,499	32259,010	120344,200	33,389	31,315
Skewness	0,960	–0,624	0,993	0,877	0,606	1,257
Kurtosis	2,268	1,924	2,353	2,040	1,899	3,182
Obs.	16	16	16	16	16	16

Note: Max.: maximum value. Min.: minimum value. Std. Dev.: standard deviation. Skewness: the measure of the probability distribution leaning to one side of the mean. Kurtosis: «peakedness» of probability distributions. Obs.: number of observations.

METHODOLOGY

For the analysis, at first, the correlations are calculated; and coefficients of linear models are calculated for the investigation of the strength of the relations between the selected variables.

Estimating the coefficients of a linear model

At first, the average value $E(x)$ of each independent variable x is calculated:

$$E(x) = 1/n \sum_{i=1}^{n} x_i ,$$

where $i = 1,2,...,n$; where n is the total number of the sample of the variable x_i .

Then, the variance $V(x)$ and covariance $C(x, y)$ of the variables x and y are calculated:

$$V(x) = E(x^2) - E^2(x) = \sigma_x^2 , \quad C(x, y) = E(x^* y^*) = \sigma_{xy} ,$$

where $x^* = x - E(x)$, $y^* = y - E(y)$; where y is also an independent variable.

Then, a linear regression model is constructed as follows.

Case of 1 independent variable

In case of 1 independent variable, the regression model is written as follows:

$$Y = c_1 + c_2 X_2 , \qquad (1)$$

[1] MCP flow rate: the flow rate of the Main Circulation Pump

where Y is a dependent variable; X_2 is an independent variable; c_1 and c_2 are constant values.

The values of those coefficients are obtained by the following equations, which are obtained by an optimization of $U = Y - (c_1 + c_2 X_2)$:

$$c_1 = E(Y) - c_2 E(X_2)\,, \tag{2}$$

$$c_2 = \frac{\sigma_{X_2 Y}}{\sigma^2_{X_2}}\,. \tag{3}$$

Case of 2 independent variables

In case of 2 independent variables, the regression model is written as follows:

$$Y = c_1 + c_2 X_2 + c_3 X_3\,, \tag{4}$$

where Y is a dependent variable; X_j are independent variables; c_1 and c_j are constant values; where, $j = 2,3$.

The values of those coefficients are obtained by the following equations, which are obtained by an optimization of $U = Y - (c_1 + c_2 X_2 + c_3 X_3)$:

$$c_1 = E(Y) - c_2 E(X_2) - c_3 E(X_3)\,, \tag{5}$$

$$c_2 = (1 \big/ (1 - \sigma^2_{X_2 X_3} \big/ \sigma_{X_2 X_3}))(\sigma_{X_2 Y} \big/ \sigma_{X_2} - \sigma_{X_2 X_3} \sigma_{X_2 Y} \big/ \sigma_{X_2} \sigma_{X_3})\,, \tag{6}$$

$$c_3 = (1 \big/ (1 - \sigma^2_{X_2 X_3} \big/ \sigma_{X_2 X_3}))(\sigma_{X_3 Y} \big/ \sigma_{X_3} - \sigma_{X_2 X_3} \sigma_{X_2 Y} \big/ \sigma_{X_2} \sigma_{X_3})\,. \tag{7}$$

Correlation coefficients

For the regression model, the independent variables X_i are independent from each other. Therefore, before formulating the model equation (1) and/or (4), the correlation (ρ) between each pair of the variables need to be investigated by the following equation:

$$\rho = \frac{C(X_i X_j)}{\sqrt{V(X_i)}\sqrt{V(X_j)}} = \frac{\sigma_{X_i X_j}}{\sigma_{X_i} \sigma_{X_j}}\,, \tag{8}$$

where $i \neq j$.

Fitting (predictability) of the linear model in the data

After obtaining the correlations ρ and the coefficients, c_2 and c_3, the fitting of the model equation (1) and/or (4) on the given data of x_i and Y needs to be investigated by the following procedure:

1. Calculate the predicted value of Y (i.e., $\hat{Y}$) with the following equation:

$$\hat{Y}_i = c_1 + \sum_{j=2}^{k} c_j x_j\,, \tag{9}$$

where $j = 2,3,\ldots.k$; i corresponds to i-th observation of the variable x_j ($k = 2$ in equation (4); $k = 3$ in equation (4)).

2. Calculate the value of R^2 by the following equation:

$$R^2 = \frac{\sum_{i=1}^{n} (\hat{Y}_i - \bar{Y})^2}{\sum_{i=1}^{n} (Y_i - \bar{Y})^2}, \tag{10}$$

where $\bar{Y} = 1/n \sum_{i=1}^{n} x_i$; where $i = 1, 2, ..., n$; n is the number of the samples of the variable, x_i .

The value of R^2 represents the fitting and predictability of the given linear model upon the given data, and when $R^2 = 1{,}0$, it is the perfect match, while the level of the matching is lower when the value of R^2 is lower. In practice, if $R^2 \geq 0{,}8 \sim 0{,}6$, the fitting of the model in the data is significant. However, the threshold value depends on the topic and the data of the concerned research question, therefore the values of R^2 need to be considered on the comparative manner.

It is noted that the coefficients of the linear model (c_i, where $i = 2,3$), R^2 of each linear model, and the correlation (ρ) of each pair of the variables are all different, as each of them is calculated by different equation from each other as shown above.

RESULTS

Formation of the linear models after calculating the correlations of the variables

Tables 3, 4 show the correlations between each pair of the variables, which are calculated by the equation (8).

Table 3. Correlation matrix 1

Variable	FUEL ENERGY*	FUEL TEMPERATURE*	FUEL TEMP2	MCP FLOW RATE	POWER	REACTIVITY
FUEL ENERGY*	1					
FUEL TEMPERATURE*	0,7202	1				
FUEL TEMP2	0,6981	0,9607	1			
MCP FLOW RATE	–0,9861	–0,7417	–0,7310	1		
POWER	0,7178	0,9883	0,9354	–0,7383	1	
REACTIVITY	0,8661	0,4322	0,3850	–0,7859	0,4300	1
TOTALENERGY*	0,7182	0,9997	0,9643	–0,7428	0,9874	0,4230
TOTALPOWER*	0,7233	0,9976	0,9450	–0,7392	0,9883	0,4523
VOID	0,9736	0,6810	0,6640	–0,9729	0,6810	0,8330
WATERENERGY*	–0,9996	–0,7201	–0,6937	0,9834	–0,7187	–0,8716
WATERPOWER*	–0,9570	–0,5393	–0,5342	0,9589	–0,5419	–0,8245

Table 4. Correlation matrix 1

Variable	TOTAL ENERGY*	TOTAL POWER*	VOID	WATER ENERGY*	WATER-POWER*
TOTALENERGY*	1				
TOTALPOWER*	0,9962	1			
VOID	0,6801	0,6859	1		
WATERENERGY*	–0,7176	–0,7244	–0,9757	1	
WATERPOWER*	–0,5396	–0,5412	–0,9647	0,9556	1

Note: Number of observations is 16 as shown in Table 1 and Table 2.

FUEL ENERGY*: Fuel energy (%) in Table 2. FUEL TEMPERATURE*: Fuel temperature (K) in Table 2. FUEL TEMP2: Fuel temperature (K) in Table 1. MCP FLOW RATE: MPC flow rate (m^3/sec) in Table 1.

POWER: Power (% nominal power) in Table 1. REACTIVITY: Reactivity (%) in Table 1. TOTAL ENERGY*: Total energy (MJ) in Table 2. TOTAL POWER*: Total power (MW) in Table 2. VOID: Void (%) in Table 1.WATER ENERGY*: Water energy (%) in Table 2. WATER POWER*: Water power (%) in Table 2.

For the formulation of the linear model as shown in equations (1) and (4), the dependent variable, Y, needs to be defined. In this analysis, it is assumed that the reactor's power indicates the transient process, inside of the nuclear reactor. Therefore, one of the following three variables: the Power (% nominal power) in Table 1, the Total energy (MJ) in Table 2, and the Total power (MW) in Table 2, should be selected as the dependent variable, Y. For this selection, the correlation between each pair of these 3 variables was examined, and the result is shown in Table 5. As the result, it was found that each pair of these 3 variables has large correlations, which are between 0,98 and 1,00. Upon this observation, it is concluded that these 3 variables are considered to be the same indicator of the reactor power. Therefore, it was assumed that any of these 3 variables could represent the reactor's power. For this research paper, the Power (% nominal power) in Table 1 is used as the dependent variable, because this value is taken from the same graph in [1] with values of the void and reactivity, which are related to the reactor transient process of Chernobyl accident[2].

Table 5. Correlations between the pairs of the candidates for dependent variables

Case	Selected pair of variables		Value of correlation
1	Power (% nominal power) in Table 1–1	Total energy (MJ) in Table 1–2	0,9874
2	Power (% nominal power) in Table 1–1	Total power (MW) in Table 1–2	0,9883
3	Total energy (MJ) in Table 1–2	Total power (MW) in Table 1–2	0,9962

And then, the independent variables (X_i , where $i = 2,3$) in the equations (1) and (4) also need to be defined. For this purpose, it is necessary to examine the

[2] The theory of the rector transient will be explained in latter part of this paper, in the section 3.3.

correlations between each pair of those variables. And, then, the variables, which are less correlated, should be selected as independent variables. As the result, the pairs of the variables with greater correlations are shown in Table 6; and, the pairs with less correlations are shown in Table 7. Those 7 pairs of the variables shown in Table 6 are not considered to be independent, therefore any of those 7 pairs cannot be put together in the same linear model for the equation (1) and (4). On the other hand, each of those 14 pairs of the variables shown in Table 7 can be regarded as independent variables.

Table 6. Correlations between pairs of variables, which hold stronger correlations ($\geq 0{,}70$)

Case	Selected pair of variables		Value of correlation
1	Fuel temperature (K) in Table 2	Fuel temperature (K) in Table 1–1	0.9607
2	MCP flow rate (m^3/sec) in Table 1	Water energy (%) in Table 1–2	0,9834
3	MCP flow rate (m^3/sec) in Table 1	Water power (%) in Table 1–2	0,9589
4	Reactivity (%) in Table 1	Void (%) in Table 1–1	0,8330
5	Water energy (%) in Table 2	Water power (%) in Table 1–2	0,9556
6	Fuel energy (%) in Table 2	Fuel temperature (K) in Table 1–2	0,7202
7	Fuel energy (%) in Table 2	Fuel temperature (K) in Table 1–1	0,6981

Table 7. Correlations between pairs of variables, which hold weaker correlations ($\leq 0{,}70$)[3]

Case	Selected pair of variables		Value of correlation
1	Fuel temperature (K) in Table 2	MCP flow rate (m^3/sec) in Table 1	–0,7417
2	Fuel temperature (K) in Table 2	Reactivity (%) in Table 1	0,4322
3	Fuel temperature (K) in Table 2	Void (%) in Table 1	0,6810
4	Fuel temperature (K) in Table 2	Water energy (%) in Table 2	–0,7201
5	Fuel temperature (K) in Table 2	Water power (%) in Table 2	–0,5393
6	Fuel temperature (K) in Table 1	MCP flow rate (m^3/sec) in Table 1	–0,7310
7	Fuel temperature (K) in Table 1	Reactivity (%) in Table 1	0,3850
8	Fuel temperature (K) in Table 1	Void (%) in Table 1	0,6640
9	Fuel temperature (K) in Table 1	Water energy (%) in Table 2	–0,6937
10	Fuel temperature (K) in Table 1	Water power (%) in Table 2	–0,5342
11	MCP flow rate (m^3/sec) in Table 1	Reactivity (%) in Table 1	–0,7859
12	MCP flow rate (m^3/sec) in Table 1	Void (%) in Table 1	–0,9729
13	Void (%) in Table 1	Water energy (%) in Table 2	–0,9757
14	Void (%) in Table 1	Water power (%) in Table 2	–0,9647

Analysis of the reactor's transient by the linear model

Before the formulation of the linear model, the following scenario was assumed to describe the process of the reactor's transient for 5 seconds before the explosion:

1. The flow rate of the Main Circulation Pump (MCP flow rate) was reduced[4].

[3] In this process of the selection, the negative correlations (with the sign of minus) were accounted as the less correlated.

2. The voids were produced in the water inside of the reactor.
3. The increased void led to the increase of the neutron flux.
4. The power increased, leading to the explosion.

And, then, the following models were formulated, which should examine the relations between the related variables:

$$\text{POWER} = c_1 + c_2 \times \text{MCPFlowRate} + c_3 \times \text{REACTIVITY}, \tag{11}$$

$$\text{POWER} = c_1 + c_2 \times \text{MCPFlowRate} + c_3 \times \text{VOID}, \tag{12}$$

$$\text{REACTIVITY} = c_1 + c_2 \times \text{VOID}. \tag{13}$$

At first, the influence of 2 variables (MCP Flow Rate, and REACTIVITY) to POWER was examined by the equation (11); and, the influence of MCP Flow Rate and REACTIVITY to POWER was examined by the equation (12). REACTIVYT and VOID are strongly correlated (the correlation value is 0,8330 as shown in Table 6), therefore these two variables could not be put in the same linear model. Then, the equations (11) and (12) were formulated as separate equations. In addition, the relation between VOID and REACTIVITY was examined by the linear model, equation (13).

The other independent variables (FUEL TEMPERATURE, FUEL TEMP2) are strongly correlated with POWER (the correlations are: 0,9883 by FEULTEMPERATURE, and 0,9354 by FUELTEMP2 as shown in Table 3); therefore, it was assumed these two variables were surrogate of the POWER, not the independent variables. On the other hand, WATER ENRGY and WATER POWER are strongly correlated with MCP Flow Rate (the correlations are: 0,9834 by WATER ENERGY, and 0,9589 by WATER POWER as shown in Table 6); therefore, it was assumed these two variables were surrogates for MCP Flow Rate, and not independent variables. Also, FUEL ENERGY was omitted from this analysis, because Table 6 suggests that it has correlations with FUEL TEMPERATURE, FUEL TEMP2, and POWER, although the values of the correlations are about 0,70[5].

And, then the coefficients (c_i, where $i = 1,2,3$) of each linear model shown in the equations (11), (12) and (13) were calculated, with the equations (2) and (3) for the cases of 1 independent variable, and the equations (5), (6) and (7) for 2 independent variables. The calculated results are shown in Table 8, together with calculated values of R^2.

Table 8. Calculated Linear Models

Model Equation	Linear models and calculated coefficients	R^2
14	$\text{POWER} = 3535019 - 351394.8 \times \text{MCPFlowRate} - 141558{,}0 \times \text{REACTIVITY}$	0,6042
15	$\text{POWER} = 4875626 - 475560{,}6 \times \text{MCPFlowRate} - 6836{,}794 \times \text{VOID}$	0,5712
16	$\text{REACTIVITY} = -0{,}1876 + 0{,}02266 \times \text{VOID}$	0,6938

[4] This action was taken for testing the plant's ability of recovering the loss of external electricity supply for the Main Circulation Pump.

[5] In practice, the value of 0,70 is considered as sufficiently a high correlation.

The R^2 of the model equation (11) is 0,6042, while its value of the equation (12) is 0,5712. The value of R^2 shows how the predicted values by the model equation fit in the sampled data. These calculated two values show that the predictability of the models is satisfactory by both models[6]; in other words, the models sufficiently fit in the data.

In both two models, the coefficients of MCP Flow Rate show similar values in their order of magnitude. But the sign of the coefficients are both negative. This sign is consistent with the scenario that led to the increase of the reactor power.

On the other hand, signs of the coefficients of REACTIIVTY and VOID are also both negative; but, the values of the coefficients are smaller than the value of the coefficient of MCP Flow Rate. This observation suggests that the increase of power was dominated by the reduction of the MCP Flow Rate, while REACTIVITY and VOID were no longer influential to the increase of the reactor power in this period of 5 seconds before the explosion.

Reactor theory

It is known that the positive void coefficient was one of the causes of the sudden increase of the reactor power, in case of the explosion of Chernobyl reactor. However, in the above observation during the period of a few seconds before the explosion, the void coefficient didn't act as the dominant factor, although the reactivity should have increased the neutron flux on theory. Therefore, in this observation, the reactivity and/or void coefficient needs to be considered as predominant factor, of which influence was not observed dynamically during the period of these 5 seconds. Therefore, here it is necessary to discuss this problem with the reactor theory.

SUMMARY, CONCLUSION AND RECOOMENDATION

Empirical method was used for analyzing the transient of the nuclear reactor core, a few second before the explosion at the time of the Chernobyl accident (between 01h 23min 38sec and 01h 23min 42,71sec). 11 parameters were taken from the literature [1] and correlations of each pair of those variables were calculated. And, then, 4 parameters were selected for further analyzing the process of transient before the explosion. A scenario was assumed for this analysis, such as the reduction of the flow rate of the Main Circulation Pump and the insertion of the void caused the increase of the power for the explosion. 2 separate linear models were made to examine this scenario. The result indicated that the reduction of the flow rate of the Main Circulation Pump was dominant over the void and the reactivity during the last few seconds to the explosion. And, then, the relation between the void and the reactivity was investigated also with this methodology, and the result was compared with the nuclear reactor theory shown in the literature [2]. The result of this comparison suggested that the value of the void coefficient was 30 pcm/%. In the literature [1], the inserted void was about 50 % before this final moment. Therefore, the reactivity was calculated as 0,015 in the literature [2], while the empirical analysis indicated its value as 0,023.

[6] R^2 is about 0.60. It means that more then half of the actual data are predicted by the model.

The empirical method calculates the degree of changes within each valuable, not the absolute value such as average; and then, this methodology further calculates and determines the relation with other variable(s). Focus is made on the changes (distribution) of the variables. In other words, empirical method emphasizes relations between the relative changes in the variables, while statistics examines the appropriateness of the estimated absolute values.

The result of the empirical analysis in this study shows a brief outlook of the process of the reactor explosion of Chernobyl. The calculated reactivity by the empirical method is not exactly as same as the value calculated by the reactor theory, but in the same order of magnitude. Rather, the empirical method calculates the strength of the relations between different types of the variables. In this study, one linear model was constructed to examine the influence to the sudden power increase by the water flow in the reactor core and by the reactivity. And, the calculated coefficients of the linear model show a significant influence of the reduction of the circulation of reactor coolant.

The predominance of reactor design, such as indicated by the void coefficient, is well known for explaining the accident of Chernobyl: although, it is not the aim of this paper to introduce a large number of published literatures about the reactor design. On the other hand, the empirical analysis in this study shows how the insertion of void and the reduction of water flow were related to the power increase.

The literature [2] calculated the value of reactivity $\left(\frac{\Delta K_{\text{eff}}}{K_{\text{eff}}}\right)$ at the time of the reactor transient of the Chernobyl accident, as 0,015, by the following equation, given 30 pcm/% of the void coefficient and 50 % void insertion by Xenon poisoning:

$$\frac{\Delta K_{\text{eff}}}{K_{\text{eff}}} = \text{Void Coefficient (pcm/\%)} \times \text{Void Insertion (\%)}.$$

On the other hand, Table 6 shows the relation between REACTIVITY and VOID in the model (13), and the coefficient of VOID for REACTIVITY is 0,023. This value is roughly on the same order of magnitude as the theory [2] indicates as 0,015. Therefore, this observation shown in Table 6 also suggests that the void coefficient is about 30 pcm/%, which was as discussed by [2].

The result of this study shows possibility of using empirical method for analysis of physical phenomena, specifically the process of transient in the nuclear reactor core.

REFERENCES

1. *Martinez J. M.* An Analysis of the Physical Causes of the Chernobyl Accident /Jose M. Aragonez, Emilio Mingues, Jose M. Peri, Guillermo Velarde // Nuclear Technology. — 1990. — Vol. 90. — P. 371–399.
2. *Reich F.* Neutron Kinetics of the Chernobyl Accident / F. Reich // ENS News. — 2005. — Issue 13.

Received 04.02.2016

From the Editorial Board: the article corresponds completely to submitted manuscript.