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Magnetooptical effects in two dimensional photonic crystals

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Magnetooptical properties of materials with periodically modulated dielectric constant — photonic crystals (or photonic band gap materials) have been examined with relation to their possible applications for the control of electromagnetic radiation in the integrated optics devices. For this investigation we propose the original theoretical approach based on the perturbation theory. Magnetooptical Faraday and Voigt effects have been studied near extremum points of photonic bands where their significant increase takes place. Basing on the elaborated theory, some experimental results are discussed. Experimental frequency dependence Faraday rotation angle agrees well with our theoretical predictions.

Магнитооптические свойства материалов с периодически модулированной диэлектрической постоянной — фотонных кристаллов (или материалов с фотонной запрещенной зоной) исследованы с точки зрения возможности их применения для управления электромагнитным излучением в устройствах интегральной оптики. Для этого исследования предложен новый теоретический подход, основанный на теории возмущений. Исследованы магнитооптические эффекты Фарадея и Фойгта вблизи экстремальных точек фотонных зон, где имеет место их значительное усиление. На основе разработанной теории обсуждаются некоторые экспериментальные результаты. Экспериментальная частотная зависимость фарадеевского угла вращения хорошо согласуется с теоретическими предсказаниями.

Recently, much attention is paid to a new kind of dielectric composites referred to as photonic crystals (PhC). The photonic crystals (also referred to as photonic band gap materials) are micro-structured materials where the dielectric constant is periodically modulated on a length scale comparable to the desired wavelength of electromagnetic radiation [1, 2]. Multiple interference between electromagnetic (EM) waves scattered from each unit cell results in a range of frequencies that do not propagate in the structure — photonic band gaps (PBGs). At these frequencies, the light is strongly reflected from the surface of the crystal, while at other frequencies, light is transmitted. This phenomenon is of great theoretical and practical importance. It can be used to study a wide range of physical problems related to the light localization and light emission [3]. The photonic crystal materials with PBGs make it possible to prepare micro-cavity lasers [4], single-mode light emitting diodes, high-efficiency wave guides [5], high-speed optical switches, etc. PhCs, even those without a PBG, possess also many other interesting properties related to the dispersion, anisotropy, and polarization characteristics of the photonic bands (PB). For example, these properties of PhCs offer an opportunity to provide efficient dispersion compensation [6], increased nonlinear frequency conversion [7], novel superprism devices [8], optical polarizers, optical filters, etc.

Tunability of PhC optical properties open new applications of these materials in the integrated optics devices. Tunability in semiconductor structures may be provided by varying temperature or by varying voltage [9]. Other ways to provide tunability are application of elastic stress [10], liquid crystal infiltration [11], application of external magnetic fields or use of magnetic constituents [12–21]. The latter two possibilities are of prime interest because they not only permit significant tunability but also may result in some new interesting phenomena of magnetooptics (such as increased magnetic circular and linear birefringence [18–21], mode conversion) essential for novel readout devices and some devices of optical microcircuits.

First works about magnetic PhC operating in near infrared and optical regions appears in 1997 when M.Inoue et al. considered theoretically Faraday effect in the random multilayer films composed of Bi-substituted YIG [18]. Such structures are in fact one-dimensional PhCs. They announced about 300-fold increase of Faraday effect when some appropriate structure parameters are chosen. Later, several other similar one-dimensional (1D) PhCs were considered theoretically and experimentally, namely, multilayer films with differently ordered magnetic and nonmagnetic layers. Thus, M. Levy et al. predicted high transmission and Faraday rotation for the 1D PhC consisting of alternating magnetic and nonmagnetic layers of quarter-wave thickness with the layer free of structural defects [20]. For such structure, Faraday rotation by 45° is realized at the distances of only 15 μm, while this length is 200 times larger in the same homogeneous magnetic medium. So such 1D magnetic PhCs are very promising in development of tiny optical isolators.

1D PhC consisting of alternating anisotropic dielectric and ferromagnetic layers was investigated theoretically by Figotin and Vitebsky [16]. They found the effect of strong spectrum irreversibility that is a property very important for practical applications: this material can be transparent for a certain EM wave traveling, for example, from the left to right, and absolutely opaque for the EMW traveling from the right to left. During the last several years, fabrication of 2D and even 3D magnetic PhCs became possible and many interesting structures were manufactured (see, e.g., review [21]).

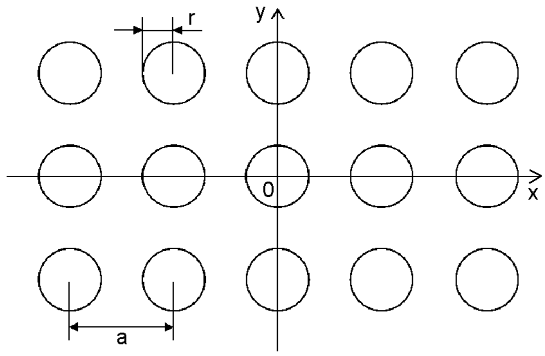

Fig. 1. Structure of a two-dimensional photonic crystal.

In this paper, we will study magnetooptical effects in two-dimensional PhCs composed either of dielectric or magnetic materials; that implies a study of the magnetic field influence on EM waves propagation in PhCs. We consider an inhomogeneous dielectric medium that can be characterized by the dielectric constant $\varepsilon_{ij}(\mathbf{r}) = \delta_{ij}\varepsilon\ (\mathbf{r})$. The function $\varepsilon(\mathbf{r})$ is a periodic one: $\varepsilon(\mathbf{r} + \mathbf{a}) = \varepsilon(\mathbf{r})$, where $\mathbf{a} = \{a_1\mathbf{e}_x + a_2\mathbf{e}_y\}$ is the unit vector of the two-dimensional PhC (Fig. 1). The influence of magnetic field is taken into account by means of polarization vector

$$\mathbf{P}_m(\{\mathbf{r}) = i\varepsilon_0\varepsilon(\{\mathbf{r})Q(\mathbf{r})[\mathbf{m},\mathbf{E}], \qquad (1)$$

where $\varepsilon_0 = 8.85 \cdot 10^{-12}$ F/m, $\mathbf{m}$ is unit vector of magnetic field (or magnetization); $Q(\mathbf{r})$, the medium magnetooptical parameter, or the Voigt one (see e.g. [22]). For ferromagnetic substances, Q is of the order of $10^{-3} \div 10^{-4}$: for yttrium iron garnets, $Q = 0.5 \cdot 10^{-3}$ ($\lambda = 1.15$ μm), and for terbium iron garnets $Q = 1.07 \cdot 10^{-3}$ ($\lambda = 1.06$ μm), [22]. For non-magnetic substances, it is proportional to the external magnetic field $\mathbf{B}_{ext}$: for Si, $Q = 1.2 \cdot 10^{-6}$ ($\lambda = 0.41$ μm, $\mathbf{B}_{ext} = 0.1$ T) [23]; for europium glass, $Q = 7 \cdot 10^{-5}$ ($\lambda = 0.435$ μm, $\mathbf{B}_{ext} = 0.1$ T) [24].

Assuming $\mu = 1$ it is straightforward to obtain from Maxwell's equations the following eigenvalue equation:

$$\left(\hat{H} + \hat{V} - \left(\frac{\omega}{c}\right)^2\right)\Psi(\mathbf{r}) = 0, \qquad (2)$$

where $\hat{H}\Psi(\mathbf{r}) = \frac{1}{\sqrt{\varepsilon(\mathbf{r})}}\nabla \times \left\{\nabla \times \frac{1}{\sqrt{\varepsilon(\mathbf{r})}}\Psi(\mathbf{r})\right\}$,

$$\hat{V}\Psi(\mathbf{r}) = -i\left(\frac{\omega}{c}\right)^2 Q \cdot \mathbf{m} \times \Psi(\mathbf{r}),$$

and $\Psi(\mathbf{r}) = \sqrt{\varepsilon}(\mathbf{r})\mathbf{E}(\mathbf{r})$.

It is worth noticing that eigenvalue equation (2) has a very important property, namely, the scaling law. This law says that PhCs which are similar to each other have essentially the same PB structure, that is, the difference between the two band structures is simply the scales of frequency and the wave vector. It is easy to prove that both operators $\hat{H}$ and $\hat{V}$ are Hermitian. The operator $\hat{H}$ has been studied rigorously elsewhere [25]. One of the main features of operator $\hat{H}$ is that its eigenfunctions can be divided into two types: quasi-longitudinal modes $\Psi^{(L)}_{n\mathbf{k}}(\mathbf{r})$ and quasi-transverse $\Psi^{(T)}_{n\mathbf{k}}(\mathbf{r})$ modes [25]. The $\Psi^{(L)}_{n\mathbf{k}}(\mathbf{r})$ modes do not satisfy the Maxwell divergence equation and, consequently, are non-existent. However, these modes are essential mathematically, since without them, the eigenfunction system $\left\{\Psi_{n\mathbf{k}}(\mathbf{r})\right\}$ is not complete. At the same time, the transversal eigenfunctions $\Psi^{(T)}_{n\mathbf{k}}(\mathbf{r})$ satisfy the Maxwell divergence equation and do really exist. Their eigen-angular frequencies $\omega^{(T)}_{n\mathbf{k}}$ are generally non-zero. Eigenfunctions $\Psi_{n\mathbf{k}}(\mathbf{r})$ form a complete set in the Hilbert space. Those are not orthogonal to each other but can be orthogonalized by Schmidt method. Eigenfunctions of $\hat{H}$ are vectorial Bloch functions

$$\Psi_{n\mathbf{k}}(\mathbf{r}) = \mathbf{u}_{n\mathbf{k}}(\mathbf{r})e^{i\mathbf{kr}}, \tag{3}$$

where $\mathbf{k}$ is quasi-momentum and n is a number of a specific PB; $u_{n\mathbf{k}}(\mathbf{r}_\|) = u_{n\mathbf{k}}(\mathbf{r}_\| + \mathbf{a})$, $\mathbf{r}_\| = x\mathbf{e}_x + y\mathbf{e}_y$. The corresponding eigenvalues ω_n form a band diagram with alternating permitted bands and band-gaps. The eigen-angular frequencies ω_n could be found by solving the eigenvalue problem for the set of linear equations obtained from Maxwell ones. There are several algorithms to that end. In our calculations, we used a procedure that was proposed in [26]. We will assume as usual that the vector $\mathbf{k}$ belongs to the first Brillouin zone. These properties of eigenfunctions and eigenvalues are well-known in the crystal physics and follow directly from periodicity of the operator $\hat{H}$.

Substitution of (3) into (2) gives

$$\left(\hat{H} + \hat{V} - \frac{\omega^2}{c^2}\right)\mathbf{u}_{n\mathbf{k}} = 0, \tag{4}$$

where

$$\hat{H}\mathbf{u}_{n\mathbf{k}} = \frac{1}{\sqrt{\varepsilon}}\nabla \times \nabla \times \frac{\mathbf{u}_{n\mathbf{k}}}{\sqrt{\varepsilon}} + \frac{i\mathbf{k}}{\sqrt{\varepsilon}} \times \nabla \times \frac{\mathbf{u}_{n\mathbf{k}}}{\sqrt{\varepsilon}} +$$
$$+ \frac{i}{\sqrt{\varepsilon}}\nabla \times \mathbf{k} \times \frac{\mathbf{u}_{n\mathbf{k}}}{\sqrt{\varepsilon}} - \frac{\mathbf{k}}{\varepsilon} \times \mathbf{k} \times \mathbf{u}_{n\mathbf{k}},$$
$$\hat{V}\mathbf{u}_{n\mathbf{k}} = -i\,\frac{\omega^2}{c^2}Q(\mathbf{r}_\|)\mathbf{m} \times \mathbf{u}_{n\mathbf{k}}. \tag{5}$$

Here, the analysis is restricted to the consideration of some highly symmetrical extreme points in the first Brillouin zone (points Γ and X) where the properties of EM radiation propagation differ substantially from those in a homogeneous medium. Let us assume that the wave packet in a PhC is constituted by Bloch functions $\Psi_{n\mathbf{k}}(\mathbf{r})$ from the one or two PBs (depending on the specific situation) and $\mathbf{k}$ lies near the critical point. This assumption is similar to the adiabatic approximation in solid state physics and is applicable when the value of perturbation $\hat{V}$ is small enough. Thus, the quasi-momentum $\mathbf{k} = (k_0 + \kappa, 0, 0)$, where $k_0 = 0$ for Γ point and $k_0 = \pi/a$ for X point. In the zero order on κ and Q, the solutions of (4) are Bloch functions for different wave zones $\mathbf{u}_{n_0k}(\mathbf{r}_\|)$ that form complete basis for the expansion of any function that possesses a translational symmetry. Hence, those could serve as a basis for the expansion of $\mathbf{u}_{n_0k}(\mathbf{r}_\|)$ into series in perturbation theory for any specified PB number n_0.

While dealing with magnetooptical effects, two main geometries are considered: (i) longitudinal, or Faraday, geometry when an EM wave propagates along magnetic field, i.e. $\mathbf{k}\|\mathbf{m}$, and (ii) transversal, or Voigt, geometry when $\mathbf{k} \perp \mathbf{m}$. Let us consider first the longitudinal geometry where $\mathbf{k}\|\mathbf{m}\|\mathbf{e}_x$ and investigate how the presence of magnetic field affects PB with arbitrary number n_0. Here, several cases are possible: (i) the n_0-th PB is not degenerate and secluded, (ii) the n_0-th PB is not degenerate, but has a close neighbor PB, (iii) the n_0-th PB is doubly degenerate (possible only for Γ and M points). The meaning of word "close" in (ii) will be discussed further. We examine two first possibilities in turn.

For two-dimensional PhC symmetry, $z \to -z$ enables to classify all eigenmodes into two kinds: TE modes (E_z, H_x, H_y) and TM modes (E_x, E_y, H_z). Each of those is characterized by additional parity with respect to the reflection in the corresponding vertical planes. Therefore, function $\mathbf{u}_{n_0 k}(\mathbf{r}_\|)$, that represents the eigenfunction of (4) for n_0-th PB, can be written in the first order of perturbation theory as

$$\mathbf{u}_{n_0 k}(\mathbf{r}_\|) = c_1 u^{TE}_{n_0 k_0}(\mathbf{r}_\|)\mathbf{e}_z + c_2 u^{TM}_{n_0 k_0}(\mathbf{r}_\|)\mathbf{e}_y + c_3 u^{L}_{n_0 k_0}(\mathbf{r}_\|)\mathbf{e}_x, \quad (6)$$

where $\mathbf{u}^{TE}_{n_0 k_0}$, $\mathbf{u}^{TM}_{n_0 k_0}$, and $\mathbf{u}^{L}_{n_0 k_0}$ are eigenfunctions of operator $\hat{H}$ with eigen-angular frequencies $\omega^{TE/TM/L}_{n_0}$. In Faraday geometry, $c_3 = 0$ because $\hat{V}\mathbf{u}^{L}_{n_0 k_0} = 0$. Substitution of (6) in (4) leads to the following equations set:

$$\begin{cases} c_1\left\{(\omega^{TE}_{n_0 k})^2 - \omega^2\right\} - c_2 i\omega^2 \langle Q\rangle = 0 \\ c_2\left\{(\omega^{TM}_{n_0 k})^2 - \omega^2\right\} + c_1 i\omega^2 \langle Q\rangle^* = 0 \end{cases} \quad (7)$$

where $(\omega^{TE(TM)}_{n_0 k})^2 = (\omega^{TE(TM)}_{n_0})^2 + c^2\kappa^2\beta^{TE(TM)}$,

$\beta^{TE(TM)} = \langle u^{TE(TM)}_{n_0 k_0}|\frac{1}{\varepsilon}|u^{TE(TM)}_{n_0 k_0}\rangle$,

$\langle Q\rangle = \langle u^{TE}_{n_0 k_0}|Q(\mathbf{r})|u^{TM}_{n_0 k_0}\rangle$.

To reveal general features of the magnetooptical effects in this geometry, we assume that

$$\omega^{TE}_{n_0} = \omega^{TM}_{n_0} \equiv \omega_{n_0} \text{ and } \beta^{TE} = \beta^{TM} \equiv \beta. \quad (8)$$

From these assumptions, Eq.(7) leads to the following solutions for :

$$\kappa_\pm = \frac{\omega}{c\sqrt{|\beta|}}\left(\left|1 - \left(\omega_{n_0}/\omega\right)^2 \pm |\langle Q\rangle|\right|\right)^{1/2} \quad (9)$$

and corresponding eigenfunctions:

$$\Psi(\mathbf{r}_\|) = e^{ik_0 x} e^{i\frac{\kappa_+ + \kappa_-}{2}x}\left(u^{TE}_{n_0 k_0}(x)\cos\frac{\Delta\kappa}{2}x \;\; u^{TM}_{n_0 k_0}(x)\sin\frac{\Delta\kappa}{2}x \right), \quad (10)$$

where $\Delta\kappa = \kappa_+ - \kappa_-$. These eigenmodes can be called "quasi-circularly polarized" modes. The prefix "quasi" means here that waves $\psi^\pm(\mathbf{r}_\|)$ in (10) are the product of fast oscillating functions $u^{TE(TM)}_{n_0 k_0}(\mathbf{r}_\|)$ and comparatively slow changing envelope functions $e^{i(k_0 + \kappa_\pm)x}$. Equation (10) shows that while light propagation along OX-axis, a mode conversion takes place. If at the PhC entry, EM radiation is a TE-wave, then, while spreading, it becomes transformed into a TM-wave because of the medium gyrotropy, and so on. Usually, condition $|\langle Q\rangle| \ll |1 - \left(\omega_{n_0}/\omega\right)^2|$ is satisfied and Faraday angle or angle of rotation of envelope wave polarization plane is

$$\Phi = \Delta k = \frac{\omega}{c^2\sqrt{|\beta|}}|\langle Q\rangle|\left|1 - \frac{(\omega_{n_0})^2}{(\omega)^2}\right|^{-1/2}. \quad (11)$$

From (11), it can be concluded that the specific Faraday angle grows sharply when $\omega \to \omega_{n_0}$. This occurs in compliance with fundamental property of PhC: near extreme points of Brillouin zone, a critical deceleration of radiation takes place that causes an increased interaction time between radiational mode and the matter system and, thus, magnetooptical effect is increased.

It is interesting to compare the result obtained with experimental measurements of Faraday rotation angle for a 3D magnetic colloidal crystal consisting of a fcc packing of silica spheres with voids that are filled with a saturated glycerol solution of dysprosium nitrate [26]. Though formula (11) is obtained for the 2D-PhC, one can expect that it is valid for some cases in 3D. For example, the propriety of its application at Γ point for cubic 3D PhC becomes intuitively clear when we conduct an analogy with the electron zones of some semiconductors (e.g. GaAs). Continuing this analogy, one can conclude that conditions (8) are satisfied there and (11) remains valid. Approximation of experimental curves by the theoretical dependence is quite good that confirms our assumption (Fig. 2). From the approximation, value of the ratio $\frac{|\langle Q\rangle|}{\sqrt{|\beta|}}$ could be found. For the curve in Fig. 2, $\frac{|\langle Q\rangle|}{\sqrt{|\beta|}} = 6.55 \cdot 10^{-9}$.

When ω becomes very close to ω_{n_0}, then inequality $|\langle Q\rangle| \ll |1 - (\omega_{n_0}/\omega)^2|$ is no longer satisfied, so to determine the specific Faraday rotation, Eq.(9) should be used directly without any approximations. More careful analysis of $\Phi(\omega)$ dependence reveals that it has extreme for $\omega = \omega_{n_0}(1 \pm ½Q)$

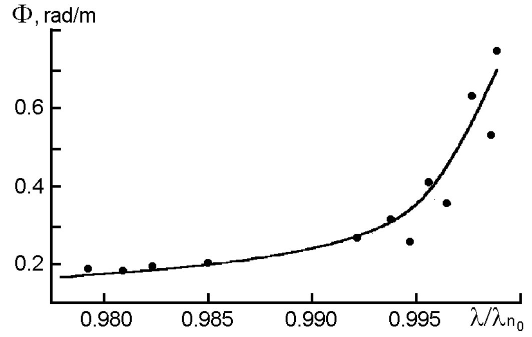

Fig. 2. Faraday rotation angle vs. λ/λ_{n_0}. Points, experimental data for 3D magnetic collodial crystal consisting of a fcc packing of silica spheres with voids filled with saturated glycerol solution of dysprosium nitrate ($d_{spheres}$ = 260 nm, ε_{silica} = 2.0, ε_{liquid} = 2.2, B_{ext} = 33.5 mT, $Q = 1 \cdot 10^7$) [26]. Solid curve, theory (in accordance to (11)), λ_{n_0} = 566.5 nm.

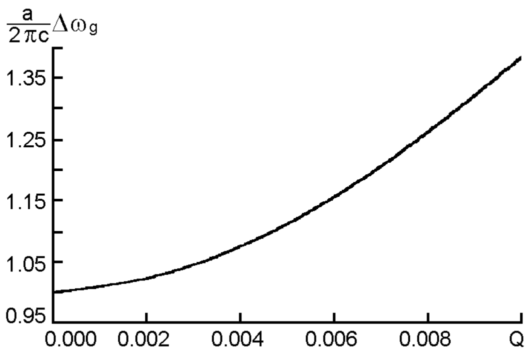

Fig. 3. Band gap width $\Delta\omega_g$ vs. magnetooptical parameter Q; $\omega_0^{TE} = \omega_0^{TM} \equiv \omega_0 = 0.32\frac{2\pi c}{a}$, $\Delta^{TE} = \Delta^{TM} = 5 \cdot 10^{-4}\,\frac{2\pi c}{a}$.

(choice between "+" and "−" here depends on the sign of β, i.e. on the sign of the second derivative of PB dispersion curve). This formula is very important because it shows that the Faraday effect takes its maximum value not exactly at the extreme angular frequency ω_{n_0}, where the transmission is negligibly low, but at its close proximity where transmission is higher. The maximum value of specific Faraday rotation is given by

$$\Phi = (\omega_{n_0}/c\sqrt{|\beta|})\sqrt{\frac{<Q>}{2}}. \qquad (12)$$

At the same time, Faraday rotation for a homogeneous medium is

$$\Phi_{uniform} = \frac{\omega}{2c}\sqrt{\varepsilon}Q. \qquad (13)$$

From (12) and (13), one can estimate the relative gain in Faraday effect in PhC as compared to the same homogeneous medium under similar conditions:

$$\frac{\Phi_{PhC}}{\Phi_{uniform}} \sim \sqrt{\frac{1}{Q}}. \qquad (14)$$

Thus, for $Q = 10^{-6}$, the Faraday effect in PhC could be increased by three decimal orders. We define the conversion coefficient R as the ratio of the maximum squared amplitudes of the TE and TM modes if at $x = 0$, the TM-wave is supposed to exist. For the conditions (8), $R \sim 1$.

In the case of different and not very close $\omega_{n_0}^{TE}$ and $\omega_{n_0}^{TM}$,

$$(|<Q>| << \left|\left[\left(\omega_{n_0}^{TE}\right)^2 - \left(\omega_{n_0}^{TM}\right)^2\right]/(\omega)^2\right|),$$

but under condition $\beta^{TE} = \beta^{TM} \equiv \beta$, the birefringence effect appears and, as in birefringent crystals (see e.g. [22]), the conversion coefficient R decreases sharply. Thus, if the EM radiation is polarized linearly at the entrance of PhC, then, while spreading in PhC, polarization remains mainly unchanged with a very little ellipticity. At the same time, the TE-TM partial conversion takes place at much smaller distances. Distinction in β^{TE} and β^{TM} values complicates the analysis. For some values of $\omega_{n_0}^{TE}/\omega_{n_0}^{TM}$, the TE mode prevails and for other values, the TM one at arbitrary entry polarization.

General effect of magnetic field on photonic band structure is expressed as an alteration of PBGs: in magnetic field, the PBs are shifted by $Q\omega_{n_0}$. For magnetooptical parameter $Q = 6 \cdot 10^{-3}$, the shift is about several tenths of percent. The shift value depends on the difference $\omega_{n_0}^{TE} - \omega_{n_0}^{TM}$ and gets smaller when the difference increases.

The situation when two PB are close together exists quite often and undoubtedly deserves special examination. Let us consider two consecutive bands numbered as n_0 and $n_0 + 1$ with corresponding frequencies $\omega_0^{TE(TM)} - \Delta^{TE(TM)}$ and $\omega_0^{TE(TM)} + \Delta^{TE(TM)}$ for TE and TM modes, respectively, where

$\omega_0^{TE(TM)}$ is the mean frequency and $\Delta^{TE(TM)}$ is the half-width of the PBG between given bands. To begin, let us define what the term "close" means at this point. The solitary PB splitting that arises in magnetic field is of the order of $\omega_{n_0} Q$. That is why we call two wave bands "close" if the distance between those is smaller than this magnet-induced splitting, i.e. $2\Delta < \omega_{n_0} Q$.

As for the close bands, the presence of magnetic field influences on the PBG width $\Delta\omega_g$:

$$\Delta\omega_g = \omega_0\left((2\Delta/\omega_0)^2 + Q^2\right)^{1/2}, \quad (15)$$

making it larger (Fig. 3). Here, the following assumptions are supposed: $\omega_0^{TE} = \omega_0^{TM} \equiv \omega_0$, $\Delta^{TE} = \Delta^{TM}$ and $\beta_i^{TE} = \beta_i^{TM} \equiv \beta$.

Another important configuration is Voigt geometry when $\mathbf{B}_{ext} \| \mathbf{e}_z$ and $\mathbf{k} \| \mathbf{e}_z$. The analysis of this case can be done in the same manner as for Faraday geometry. The eigenfunction $\mathbf{u}_{n_0 k}(\mathbf{r}_\|)$ of (4) for n_0-th PB is again set by (6), but in Voigt geometry, coefficient c_3 in (6) does not disappear. Substitution of (6) in (4) leads to the two independent subsets: one for $u^{TE}_{n_0 k_0}$, and the other for $u^{TM}_{n_0 k_0}$ and $u^{L}_{n_0 k_0}$. From these equations, it can be derived that the relative phase shift between TE and TM modes takes place. The phase shift at the unit distance is

$$B_{mb} = \mathrm{Re}(\kappa_\| - \kappa_\perp) = \quad (16)$$

$$= \frac{\omega}{2c\sqrt{|\beta|}} \left|\langle Q_L \rangle\right|^2 \left(1 - \frac{\omega_{n_0}^2}{\omega^2}\right)^{-1/2},$$

where $\langle Q_L \rangle = \langle u^{L}_{n_0 k_0} | Q(\mathbf{r}) | u^{TM}_{n_0 k_0} \rangle$. This effect of magnetic birefringence is similar to the magnetooptical Voigt effect (see e.g. [22]). In comparison to the latter, formula (16) demonstrates a sharp increase of the phase shift B_{mb} near the extreme points of the Brillouin zone. It is largely due to the same reasons as the increase of Faraday rotation. Magnetic field affects only on TM mode shifting its corresponding PB. The shift value is of the order of $\omega_{n_0} Q^2$ and, consequently, much smaller than in Faraday geometry.

To conclude, we have studied magnetooptical properties of two-dimensional PhCs composed either of dielectric or magnetic materials that implies an investigation of the magnetic field influence on the electromagnetic waves propagation in PhCs. Theoretical investigation has been performed basing on solving the eigenvalue problem obtained from Maxwell equations. Magnetic part of the medium polarization has been considered as a perturbation and related magnetooptical effects were calculated in the first order of perturbation theory. Two main geometries have been examined, namely, the Faraday and Voigt configurations. In Faraday geometry where $\mathbf{k} \| \mathbf{m} \| \mathbf{e}_x$ the TE-TM mode conversion takes place, an effect similar to the magnetooptical Faraday effect. The Faraday angle depends on the wave frequency ω and increases sharply when ω approaches extreme frequencies ω_{n_0} of wave bands. The Faraday effect takes its maximum value not exactly at ω_{n_0}, but close thereto where transmission coefficient is not too small. The increase of the Faraday rotation in PhC in comparison to the uniform medium is larger for smaller Q (see (14)). That makes PhC applications for non-magnetic substances with magnetic field induced gyrotropy (at $Q \sim 10^{-5}$– 10^{-7}) the most valuable. At the same time, a substantial increase in the Faraday effect also occurs for ferromagnetic constituents. Thus, for the magnetic material with magnetooptical parameter $Q \sim 10^{-3}$, the Faraday rotation angle can be as large as 20°/μm for near infrared radiation. This effect is very promising for construction of the miniature optical isolators in integrated optics. For the case of close wave bands, effect of magnet-induced band gap widening is predicted. It is about 10 percent for $Q \sim 5 \cdot 10^{-3}$. Comparison of the theoretical formula for Faraday rotation with experiment for 3D opal-like magnetic PhC gives good results approving more wide validity of the elaborated theory.

The relative phase shift between TE and TM modes that originates in the Voigt configuration shows a similar sharp frequency dependence. This effect is similar to the linear magnetic birefringence.

Thus, magnetic PhCs evince giant magnetooptical effects (circular and linear birefringence) for radiation frequencies close to the extreme PB frequencies at the vicinity of high-symmetry points in the Brillouin zone. Besides, magnetic field can influence the PBG structure changing their width substantially. All this proves that magnetic

PhCs are of importance for light managing in modern devices of integrated optics.

Theoretical approach presented here can be applied to investigate magnetooptical effects in other configurations. It can also be utilized to study electrooptical effects, for example, the Pockels effect, that appears when external electric field is applied. In this case, one should use electric field dependent polarization term.

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Магнітооптичні ефекти у двовимірних фотонних кристалах

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Магнітооптичні властивості матеріалів з періодично модульованою діелектричною константою — фотонних кристалів (або матеріалів з фотонною забороненою зоною) досліджено з точки зору можливості їх застосування для управління електромагнітним випромінюванням у пристроях інтегральної оптики. Для цього дослідження запропоновано новий теоретичний підхід, оснований на теорії збурень. Досліджено магнитооптичні ефекти Фарадея та Фойгта поблизу екстремальних точок фотонних зон, де має місце їх значне посилення. На основі розробленої теории обговорюються деякі експериментальні результати. Експериментальна частотна залежність фарадеївського кута обертання добре узгоджується з теоретичними передбаченнями.