

Refraction and reflection of bulk spin waves on a boundary of two homogeneous dielectric ferromagnets having biaxial anisotropy

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Refraction index of spin waves propagating in a dielectric ferromagnetic medium consisting of two homogeneous parts with different parameters of exchange interaction, uniaxial and rhombic anisotropy as well as saturation magnetization, has been calculated in the formalism of spin density, taking into account the defect of exchange interaction on the interface of this media. The coefficients of spin wave reflection and transmission have been computed. The dependences of both reflected wave intensity and refraction index on the wave frequency and value of external permanent homogeneous magnetic field have been obtained.

В формализме спиновой плотности рассчитывается показатель преломления объемных спиновых волн, распространяющихся в ферромагнитной среде, состоящей из двух частей с различными значениями параметров обменного взаимодействия, одноосной и ромбической анизотропии, а также намагниченности насыщения, с учетом дефекта обменного взаимодействия на границе этих сред. Вычислены коэффициенты отражения и прохождения спиновых волн. Получены зависимости интенсивности отраженной волны и показателя преломления от частоты волны и величины внешнего постоянного однородного магнитного поля.

As a rule, the wave concept is traditionally used to describe theoretically the peculiarities of spin wave propagation. This concept is useful, for example, to find spectral and some other characteristics of magnetic materials [1–5].

This paper is devoted to application of the geometrical optics formalism to describe the behaviour of spin waves propagating in a ferromagnetic medium with inhomogeneous distribution of magnetic parameters. The use of this approach enables to vary necessarily the propagation direction of spin waves (in particular, a focusing) by forming artificial inhomogeneities of the medium magnetic parameters of the set configuration, and also by changing the external magnetic field strength.

In the paper [6], refractive index of a spin beam has been determined, and its behaviour was investigated at the boundary of two homogeneous magnetic media with different parameters of exchange interaction and uniaxial magnetic anisotropy. In this work, the case of biaxial magnetic medium is considered taking into account the jump of the rhombic anisotropy parameter as well as the exchange interaction defect. Besides, refractive index and reflection intensity of a spin wave are calculated at the boundary of two homogeneous ferromagnetic materials with diffrerent values of uniaxial and rhombic magnetic anisotropy, as well as constants of exchange interaction and saturation magnetisation.

Let us consider an unbounded ferromagnetic medium consisting of two half-infinite homogeneous parts. Those are in contact along the yOz plane and have in half-spaces the values of saturation magnetization M_{01} and M_{02}, values of exchange interaction parameters α_1, α_2, uniaxial β_1, β_2 magnetic anisotropy and rhombic one ρ_1, ρ_2. The easy axis of such structure and an external permanent magnetic field are directed along Oz axis.

The energy density of such magnetic structure in exchange mode looks as

$$w = \sum_{j=1}^{2} \theta[(-1)^j x] w_j + A\delta(x)\mathbf{M}_1\mathbf{M}_2, \quad (1)$$

where

$$w_j = \frac{\alpha}{2}\left(\frac{\partial m_j}{\partial x_k}\right)^2 + \frac{\beta}{2}\left(m_{jx}^2 + m_{jy}^2\right) + \rho m_{jx}^2 - H_0 M_{jz}, \quad (2)$$

$\theta(x)$ is the Heavyside step function; A, the parameter characterizing the exchange interaction between half-spaces at $x = 0$; $\mathbf{M}_j = M_{0j}\mathbf{m}_j$; $\mathbf{m}_j$, unit vectors in the direction of magnetization, $j = 1,2$.

We shall use the formalism of spin density [7], according to which magnetization can be presented as

$$\mathbf{M}_j(\mathbf{r},t) = M_{0j}\Psi_j^{+}(\mathbf{r},t)\sigma\Psi_j(\mathbf{r},t), \quad j = 1,2, \quad (3)$$

where Ψ_j are quasi-classic wave functions playing the part of spin density order parameter; $\mathbf{r}$, the radius-vector of Cartesian coordinates; t, time; σ, Pauli matrices.

The Lagrange equations for Ψ_j have the form

$$i\hbar\frac{\partial\Psi_j(\mathbf{r},t)}{\partial t} = -\mu_0\mathbf{H}_{ej}(\mathbf{r},t)\sigma\Psi_j(\mathbf{r},t), \quad (4)$$

where μ_0 is a Bohr magneton;

$$\mathbf{H}_{ej} = -\frac{\partial w_j}{\partial \mathbf{M}_j} + \frac{\partial}{\partial x_k}\frac{\partial w_j}{\partial(\partial\mathbf{M}_j/\partial x_k)}.$$

Taking into account that the material magnetization in the ground state is parallel to $\mathbf{e}_z$ and assuming $M_j^2(\mathbf{r},t) = const$ in both half-spaces, we shall search the solution of (4) as

$$\Psi_j(\mathbf{r},t) = \exp(i\mu_0 H_0 t/\hbar)\cdot\begin{pmatrix} 1 \\ \chi_j(\mathbf{r},t)\end{pmatrix} \quad (5)$$

where $\chi_j(\mathbf{r},t)$ is a small addition characterizing the deviation of magnetization from the ground state. Linearizing the equations (4) and taking into account (5), we obtain:

$$-\frac{i\hbar}{2\mu_0 M_{0j}}\frac{\partial\chi_j(\mathbf{r},t)}{\partial t} = $$
$$= \left(\alpha_j\Delta - \beta_j - \frac{\rho_j}{2} - \tilde{H}_{0j}\right)\chi_j(\mathbf{r},t) - \frac{\rho_j}{2}\chi_j^*(\{\mathbf{r},t),$$

$$\frac{i\hbar}{2\mu_0 M_{0j}}\frac{\partial\chi_j^*(\mathbf{r},t)}{\partial t} = \quad (6)$$
$$= \left(\alpha_j\Delta - \beta_j - \frac{\rho_j}{2} - \tilde{H}_{0j}\right)\chi_j^*(\mathbf{r},t) - \frac{\rho_j}{2}\chi_j(\mathbf{r},t),$$

where $\tilde{H}_{0j} = H_0/M_{0j}$, $j = 1,2$.

Expressing $\chi^*(\mathbf{r},t)$ from one of the equations (6) and substituting it into another, we obtain the following equation for the magnetization dynamics:

$$-\frac{\hbar^2}{(2\mu_0 M_{0j})^2}\frac{\partial^2\chi_j(\mathbf{r},t)}{\partial t^2} = \quad (7)$$
$$= \left[\alpha_j^2\Delta^2 - 2\alpha_j\left(\beta_j + \frac{\rho_j}{2} + \tilde{H}_{0j}\right)\Delta + \right.$$
$$\left. + \left(\beta_j + \tilde{H}_{0j}\right)\left(\beta_j + \rho_j + \tilde{H}_{0j}\right)\right]\chi_j(\mathbf{r},t).$$

Representing in (7) $\chi = \chi_{\mathbf{k}\omega}\exp[i(\mathbf{kr} - \omega t)]$ where ω is the spin wave frequency, we obtain an expression for spin wave spectrum in biaxial magnetic medium:

$$\Omega_j^2 = \left(\alpha_j k_j^2 + \beta_j + \tilde{H}_{0j}\right)\left(\alpha_j k_j^2 + \beta_j + \rho_j + \tilde{H}_{0j}\right), \quad (8)$$

where $\Omega_j = \omega\hbar/2\mu_0 M_{0\mathrm{j}}$.

We shall use the JWKB method [8, 9] to simplify the equation (7).

Let us present in (7) $\chi_j(\mathbf{r},t) = C\exp[i(k_0 s_j(\mathbf{r}) - \omega t)]$, where k_0 is the module of wave vector corresponding to spin wave incident the boundary, C is the slowly changing amplitude. It follows from (8) that

$$\alpha_j k_j^2 = \sqrt{\Omega_j^2 + \rho_j^2/4} - \beta_j - \rho_j/2 - \tilde{H}_{0j}.$$

If the spin wave length λ satisfies the transition condition of geometrical optics:

$$\lambda \ll a, \quad (9)$$

where a is a characteristic size of an inhomogeneity present in the medium, then we obtain from (7) an analogue of classic Hamilton-Jacoby equation:

$$(\Delta s_j)^2 = n_j^2, \quad (10)$$

where

$$n_j^2 = k_j^2/k_0^2. \quad (11)$$

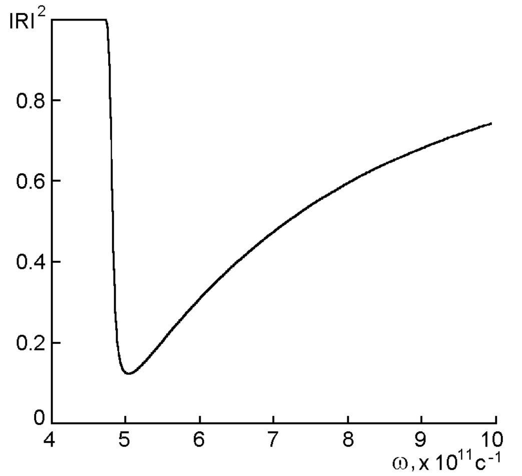

Fig. 1. The dependence of reflection coefficient $|R|^2$ on the spin wave frequency ω at $\alpha_1 = 10^{-8}$ cm^2, $\alpha_2 = 2 \cdot 10^{-8}$ cm^2, $\beta_1 = 50$, $\beta_2 = 100$, $\rho_1 = 10$, $\rho_2 = 20$, $M_{10} = 125$ Gs, $M_{20} = 175$ Gs, $A = 100$, $\theta_1 = \pi/80$, $H_0 = 800$ Oe.

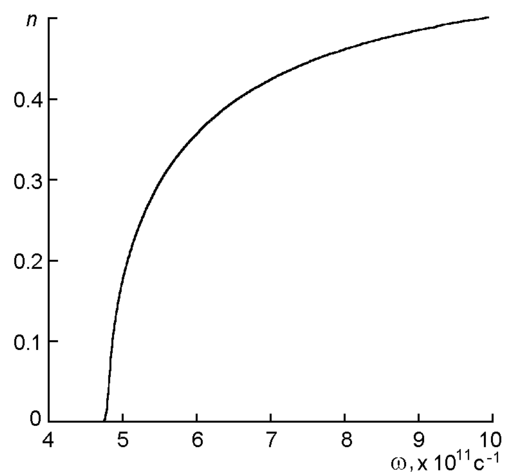

Fig. 2. The dependence of refraction index n on the frequency ω at $\alpha_1 = 10^{-8}$ cm^2, $\alpha_2 = 2 \cdot 10^{-8}$ cm^2, $\beta_1 = 50$, $\beta_2 = 100$, $\rho_1 = 10$, $\rho_2 = 20$, $M_{10} = 125$ Gs, $M_{20} = 175$ Gs, $H_0 = 800$ Oe.

Like to optics [10], we consider, that right part of the equation (10) is a square of refraction index:

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{k_2}{k_0} = \left(\frac{\alpha_1\sqrt{\Omega_2^2 + \rho_2^2/4} - \beta_2 - \rho_2/2 - \tilde{H}_{02}}{\alpha_2\sqrt{\Omega_1^2 + \rho_1^2/4} - \beta_1 - \rho_1/2 - \tilde{H}_{01}}\right)^{1/2} = n, \tag{12}$$

where θ_1 is the incidence angle, θ_2, the refraction one.

The critical total reflection angle is defined as

$$\sin\theta_0 = \left(\frac{\alpha_1\sqrt{\Omega_2^2 + \rho_2^2/4} - \beta_2 - \rho_2/2 - \tilde{H}_{02}}{\alpha_2\sqrt{\Omega_1^2 + \rho_1^2/4} - \beta_1 - \rho_1/2 - \tilde{H}_{01}}\right)^{1/2}.$$

Let us use boundary conditions for $\chi(\mathbf{r},t)$, which follow from (1)–(2):

$$[A\gamma(\chi_2 - \chi_1) + \alpha_1\chi'_1]_{x=0} = 0, \tag{13}$$
$$[A(\chi_1 - \chi_2) - \gamma\alpha_2\chi'_2]_{x=0} = 0.$$

Here $\gamma = M_{20}/M_{10}$. We shall obtain the expressions for amplitudes of the spin wane reflection and transmission. Let the incident, reflected and transmitted waves be defined as $\chi_I = \exp(i(\mathbf{k}_0\mathbf{r} - \omega t))$, $\chi_R = R\exp(i(\mathbf{k}_1\mathbf{r} - \omega t))$ and $\chi_D = D\exp(i(\mathbf{k}_2\mathbf{r} - \omega t))$, respectively. Here R is the complex reflection amplitude, D, the transmission amplitude; $\mathbf{k}_0$, $\mathbf{k}_1$, wave vectors of incident and reflected waves, respectively; $\mathbf{k}_2$, the wave vector of transmitted wave.

Then

$$R = \frac{k_0\alpha_1\alpha_2\gamma\cos\theta_1\sqrt{n^2 - \sin^2\theta_1} - iA\left(\alpha_1\cos\theta_1 - \alpha_2\gamma^2\sqrt{n^2 - \sin^2\theta_1}\right)}{k_0\alpha_1\alpha_2\gamma\cos\theta_1\sqrt{n^2 - \sin^2\theta_1} - iA\left(\alpha_1\cos\theta_1 + \alpha_2\gamma^2\sqrt{n^2 - \sin^2\theta_1}\right)}, \tag{14}$$

$$D = \frac{-2iA\alpha_1\cos\theta_1}{k_0\alpha_1\alpha_2\gamma\cos\theta_1\sqrt{n^2 - \sin^2\theta_1} - iA\left(\alpha_1\cos\theta_1 + \alpha_2\gamma^2\sqrt{n^2 - \sin^2\theta_1}\right)}.$$

Let us estimate the material parameters when a lens is thin and incident angles are small. Obviously, we have to provide a necessary lens transparency. The reflected wave intensity is defined by a square of reflection amplitude module and, according to (14), is given by $|R|^2 \approx \left[\left(\alpha_1 - \alpha_2\gamma^2 n\right)/\left(\alpha_1 + \alpha_2\gamma^2 n\right)\right]^2$ (for small incident angles and $A \to \infty$). Demanding a conformity to the condition $|R|^2 < \eta$, where η is a necessary smallness of reflection coefficient, we obtain a limitation on n and, therefore, on α, β, ρ, ω, M_0 and H_0:

$$\frac{1 - \sqrt{\eta}}{1 + \sqrt{\eta}} < \frac{\alpha_2}{\alpha_1}n < \frac{1 + \sqrt{\eta}}{1 - \sqrt{\eta}}.$$

In particular, at $\alpha_1 = \alpha_2$, $M_{01} = M_{02}$, the reflection coefficient is less than 10 % if $0.52 < n < 1.92$. The respective limitations for a mirror are given by or

$$\frac{\alpha_2}{\alpha_1}n < \frac{1 - \sqrt{\eta}}{1 + \sqrt{\eta}} \text{ or } \frac{\alpha_2}{\alpha_1}n > \frac{1 + \sqrt{\eta}}{1 - \sqrt{\eta}}.$$

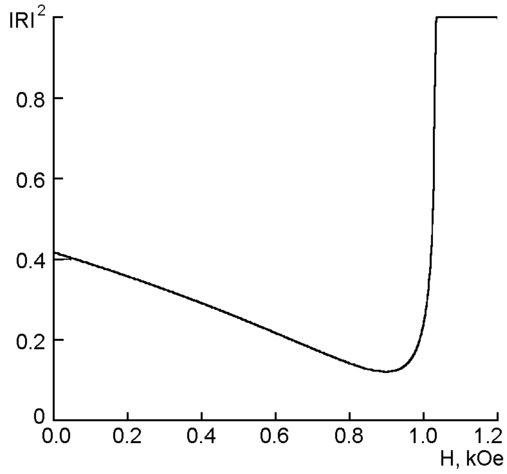

Fig. 3. The dependence of reflection coefficient $|R|^2$ on the external permanent magnetic field strength H_0 at $\alpha_1 = 10^{-8}$ cm^2, $\alpha_2 = 2 \cdot 10^{-8}$ cm^2, $\beta_1 = 50$, $\beta_2 = 100$, $\rho_1 = 10$, $\rho_2 = 20$, $M_{10} = 125$ Gs, $M_{20} = 175$ Gs, $A = 100$, $\theta_1 = \pi/80$, $\omega = 4.6 \cdot 10^{11}$ s^{-1}.

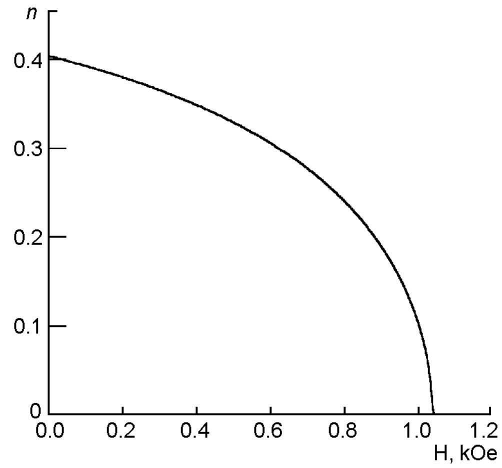

Fig. 4. The dependence of refraction index n on the external permanent magnetic field strength H_0 at $\alpha_1 = 10^{-8}$ cm^2, $\alpha_2 = 2 \cdot 10^{-8}$ cm^2, $\beta_1 = 50$, $\beta_2 = 100$, $\rho_1 = 10$, $\rho_2 = 20$, $M_{10} = 125$ Gs, $M_{20} = 175$ Gs, $\omega = 4.6 \cdot 10^{11}$ s^{-1}.

For example, $|R|^2 > 0.9$ is reached for $\alpha_1 = \alpha_2$, $M_{01} = M_{02}$ at $n < 0.03$ or $n > 37.97$.

To satisfy to the condition of geometrical optics (9), a thickness of lens or mirror is restricted by

$$a \gg 2\pi\left(\alpha/\left(\sqrt{\Omega^2 + \rho^2/4} - \beta - \rho/2 - \tilde{H}_0\right)\right)^{1/2}. \qquad (15)$$

As it is seen from (15), parameters for making lens or mirror can be easy provided using a wide variety of magnetic materials [11]. In particular, in the case of ferrite garnets, the condition (15) for thin lens gives permissible values of $a > 10^{-4} \div 10^{-6}$ cm.

Figures 1, 2 show the dependences of reflection intensity $I_R = |R|^2$ and refraction index n on the spin wave frequency at characteristic values of material parameters [11]. It is seen well that necessary correlation between intensities of reflected and transmitted waves for a selected frequency can be achieved by choosing the material parameters. Besides, Fig. 3 shows an essential dependence of reflection intensity on the value of external homogeneous magnetic field. This makes it possible to vary the reflected wave intensity within a wide range by of changing only the magnetic field value at fixed material parameters. In this case, the refraction index changes like to shown in Fig. 4.

Thus, there is a possibility to obtain a necessary value of reflection coefficient from an inhomogeneity acting as a lens or mirror by means of external field change. Thereby, the reflection coefficient can change considerably without change of medium parameters. This fact allows to use the same inhomogeneity both as lens and as mirror at fixed parameters of structure.

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Заломлення та відбиття об'ємних спінових хвиль на межі розділу двох однорідних діелектричних магнетиків з двовісною анізотропією

Ю.І.Горобець, С.О.Решетняк

У формалізмі спінової густини розраховується показник заломлення об'ємних спінових хвиль, що поширюються у феромагнітному середовищі, яке складається з двох однорідних частин з різними параметрами обмінної взаємодії, одновісної та ромбічної анізотропії, а також намагніченості насичення, з урахуванням дефекту обміну на межі розділу цих середовищ. Обчислено коефіцієнти відбиття та проходження спінових хвиль. Отримано залежності інтенсивності відбитої хвилі та показника заломлення від частоти хвилі та величини зовнішнього постійного однорідного магнітного поля.