Fizika Nizkikh Temperatur, 1999, v. 25, No 11, p.1154–1160

On the self-consistent theory of Josephson effect in ballistic superconducting microconstrictions

M. Zareyan

Institute for Advanced Studies in Basic Sciences, 45195-159, Gava Zang, Zanjan, Iran

Yu. A. Kolesnichenko and A. N. Omelyanchouk

B. Verkin Institute for Low Temperature Physics and Engineering, National Academy of Sciences of Ukraine, 47 Lenin Ave., 310164 Kharkov, Ukraine
E-mail: omelyanchouk@ilt.kharkov.ua

Received March 22, 1999

The microscopic theory of current-carrying states in the ballistic superconducting microchannel is presented. The effects of the contact length L on the Josephson current are investigated. For the temperatures T close to the critical temperature T_c the problem is treated self-consistently, with allowance for the distribution of the order parameter $\Delta(\mathbf{r})$ inside the contact. The closed integral equation for Δ in strongly inhomogeneous microcontact geometry ($L \lesssim \xi_0$, where ξ_0 is the coherence length at $T = 0$) replaces the differential Ginzburg–Landau equation. The critical current $I_c(L)$ is expressed in terms of the solution of this integral equation. The limiting cases of $L << \xi_0$ and $L >> \xi_0$ are considered. With increasing length L, the critical current decreases, although the ballistic Sharvin resistance of the contact remains the same as at $L = 0$. For ultrashort channels with $L \lesssim a_D$ ($a_D \sim v_F/\omega_D$, where ω_D is the Debye frequency) the corrections for the value of the critical current I_c ($L = 0$) are sensitive to the strong-coupling effects.

PACS: **73.23.–b**, **74.50.+r**

1. Introduction

Weak superconducting links [1] include the tunnel structures *SIS* (superconductor—insulator—superconductor) and the contacts with direct conductivity, *SNS* (*N* is the normal layer) and *ScS* (*c* is a geometrical constriction). Superconducting constrictions can be modeled as an orifice with diameter d in an impenetrable sheet for electrons between two superconducting half spaces (point contact) or as a narrow channel with length L in contact with superconducting banks (microbridge). Aslamazov and Larkin [2] have shown on the basis of a solution of the Ginzburg-Landau (GL) equations that in the dirty limit and for small sizes of the constriction L, $d << \xi(T)$ [$\xi(T)$ is the GL coherence length] the *SCS* contact can be described by a Josephson model with the current-phase relation

$$I = I_c \sin\varphi\ , \quad I_c = \pi\Delta_0^2(T)/(4eR_N T_c)\ , \qquad (1)$$

where I_c is the Josephson critical current; Δ_0 is the absolute value of the order parameter in the bulk banks; T_c is the critical temperature and R_N is the normal-state resistance of the dirty microbridge. The critical current of the microbridge (1) depends on the bridge length as $I_c \sim 1/L$. The expression (1) is valid within the domain of applicability of the GL approach, i.e., for temperatures T close to T_c and L, $d >> \xi_0$ ($\xi_0 \simeq v_F/T_c$ is the coherence length at $T = 0$, and v_F is the Fermi velocity).

The present level of technology has made it possible to study the ultrasmall Josephson weak links with the dimensions up to interatomic size. For example, they can be nanosize microchannels produced by means of a scanning tunneling microscope [3] or point contacts and microchannels obtained by using the mechanical controllable break technique [4–6]. The microchannels between two superconductors can also arise spontaneously as microshorts in tunnel junctions [7], with the length L determined by the thickness of an insulator layer. The value of the critical current I_c of such microshorts is of special interest in the case of tunnel structures based on high-T_c metal-oxide compounds. Small microconstrictions with dimensions of the order or smaller than the coherence length ξ_0, when the expression (1) for the critical current $I_c \sim 1/L$ is not valid, require the microscopic consideration even for T near T_c. Such microscopic theory of stationary Josephson effect in microconstrictions was developed in Ref. 8 for the ballistic channel of zero length $L = 0$ in the model of the orifice with diameter $d << \xi_0$. The Josephson current in this case is given by

$$I = \frac{\pi\Delta_0(T)}{eR_0} \sin\frac{\varphi}{2} \tanh\frac{\Delta_0(T)\cos(\varphi/2)}{2T} , \quad -\pi < \varphi < \pi , \qquad (2)$$

$$R_0^{-1} = \frac{1}{2} S e^2 v_F N(0) , \qquad (3)$$

where $S = \pi d^2/4$ is the contact cross-sectional area, and $N(0) = m p_F/(2\pi^2)$ is electron density of states at the Fermi surface. At temperatures $T_c - T << T_c$ expression (2) coincides with the Aslamazov-Larkin result [Eq. (1)], in which instead of the normal resistance R_N for dirty metal, the ballistic Sharvin resistance [9] R_0 (3) is substituted.

In this article we present a microscopic theory of current-carrying states in the ballistic microbridges of arbitrary length L in the scale of the coherence length ξ_0. We have investigated the dependence of the Josephson critical current on the ratio L/ξ_0 and analyzed the transition from the case of $I_c(L = 0)$ [Eq. (2)] to $I_c \sim 1/L$ [Eq. (1)] with increasing length L.

In Sec. 2 we formulate the model of a microbridge and the microscopic equations for Green's functions with boundary conditions at the bridge edges. In studying the effects on the critical current of the lenght of the microconstriction, the crucial point, as always, in the inhomogeneous superconducting state is the self-consistent treatment of the order parameter distribution $\Delta(\mathbf{r})$ inside the weak link. In Sec. 3 the closed integral equation for the order parameter Δ in the microchannel is derived for temperatures near T_c, which in a strongly inhomogeneous ($L \sim \xi_0$) microcontact geometry replaces the differential GL equation. The critical current $I_c(L)$ is expressed in terms of the solution of this integral equation. The limiting cases of $L << \xi_0$ and $L >> \xi_0$ are considered. We will show that in addition to the characteristic scale ξ_0, there is the length $a_D \simeq v_F/\omega_D$ (ω_D is the Debye frequency) in the case of an ultrasmall channel. The length $L \sim a_D$ is the length at which the frequency of the ballistic flight of an electron from one bank to another becomes comparable with the frequency ω_D, which characterizes the retardation of the electron-phonon interaction. In conventional superconductors the value of the coherence length ξ_0, about 10^{-4} cm, is much larger than $a_D \sim 100$ Å. In high-T_c metal-oxide compounds, howerer, we have a situation in which ξ_0 is comparable with a_D. Thus, in high-T_c compounds the critical current of the contact with dimensions $\sim a_D \sim \xi_0$ is sensitive to the effects of strong coupling.

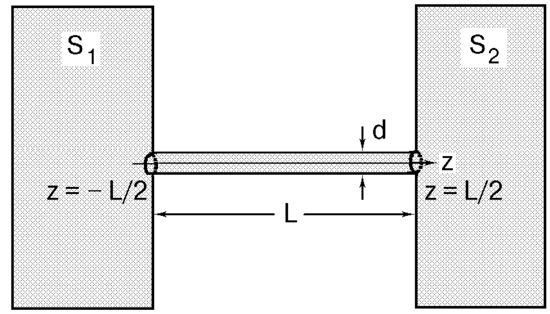

Fig. 1. Model of ScS contact as narrow superconducting channel is in contact with bulk superconductors S_1 and S_2.

2. Model and basic equations

We consider the model of a contact in the form of a filament (narrow channel) that joins two superconducting half-spaces (massive banks) (Fig. 1). The length L and the diameter d of the channel are assumed to be large as compared with the Fermi wavelength λ_F, so we can apply the quasi-classical approximation. In the ballistic case, we proceed from the quasi-classical Eilenberger equation for the energy-integrated Green's function [10]:

$$\mathbf{v}_F \frac{\partial \hat{G}}{\partial \mathbf{r}} + [\omega\hat{\tau}_3 + \hat{\Delta}, \hat{G}] = 0 , \qquad (4)$$

where

$$G(\omega, \mathbf{v}_F, \mathbf{r}) = \begin{pmatrix} g_\omega & f_\omega \\ f_\omega^+ & -g_\omega \end{pmatrix}$$

is the matrix Green's function which depends on the Matsubara frequency ω, the electron velocity on the Fermi surface $\mathbf{v}_F$, and the spatial variable $\mathbf{r}$;

$$\hat{\Delta}(\mathbf{r}) = \begin{pmatrix} 0 & \Delta \\ \Delta^* & 0 \end{pmatrix}$$

is the superconducting pair potential; $\hat{\tau}_i$ ($i = 1, 2, 3$) are Pauli matrices. Equation for the matrix Green's function (4) is supplemented by the normalization condition [11]

$$\hat{G}^2 = 1 . \qquad (5)$$

The off-diagonal potential $\Delta(\mathbf{r})$ must be determined from the self-consistency equation

$$\Delta(\mathbf{r}) = \lambda 2\pi T \sum_{\omega > 0} \langle f \rangle \ , \quad (6)$$

in which $\langle ... \rangle$ stands for averaging over directions of $\mathbf{v}_F$ on the Fermi surface, and λ is the electron-phonon coupling constant. In the BCS model the summation over ω contains the cutoff on the frequency ω_D, which is of the order of the Debye frequency.

The equations (4) and (6) are supplemented by the values of the Green's functions and Δ in the bulk superconductors S_1 and S_2 far from the channel ends:

$$\hat{G}_{1,2} = \frac{\omega\hat{\tau}_3 + \hat{\Delta}_{1,2}}{\Omega} \ , \quad (7)$$

$$\hat{\Delta}_{1,2} = \Delta_0 \left(\cos(\varphi/2)\hat{\tau}_1 \pm \sin(\varphi/2)\hat{\tau}_2\right) .$$

Thus the phase φ is the total phase difference at the contact. We also must determine the boundary conditions concerning the reflection of the electrons from the surface of the superconductors $\mathbf{r}_S$. For simplicity we assume that at $\mathbf{r}_S$ electrons undergo the specular reflection. Then for quasiclassical Green's function we have the boundary condition (Ref. 8)

$$G(\mathbf{v}_F , \mathbf{r}_S) = G(\mathbf{v}'_F , \mathbf{r}_S) \ , \quad (8)$$

in which $\mathbf{v}_F$ and $\mathbf{v}'_F$ are the velocities of the incident and specular reflected electron. These velocities are related by the conditions, which conserve the component of $\mathbf{v}_F$ parallel to the reflecting surface $\mathbf{r}_S$ and changes the sign of the normal component.

The solutions of Eqs. (4) and (6) allow us to calculate the current density $\mathbf{j}$:

$$\mathbf{j}(r) = -4i\pi e N(0) T \sum_{\omega > 0} \langle \mathbf{v}_F \, g_\omega \rangle \ . \quad (9)$$

In the case of the microconstriction shown in Fig. 1, under the conditions $d << \xi_0$ and $L >> d$ (d is the contact diameter) inside the filament we can solve the one-dimensional Eilenberger equations with $\Delta = \Delta(z)$. The banks of the bridge are equivalent here to certain boundary conditions for the Green's function $\hat{G}(v_z , z)$ at the points $z = \pm L/2$. Following the procedure which was described in Ref. 8, we find the Green's functions at the end points ($z = \pm L/2$) from the general solutions of Eq. (4) in superconducting half-spaces S_1 and S_2 with conditions (5). They are given by

$$\hat{G}(z = \mp L/2) = \hat{G}_{1,2} +$$

$$+ A_{1,2}[\Delta_0 \hat{\tau}_3 - [\omega \cos(\varphi/2) + i\eta\Omega \sin(\varphi/2)] \hat{\tau}_1 \mp$$

$$\mp [\omega \sin(\varphi/2) - i\eta\Omega \cos(\varphi/2)] \hat{\tau}_2 \ , \quad (10)$$

where $\Omega = \sqrt{\omega^2 + \Delta_0^2}$ and $\eta = \mathrm{sign}\,(v_z)$. The arbitrary constants $A_{1,2}$ must be determined by matching these boundary conditions with the solution for $\hat{G}(v_z , z)$ inside the channel.

Taking the off-diagonal components in Eq. (4), we have the following first-order differential equations for the anomalous Green's functions:

$$v_z \frac{df_\omega}{dz} + 2\omega f_\omega = 2\Delta(z) g_\omega \ ,$$

$$- v_z \frac{df^+_\omega}{dz} + 2\omega f^+_\omega = 2\Delta^*(z) g_\omega \ . \quad (11)$$

The normal Green's function g_ω, as follows from condition (5), is expressed in terms of f_ω and f^+_ω:

$$g_\omega = \sqrt{1 - f_\omega f^+_\omega} \ . \quad (12)$$

From Eqs. (6), (9), (11), and (12) we obtain the symmetry relations

$$f^+_\omega(v_z , z) = [f_\omega(-v_z , z)]^*, \ \ \Delta^*(z) = \Delta(-z) \quad (13)$$

and the current conservation inside the channel $dj/dz = 0$.

3. Josephson current and order parameter distribution in superconducting microchannel

In present paper we consider the case of temperatures T close to the critical temperature T_c. Near the phase transition curve the order parameter $\Delta_0(T)$ in the banks is small. In order to find the Josephson current in the lowest order in Δ_0 we linearize Eqs. (11) for Δ and obtain $f_\omega \sim \Delta_0(T)$, $g_\omega \simeq 1 - ½ f_\omega f^+_\omega \sim 1 - O(\Delta_0^2)$, $j \sim \Delta_0^2$. The equation for f_ω near T_c takes the form

$$v_z \frac{df_\omega}{dz} + 2\omega f_\omega = 2\Delta(z) \ , \quad (14)$$

with linearized boundary conditions (10)

$$f_\omega(v_z > 0, \ z = -L/2) = \frac{\Delta_0}{\omega} e^{-i\varphi/2} \ ,$$

$$f_\omega(v_z < 0, \ z = +L/2) = \frac{\Delta_0}{\omega} e^{+i\varphi/2} \ . \quad (15)$$

Its solution for arbitrary function $\Delta(z)$ is given by

$$f_\omega(v_z\,,\,z) = \frac{\Delta_0}{\omega}\,\mathrm{e}^{-i\eta\varphi/2}\mathrm{e}^{-(2\omega/v_z)(z+\eta L/2)} +$$

$$+\,\mathrm{e}^{-2\omega z/v_z}\int\limits_{-\eta L/2}^{z} dz'\,\frac{2\Delta(z')}{v_z}\,\mathrm{e}^{2\omega z'/v_z}\,. \qquad (16)$$

The Green's function $f^{+}_{\omega}(v_z\,,\,z)$ is obtained from expression (14) with the help of relations (15).

Substituting the function $f_\omega(v_z\,,\,z)$ (16) in the self-consistency equation (6), we obtain the integral equation for the space-dependent order parameter inside the contact

$$\Delta(z) = A(z) + \int\limits_{-L/2}^{L/2} dz'\,\Delta(z')\,K(|z-z'|)\,, \qquad (17)$$

where

$$A(z) = \lambda 2\pi T\sum_{\omega>0}\frac{\Delta_0}{\omega}\left\langle \mathrm{e}^{-\omega L/v_z}\cosh\left(\frac{2\omega z}{v_z} + i\,\frac{\varphi}{2}\right)\right\rangle_{v_z>0}, \qquad (18)$$

$$K(z) = \lambda 2\pi T\sum_{\omega>0}\left\langle\frac{1}{v_z}\,\mathrm{e}^{-2\omega z/v_z}\right\rangle_{v_z>0}. \qquad (19)$$

The averaging $\langle\ldots\rangle_{v_z>0}$ denotes

$$\langle F(v_z = v_F\cos\theta)\rangle_{v_z>0} = \int\limits_0^1 d(\cos\theta)\,F(\cos\theta)\,.$$

In the case of strongly inhomogeneous microcontact problem the integral equation for the order parameter Δ replaces the differential Ginzburg-Landau equation. It contains the needed boundary conditions at the points of contact between the filament and the bulk superconductors. Some general properties of the solution $\Delta(z)$ of Eq. (17) follow from the form of the functions (18) and (19). Let us write $\Delta(z)$ in the form

$$\Delta(z) = \Delta_0(T)\left(\cos\frac{\varphi}{2} + iq(z)\sin\frac{\varphi}{2}\right) \qquad (20)$$

and substitute it in Eq. (17). For the function $q(z)$ we obtain the equation

$$q(z) = b(z) + \int\limits_{-L/2}^{L/2} dz'\,q(z')K(|z-z'|)\,, \qquad (21)$$

with $K(z)$ defined by (19) and the new out-integral function $b(z)$,

$$b(z) = \lambda 2\pi T\sum_{\omega>0}\frac{1}{\omega}\left\langle \mathrm{e}^{-\omega L/v_z}\sinh\left(\frac{2\omega z}{v_z}\right)\right\rangle_{v_z>0}. \qquad (22)$$

In obtaining Eq. (21) we have used the relation

$$\lambda 2\pi T\sum_{\omega>0}^{\omega_D}\frac{1}{\omega} = 1 \quad,\quad \text{for}\quad T\to T_c\,. \qquad (23)$$

It follows from (19), (21), and (22) that the function $q(z)$ has such properties:

i) the function $q(z)$ is real,

ii) $q(z)$ does not depend on the phase φ,

iii) $q(-z) = -\,q(z)$, $q(0) = 0$.

Thus, the value of the order parameter Δ at the center of the contact always is equal to $\Delta_0(T)\cos(\varphi/2)$. Also, the universal phase dependence of $\Delta(z,\varphi)$, which is determined by (20) and i)–iii), leads (see below) to the sinusoidal current-phase dependence $j = j_c\sin\varphi$. It is emphasized that these general properties of the ballistic microchannel [within the considered case of "rigid" boundary conditions (10) and temperatures close to T_c] *does not depend* on the contact length L, in particular, on the ratio of L/ξ_0.

Now we are going to obtain the Josephson current in the system. To calculate the total current $I = Sj$ that flows through the channel at the given phase difference φ, we use the equation for the current density (9) and the anomalous Green's function f_ω (16) obtained above. The normal Green's function q_ω (12) in the second order in $\Delta_0(T)$ is $g_\omega(v_z\,,\,z) = 1 - ½\,f_\omega(v_z\,,\,z)[f_\omega(-v_z\,,\,z)]^*$. It is convenient to calculate the current density at the point $z = 0$. Using the expression for $\Delta(z)$ (20), we obtain the general formula for the Josephson current $I(\varphi)$ in terms of the function $q(z)$:

$$I(\varphi) = I_c\sin\varphi\,, \qquad (24)$$

$$I_c = I_0\,\frac{16T^2}{v_F}\times$$

$$\times\sum_{\omega>0}\left[\frac{1}{\omega^2}\,\langle v_z\,\mathrm{e}^{-\omega L/v_z}\rangle_{v_z>0} + \frac{2}{\omega}\int\limits_0^{L/2} dz\,q(z)\langle \mathrm{e}^{-2\omega z/v_z}\rangle_{v_z>0}\right]. \qquad (25)$$

Here $I_0 = \pi\Delta_0^2(T)/(4eR_0T_c)$ is the critical current at $L = 0$. It coincides with the result of Ref. 8 for the orifice (2) at T near T_c. Expression (25) jointly with Eq. (21) for $q(z)$ describes the dependence of the critical current on the contact length $I_c(L)$. It is valid for arbitrary value of the ratio L/ξ_0. Note that in our case $T \to T_c$, we have the relation $\xi_0, L << \xi(T)$.

Let us introduce the dimensionless quantities

$$x = z/L\ , \quad l = \frac{\pi T_c L}{v_F}\ , \quad \frac{\omega}{\pi T_c} = 2n + 1\ , \quad J_c = \frac{I_c}{I_0}\ . \tag{26}$$

In reduced units (26), after taking the average $\langle \ldots \rangle_{v_z > 0}$, the equations for $q(x)$ and J_c take the form

$$q(x) = b(x) + l \int_{-1/2}^{1/2} dx'\ q(x') K(|x - x'|)\ , \tag{27}$$

$$J_c = \frac{8}{\pi^2} \sum_{n=0}^{N} \left\{ \frac{\exp[-l(2n+1)][1 - l(2n+1)]}{(2n+1)} - l^2 Ei[-l(2n+1)] + \right.$$

$$\left. + 4l \int_0^{1/2} dx\ q(x) \frac{\exp[-2l(2n+1)x]}{(2n+1)} + 2lxEi[-2l(2n+1]x \right\}\ , \tag{28}$$

where

$$b(x) = \lambda \sum_{n=0}^{N} \left\{ \frac{2\exp[-l(2n+1)]\sinh[2l(2n+1)x]}{(2n+1)} + l(2n+1)(1-2x)Ei[-l(2n+1)(1-2x)] \right.$$

$$\left. + l(2n+1)(1+2x)Ei[-l(2n+1)(1+2x)] \right\}\ , \tag{29}$$

$$K(x) = -2\lambda \sum_{n=0}^{N} Ei[-2l(2n+1)x]\ . \tag{30}$$

The function $Ei(x) = \int_{-\infty}^{x} [(\exp t)/t]\, dt$ is the integral exponent. The upper limit N in the sums over n is related to the cutoff frequency ω_D in the BCS model, $N \simeq \omega_D/T_c$. The value of the coupling constant λ is related to N by Eq. (23) or, in reduced units,

$$2\lambda \sum_{n=0}^{N} \frac{1}{(2n+1)} = 1\ .$$

In the weak-coupling limit of $\lambda << 1$, we have $N >> 1$.

In the general case of the arbitrary value of the parameter l ($l \simeq L/\xi_0$) Eq. (27) is a convenient starting point for the numerical calculation of the function $J_c(l)$. We consider here two limiting cases, $l >> 1$ and $l << 1$.

For a long microbridge with $l >> 1$ we seek a solution of Eq. (27) in the form $q(x) = \alpha x$. Substituting this $q(x)$ in Eq. (27), we find $\alpha = 2 + O(1/l)$. Calculating J_c (28) with $q(x) = 2x$, we find that the order parameter and the critical current are

$$\Delta(z) = \Delta_0 \left(\cos\frac{\varphi}{2} + i\,\frac{2z}{L}\sin\frac{\varphi}{2} \right), \quad L >> \xi_0\ , \tag{31}$$

$$I_c(L) = \frac{14}{3\pi^2}\,\zeta(3) I_0 \frac{\hbar v_F}{T_c L}\ , \quad L >> \xi_0\ . \tag{32}$$

Expressions (31) and (32) coincide with the solution of GL equations (with effective boundary conditions for the order parameter Δ) for the clean superconducting microbridge [12]. Thus, our microscopic approach with the boundary conditions (10) for the Green's functions (not for Δ) gives the results of the phenomenological theory at $L >> \xi_0$.

For a short microbridge with $l << 1$, in zero approximation on l we find that $q(x) = 0$ [$\Delta(z) = \Delta_0 \cos(\varphi/2)$], $J_c = 1$ or, in dimension units, $I_c(0) = I_0$, in agreement with formula (2). The corrections for the zero approximation depend on the value of the product lN. For very small $l << T_c/\omega_D$ (i.e., $L << a_D \simeq v_F/\omega_D$), the product lN is small, although $N >> 1$. As a result, when $q(x, l)$ and $J_c(l)$ are calculated in the region $L < a_D$, the cutoff in the sums over n must be taken into account. Apparently, when the cutoff frequency appears explicitly but not through the value of T_c, the applicability of the BCS theory becomes questionable. More rigorous consideration, based on the Eliashberg theory of superconductivity [13], is needed in this case. Nevertheless, by using the BCS model with cutoff frequency we assume qualitatively to take into account the retardation effects of electron-phonon coupling in our problem. In the domain, defined by the following inequalities: $lN << 1$, $N >> 1$, $l << 1$, the functions $b(x)$ (29) and $K(x)$ (30) have the asymptotic behavior:

$$b(x) = 4\lambda lN\left\{x \ln (lN) + x(C + \ln 2) + \right.$$

$$\left. + \frac{1}{4}\left[\ln\left(\frac{1+2x}{1-2x}\right) + 2x \ln (1 - 4x^2)\right]\right\}, \quad (33)$$

$$K(|x|) = -2\lambda N[\ln (2lN|x|) - 1] . \quad (34)$$

Where $C \simeq 0.577$ is the Euler constant. As follows from Eqs. (33) and (34), in this case the integral term in Eq. (27) is small, and calculating the critical current in the first approximation on the small parameter lN, we can set $q(x) = b(x)$. As a result, we have

$$\Delta(z) = \Delta_0(T)\left(\cos\frac{\varphi}{2} + ib(z/L)\sin\frac{\varphi}{2}\right), \quad L << a_D, \quad (35)$$

with $b(x)$ is defined by expression (33),

$$I_c(L) = I_0\left(1 - \frac{8}{\pi\lambda}\frac{T_c L}{v_F}\right), \quad L << a_D . \quad (36)$$

In the region $\{l << 1$ and $lN \lesssim 1\}$ the integral term in Eq. (27) is numerically small as compared with the out-integral term $b(x)$. Using in Eq. (27) the $q(x) = b(x)$ as a rough approximation, we calculate the function $J_c(l)$ shown in Fig. 2.

For the case $l << 1$ and $lN >> 1$, we set $N = \infty$ in the equation for $q(x)$ and $J_c(l)$. The corrections for the critical current in this region of length L can be estimated as

$$I_c \approx I_0\left(1 - \text{const}\,\frac{L}{\xi_0}\ln\frac{\xi_0}{L}\right), \quad a_D << L << \xi_0 . \quad (37)$$

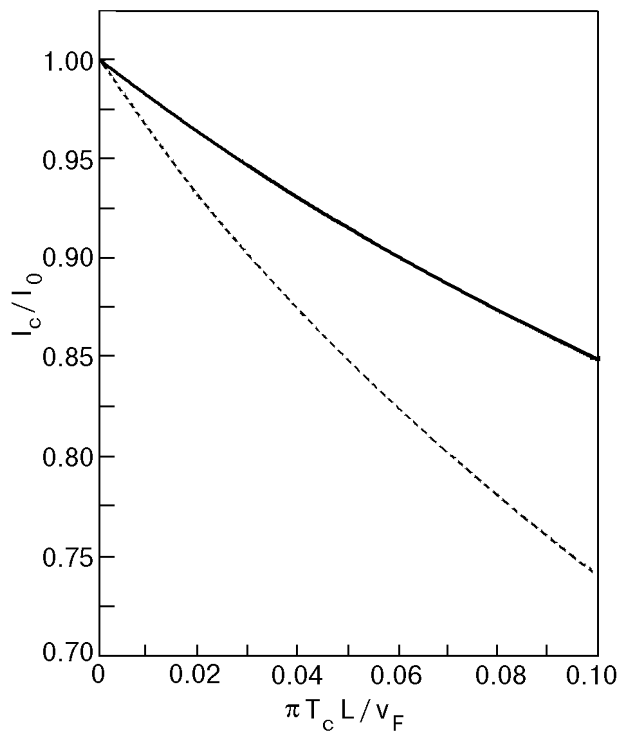

Fig. 2. Dependence of the critical current I_c on the contact length L for the microbridge (solid line). The coupling constant $\lambda = 0.2$. For comparison, the dependence $I_c(L)$ for *SNS* contact ($\lambda = 0$ inside the channel) is shown (dashed line).

The expressions (32), (36), and (37) describe the dependence of the critical current on the contact length in the limiting cases of short and long channels. With increasing length L, the critical current decreases. For ultrasmall $L \lesssim a_D$ the value of $\delta I_c/I_0 \sim (1/\lambda)(L/\xi_0)$ directly depends on the BCS coupling constant λ, and consequently it is sensitive to the effects of the strong electron-phonon coupling.

4. Conclusion

We have studied the size dependence of the Josephson critical current in ballistic superconducting microbridges. Near the critical temperature T_c, the Eilenberger equations have been solved self-consistently. The closed integral equation for the order parameter Δ (17) and the formula for the critical current I_c (25) are derived. Equations (17) and (25) are valid for the arbitrary microbridge length L in the scale of the coherence length, $\xi_0 \sim v_F/T_c$. In strongly inhomogeneous microcontact geometry they replace the differential Ginzburg–Landau equations and can be solved numerically. In the limiting cases $L >> \xi_0$ and $L << \xi_0$, we obtained the analytical expressions for Δ inside the weak link and for the $I_c(L)$. The dependence of I_c on L is shown schematically in Fig. 3. For a long

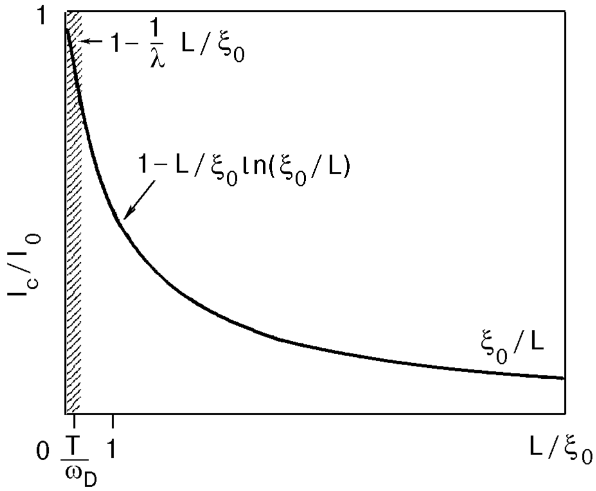

Fig. 3. Dependence of the critical current on the length of the bridge. The asymptotic behavior for short and long bridges is shown. The hatched region corresponds to the ultrashort microbridge, $L \lesssim v_F/\omega_D$.

microbridge, $L >> \xi_0$, the critical current $\sim 1/L$ is in correspondence with the phenomenological analysis. The main interest lies in the region $L \lesssim \xi_0$, where a microscopic theory is needed. We have calculated the corrections for the KO theory [8], which are connected with the finite value of the contact size. The expression (2) for the Josephson current was obtained in Ref. 8 in zeroth approximation on the contact size. For the $L << \xi_0$ we find that $\delta I_c/I_0 \sim (-L/\xi_0)\ln(\xi_0/L)$, where I_0 is the value of the critical current in KO theory. Thus, the corrections for the value I_0 are small when $L << \xi_0$, but the derivative dI_c/dL has a singularity at $L = 0$. This singularity is smeared if we take into account the finite value of the ratio T_c/ω_D. For an ultrashort microchannel, $L \lesssim a_D \sim v_F/\omega_D$ (the hatched region in Fig. 3), the length dependence of the critical current is $\delta I_c/I_0 \sim -L/(\lambda\xi_0)$ (λ is the constant of electron-phonon coupling). In the very small microcontacts we have a unique situation in which the disturbance of the superconducting order parameter can be localized on the length a_D, making essential the effects of retardation of electron-phonon interaction. The ballistic flight of electrons through the channel is a dynamic process with characteristic frequency $\omega_0 \sim v_F/L$. For L smaller than a_D this frequency is comparable with the Debye frequency ω_D.

In summary, the critical current I_c for the finite contact's size is smaller than I_0. At the same time, the normal-state resistance R_N of the ballistic microchannel does not depend of the length L and remains equal to the Sharvin resistance R_0 (3). As a result, the value of the product $I_c R_N$ is not equal $\pi\Delta_0^2/4eT_c$ and depends on the contact size. We have considered here the quasi-classical case $L >> \hbar/p_F$. In the quantum regime, $L \sim \hbar/p_F$, the Sharvin resistance R_0 in Eq. (2) is substituted by the quantized resistance of the contact, as was first shown by Beenakker and Houten [14]. It follows from our analysis that for such small microcontacts with $L \lesssim a_D$ the rigorous calculation of the Josephson current requires taking into account the retardation effects.

We wish to thank M. R. H. Khajehpour for useful discussions. This work has been supported in part by the Institute for Advanced Studies in Basic Sciences at Zanjan, IRAN.

1. B. D. Josephson, *Weakly Coupled Superconductors, Superconductivity*, R. D. Parks (ed.), Marcel Dekker (1969), Vol. 1.
2. L. G. Aslamazov and A. I. Larkin, *JETP Lett.* **9**, 87 (1969). See also K. K. Likharev, *Rev. Mod. Phys.* **51**, 101 (1979).
3. M. Poza et al., *Phys. Rev.* **B58**, 11173 (1998).
4. C. J. Muller, J. M. van Ruitenbeek, and L. J. de Jongh, *Physica* **C191**, 485 (1992).
5. A. I. Yanson, G. Rubio Bollinger, H. E. van der Brom, N. Agrait, and J. M. van Ruitenbeek, *Nature* **395**, 783 (1998).
6. H. Ohishi, Yu. Kondo, and K. Takayanagi, *Nature* **395**, 780 (1998).
7. I. K. Yanson, *Fiz. Nizk. Temp.* **4**, 141 (1978).
8. I. O. Kulik and A. N. Omelyanchouk, *Sov. J. Low Temp. Phys.* **4**, 142 (1978).
9. Yu. V. Sharvin, *Sov. Phys. JETP* **21**, 655 (1965).
10. G. Eilenberger, *Z. Phys.* **214**, 195 (1968).
11. A. I. Larkin and Yu. N. Ovchinnikov, *Sov. Phys. JETP* **46**,155 (1977).
12. I. O. Kulik and A. N. Omelyanchouk, *Sov. Phys. JETP* **41**, 1071 (1976).
13. G. M. Eliashberg, *Sov. Phys. JETP* **38**, 969 (1960); **39**, 1437 (1960).
14. C. W. J. Beenakker and H. van Houten, *Phys. Rev. Lett.* **66**, 3056 (1991).