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HIGHLY EFFICIENT METHODS FOR REGIONAL WEATHER FORECASTING

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A model and computational method is offered for the high performance forecasting regional meteorological processes. Relying on «unilateral influence» relationship of macro- and mesoscale models it suggests avoiding the Cauchy problem in the atmospheric model and replacing it by a boundary-value problem with specific interpolation technique that has a number of advantages of computational efficiency and good suitability for parallelization. The method and its parallel implementation on multiprocessor cluster architecture are considered.

INTRODUCTION

In recent years a great attention is taken by mesoscale weather events (floods, tornadoes, strong winds and others) as they can cause many deaths and result in huge economic losses [1]. Mitigating the impacts of such events would yield enormous economic and societal benefits, so models and methods of high performance large scale computation leading to regional forecasting regional atmospheric processes are of great importance to provide accommodation the real time, on-demand, and dynamically-adaptive needs of mesoscale weather research.

Regional atmospheric processes are influenced by macroscale atmospheric circulation, so modeling meteorological values in restricted area is to be considered as a task with transitional boundary conditions. To achieve a prescribed level of accuracy of the solutions for a model in places of heavy gradients of related functions it is often necessary to apply a numerical method with variable grid steps for restricted terrains. However the common techniques of mathematical physics [2] cannot often satisfy these requirements because of low accuracy, slow divergence and stability problems, so some dedicated numerical methods are needed to make computation more time- and cost-effective.

Following «unilateral influence» approach to combine macro- and mesoscale models [3,4] in this paper we describe our technique for modeling and forecasting atmospheric processes over a region [5,6] that replaces the Cauchy problem in the atmospheric model by a boundary-value problem and introduces a specific interpolation method that has advantages of computational efficiency and good parallelization. The methodology is well tested and approved in complex regional ecological-meteorological modeling in Ukraine [5, 6, 7].

REGIONAL WEATHER FORECASTING PROBLEM STATEMENT AND A METHOD OF ITS NUMERICAL SOLUTION

For forecasting values of meteorological quantities (components v_1, v_2, v_3 of velocity $\mathbf{V}$, pressure, temperature, specific humidity, specific liquid water content, concentration of pollutants and others) in the atmosphere in a bounded territory $\overline{G}$ we will follow the basics of the method of «unilateral influence» [3], where results of analyses and forecasts received from a macroscale (hemisphere or global) model are used as boundary conditions in a regional model.

Let the state of the atmosphere at spatial point $r=(\lambda, \varphi, \sigma)$ of the macro-space area G be defined by a vector of meteorological quantities $\Re(r,t)$ of discrete values of the analysis and, similarly, forecast $\Re\left(r,t^{m+1}\right)=\Re^{m+1}(r)$ received from a macroscale model at time $t=t^{m+1}$ $(m=0,1,\dots,M)$ with a step $\tau=t^{m+1}-t^{m}$.

Then for determining the atmospheric state in the bounded domain $\overline{G}(r)\subset G(r)$ at $\forall t\in\left[t^{m}, t^{m+1}\right]$ we will solve a task of the following kind in vector representation:

$$\frac{\partial\Re}{\partial t}=D\Re\,,\quad \forall t\in\left[t^{m},\, t^{m+1}\right],\quad \forall r\in\overline{G}\,, \tag{1}$$

$$\Re\left(r,\, t^{m+1}\right)=\Re^{m+1}(r)\,,\quad m=0,\,1,\,\dots,M\,, \text{ where}$$

$$D\Re=\frac{1}{r\cos\varphi}\frac{\partial}{\partial\lambda}\left(\frac{v_1}{r\cos\varphi}\frac{\partial\Re}{\partial\lambda}\right)+\frac{1}{r}\frac{\partial}{\partial\varphi}\left(\frac{v_2}{r}\frac{\partial\Re}{\partial\varphi}\right)+\frac{\partial}{\partial r}\left(v_3\frac{\partial\Re}{\partial r}\right)-$$

$$-\frac{v_1}{r\cos\varphi}\frac{\partial\Re}{\partial\lambda}-\frac{v_2}{r}\frac{\partial\Re}{\partial\varphi}-v_3\frac{\partial\Re}{\partial r}+F$$

is the right-hand side function describing the momentum, heat and mass transmission in spherical coordinates with sink/source term F.

Now replace continuum $\overline{G}$ by a spatial grid of points gained by a spatial grid of points obtained by discretization of the domain $\overline{G}$ with a set of $J-1$ elements $\Delta\lambda_j$, $K-1$ elements $\Delta\varphi_k$ and $L-1$ elements $\Delta\sigma_l$. Let us construct a vector $\left\{r_{jkl}\right\}$, defining the continuous variable r only in points j $(1\le j\le J)$, k $(1\le k\le K)$, l $(1\le l\le L)$. As a result we will have

$$\lambda_J=\lambda_1+\sum_{\mu=2}^{J-1}\Delta\lambda_\mu \quad \varphi_K=\varphi_1+\sum_{\mu=2}^{K-1}\Delta\varphi_\mu \quad \sigma_L=\sigma_1+\sum_{\mu=2}^{L-1}\Delta\sigma_\mu\,.$$

In the domain $\overline{G}$ instead of function $\Re(r,t)$ defined on a macroscale grid, we will construct below a function of discrete argument on a regional grid in the nodes $\left(\lambda_j,\varphi_k,\sigma_l,t^{m}\right)\in R$, $1\le j\le J$, $1\le k\le K$, $1\le l\le L$, $1\le l\le L$. Our aim is to put in correspondence the differential operator D in (1) and the grid operator Λ (see the next section). After filling up function $\Re_{jkl}\left(t^{m+1}\right)=\Re_{jkl}^{m+1}$ in the nodes of the regional grid and computing the right parts $f\left(t^{m+1}\right)=$ $=f^{m+1}=\Lambda\Re^{m+1}$, $m=1,2,\dots,M$, in all nodes of the grid, $\left(\lambda_j,\varphi_k,\sigma_l\right)$, $1\le j\le J$,

$1 \le k \le K$, $1 \le l \le L$, we will search for a solution of the problem (1) for $\forall\, t \in \left[t^m, t^{m+1}\right]$ with the help of a Hermite polynomial like above for number of points $M = 3$:

$$\begin{aligned}
\Re(t) = \Re^m + \frac{t - t^m}{\tau}\Bigg[\tau f^m + \frac{t - t^m}{4\tau}\Bigg[4\left(\Re^{m+1} - 2\Re^m + \Re^{m-1}\right) - \\
- \tau\left(f^{m+1} - f^{m-1}\right) + \frac{t - t^m}{4\tau}\Bigg[5\left(\Re^{m+1} - \Re^{m-1}\right) - \tau\left(f^{m+1} + 8f^m + f^{m-1}\right) - \\
- \frac{t - t^m}{4\tau}\Bigg[2\left(\Re^{m+1} - 2\Re^m + \Re^{m-1}\right) - \tau\left(f^{m+1} - f^{m-1}\right) + \\
+ \frac{t - t^m}{4\tau}\Bigg[3\left(\Re^{m+1} - \Re^{m-1}\right) - \tau\left(f^{m+1} + 4f^m + f^{m-1}\right)\Bigg]\Bigg]\Bigg]\Bigg]\Bigg]
\end{aligned} \qquad (2)$$

for each node of the grid $\left(\lambda_j, \varphi_k, \sigma_l\right)$, $1 \le j \le J$, $1 \le k \le K$, $1 \le l \le L$.

It is easy to check up that the scheme (2) has interpolation properties, i.e. at $t = t^m$ or $\left(\tau = t - t^m = 0\right)$ and $t = t^{m+1}$ or $\left(\tau = t^{m+1} - t = 0\right)$ the equalities $\Re\left(t^m\right) = \Re^m$ and $\Re\left(t^{m+1}\right) = \Re^{m+1}$ hold, respectively. So the maximal error of the solution of problem (1) with the help of (2) is inside the interval $t^m \le t \le t^{m+1}$ and it has an order of approximation $O[(\tau)^4]$.

It was shown in [5] that constructed interpolation formulae involving a function and its derivative $f^{(\alpha)}(\eta_i)$, $i = 1,2,...,N$, $\alpha = 0,1$, have following advantages:

- they have greater accuracy than any of the formulae using only function values $f(\eta_i)$;
- no data are required on the right border of the interpolation interval, and so the formulae can also be used for the rightmost interval;
- the values of function $f(\eta_i)$ and its derivatives $f^{(\alpha)}(\eta_i)$ can be given through unequal intervals.

APPROXIMATIONS OF DIFFERENTIAL OPERATORS

To provide a fourth-order approximation of a differential operator D in (1) by a grid operator Λ we need to guarantee the accuracy of the same order in the interpolation method for smooth filling up of the given discrete function in the nodes of the regional grid. To this aim we propose in this section the following computational scheme.

Designate with η one of the horizontal axes of the system of coordinates $r = (\lambda, \varphi, \sigma)$ and with interval $a \le \eta \le b$ the linear size of the area of the solutions of the macroscale model along this axis. Let any points $a < \eta_1 < \eta_2 < ... < \eta_{N-1} < b$, form a non-uniform macroscale grid $\omega_h[a,b]$

with grid step $h_{i-1} = \eta_i - \eta_{i-1}$. Let us enumerate all nodes in some order $\eta_0, \eta_1, \eta_2, ..., \eta_N$ and consider the values of macroscale function $\Re(\eta_i, t^m)$ in the nodes of a grid as components of a vector $\Re = \{\Re_i(t^m), \ i = 0,1,...,N\}$.

The task of filling up values of a function defined on a macroscale grid in nodes of a regional grid on each interval $[\eta_i, \eta_{i+1}]$ will be performed with the help of a polynomial of the fifth degree:

$$Q_i(\eta) = a_0 + a_1(\eta - \eta_i) + a_2(\eta - \eta_i)^2 +$$

$$+ a_3(\eta - \eta_i)^3 + a_4(\eta - \eta_i)^4 + a_5(\eta - \eta_i)^5, \qquad (3)$$

where

$$a_0 = \Re_i, \ a_1 = \frac{h_{i-1}}{h_i(h_i + h_{i-1})}\left[\Re_{i+1} - \left(1 - \frac{h_i^2}{h_{i-1}^2}\right)\Re_i - \frac{h_i^2}{h_{i-1}^2}\Re_{i-1}\right],$$

$$a_2 = \frac{1}{h_i(h_i + h_{i-1})}\left[\Re_{i+1} - \left(1 + \frac{h_i}{h_{i-1}}\right)\Re_i + \frac{h_i}{h_{i-1}}\Re_{i-1}\right],$$

$$a_3 = -h_i(a_4 + h_i a_5), \ \ a_4 = -\frac{5}{2}h_i a_5,$$

$$a_5 = \frac{2}{h_i^3}\left[a_2 - \frac{1}{h_{i+1}(h_i + h_{i+1})}\left(\Re_{i+2} - \left(1 - \frac{h_{i+1}}{h_i}\right)\Re_{i+1} + \frac{h_{i+1}}{h_i}\Re_i\right)\right].$$

As for vertical changes of the meteorological values near the underlying surface, where they have the heaviest gradients, it is needed to use grids with small steps. On the other hand, to save computer memory and time it is expedient to make use of a rough grid far from the land surface. So, irregular grids are needed for solving mesoscale problems. However macroscale models are usually determined on the standard levels of pressure σ $(z_0, 850, 700, 500, ..., \text{hPa})$ where z_0 stands for sea level. Evidently, there is no unique interpolating formula which provides necessary accuracy of interpolation in the segment $[z_0, 850]$ of the atmospheric boundary layer.

Let us divide the domain height $\sigma = H$ into two pieces: $0 \le \sigma \le h$ and $h \le \sigma \le H$, where h is the 850 hPa pressure level. Values of the meteorological quantities in the nodes of the vertical grid $h \le \sigma \le H$ will be filled in with an interpolation polynomial spline like (3) above, and values on another layer $0 \le \sigma \le h$ will be based on the commonly known theory of the turbulent atmospheric boundary layer [7].

We will adopt the conditions of horizontal homogeneity of the meteorological fields, the absence of heating or chilling effects and other factors except turbulent exchange in the atmosphere. Then a system of equations for the mesoscale processes in the layer $0 \le \sigma \le h$ can be written as follows:

$$\frac{\partial v_1}{\partial t} = \frac{\partial}{\partial z}\left[\nu_T \frac{\partial v_1}{\partial z}\right] + \ell(v_2 - v_{2g}), \qquad (4)$$

$$\frac{\partial v_2}{\partial t} = \frac{\partial}{\partial z}\left[\nu_T \frac{\partial v_2}{\partial z}\right] - \ell\left(v_1 - v_{1g}\right),$$

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left[\frac{\nu_T}{\Pr}\frac{\partial \theta}{\partial z}\right] + S_\theta,$$

$$\frac{\partial q}{\partial t} = \frac{\partial}{\partial z}\left[\frac{\nu_T}{Sc}\frac{\partial q}{\partial z}\right] + S_q,$$

$$\frac{\partial p}{\partial z} = -g\rho,$$

$$\theta = T\left(1 + 0.608q\right),$$

$$\rho = \frac{p}{R\theta},$$

where t is time, playing the role of the iteration parameter; v_1 and v_2 are the components of the wind velocity; v_{1g} and v_{2g} are those at the height $\sigma = h$ (geostrophic wind); S_θ and S_q the sources and outflows of enthalpy and humidity, respectively; ν_T is the turbulent viscosity; $\Pr$ is the Prandtl number; Sc is the Schmidt number, ℓ is a Coriolis parameter. The further designations are commonly known.

We construct a vertical grid of M levels with uneven grid steps estimated as

$$z = 1 - \frac{\ln\{[\beta + 1 - (\sigma/h)]/[\beta - 1 + (\sigma/h)]\}}{\ln[(\beta+1)/(\beta-1)]}, \tag{5}$$

where $1 < \beta < \infty$ should hold and the closer parameter β to 1 the more nodes are collected nearby the level $z = 0$.

The formulated nonlinear problem (4) has a numerical solution on the grid (5) [7].

Equation system (4) concerns all internal points of the whole layer $z_0 < \sigma < H$. Particularly, for the sub-domain $h < \sigma < H$, where the turbulence viscosity coefficient can be considered as constant, system (4) has an analytical solution [7]. Combining the numerical solution on the segment $z_0 < \sigma < h$ with the analytical solution on the other segment $h < \sigma < H$ and imposing respective boundary conditions one can define a divergent iterative process to reproduce the vertical profiles of the meteorological fields based on their known values on the standard levels $\left(z_0, 850, 700, 500, ..., \text{hPa}\right)$.

The offered method of filling up the vertical grid allows us to take into account the heterogeneity of the underlying surface, which can disturb the macro-scale flow.

Now the computation of the grid values of the partial derivatives of the first order $\psi_i = \left(\partial\Re/\partial\eta\right)_i$ and of the second order $\zeta_i = \left(\partial^2\Re / \partial\eta^2\right)_i$ included in $f_{jkl}^m = \Lambda\Re_{jkl}^m$, will be performed on the basis of the following relations:

$$\psi_{i+1} + 2\left(1 + \frac{h_i}{h_{i-1}}\right)\psi_i + \frac{h_i}{h_{i-1}}\psi_{i-1} =$$

$$= \frac{3}{h_i}\left\{\Re_{i+1} - \left[1 - \left(\frac{h_i}{h_{i-1}}\right)^2\right]\Re_i - \left(\frac{h_i}{h_{i-1}}\right)^2 \Re_{i-1}\right\} - \frac{h_i h_{i-1}^2}{24}\left[1 - \left(\frac{h_i}{h_{i-1}}\right)^2\right]\frac{\partial^4 \Re}{\partial \eta^4}, \quad (6)$$

$$\frac{h_{i-1}}{h_i}\left[\frac{h_{i-1}}{h_i}\left(1 - \frac{h_{i-1}}{h_i}\right) + 1\right]\xi_{i+1} + \left(1 + \frac{h_{i-1}}{h_i}\right)\left[\frac{h_{i-1}}{h_i}\left(3 + \frac{h_{i-1}}{h_i}\right) + 1\right]\xi_i +$$

$$+ \left[\frac{h_{i-1}}{h_i}\left(1 + \frac{h_{i-1}}{h_i}\right) - 1\right]\xi_{i-1} = \frac{12}{h_i^2}\left[\frac{h_{i-1}}{h_i}\Re_{i+1} - \left(1 + \frac{h_{i-1}}{h_i}\right)\Re_i + \Re_{i-1}\right] +$$

$$+ \frac{h_i^2 h_{i-1}}{360}\left[1 - \left(\frac{h_{i-1}}{h_i}\right)^2\right]\left\{5\frac{h_{i-1}}{h_i} + 2\left[1 - \left(\frac{h_{i-1}}{h_i}\right)^2\right]\right\}\frac{\partial^5 \Re}{\partial \eta^5}. \quad (7)$$

It is obvious that the relations (6), (7) have the third order at $h_i \neq h_{i-1}$ and the fourth order at $h_i = h_{i-1}$. Derivatives $\psi_i = (\partial \Re / \partial \eta)_i$ and $\xi_i = \left(\partial^2 \Re / \partial \eta^2\right)_i$ belong to (4), (5) implicitly. But these are systems of algebraic equations with tridiagonal matrices, so solutions can be found effectively with the help of the sweep method [8] with the boundary conditions

$$-\frac{h_1}{6}(\xi_2 - \xi_1) + \psi_1 + \psi_2 = 2\frac{\Re_2 - \Re_1}{h_1} + O\left[h_1^4\right], \quad (8)$$

$$-\frac{h_{N-1}}{6}(\xi_N - \xi_{N-1}) + \psi_{N-1} + \psi_N = 2\frac{\Re_N - \Re_{N-1}}{h_{N-1}} + O\left[h_{N-1}^4\right]. \quad (9)$$

The main advantage of the offered method for the approximation of derivatives is that the solution of the system of algebraic equations (6)–(7) at all points depends on values at other points, i.e., it depends on $\Re_i$ globally, which means smooth filling up and approximation of the differential operators by the grid operators.

SOFTWARE IMPLEMENTATION AND EXPERIMENTS

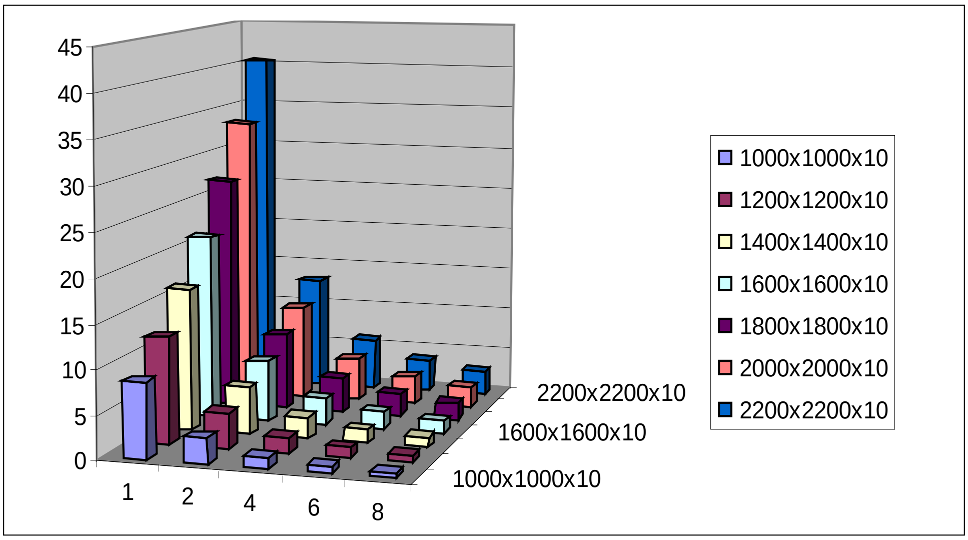
A software package of the method considered above was implemented and experienced in short- and intermediate-term regional meteorological forecasting for the territory of Ukraine and nearby areas. The meteorological functionality of the package includes following options:

- setting up an area of the initial data and weather forecast;
- downloading and decoding initial meteorological data;
- adaptation of the initial meteorological data;
- weather forecast on required term;
- visualization of results of the forecast.

The size of area and the parameters of a grid depend on parameters of model of numerical weather forecast of hydrometeorological service DWD Offenbach (Germany) from which the initial meteorological data are accepted.

Initial data are downloaded via Internet channels from Offenbach in GRIB binary representation and then decoding is performed. This task is launched twice per day after 5.00 and 17.00 at local time. The program of weather forecast on required term (from 1 to 5 days with a step of 1 hour) can be started on demand any times. The execution begins with the analysis of presence of files with the initial data, and process is visually supervised.

Experiments on parallel implementation of the package were carried out on cluster multiprocessor (2.6 MHz, 512Mb of main memory for each Intel Xeon processor, Dolphin SCI interconnection) and exposed good suitability of the task to parallelization. Below a diagram is depicted (see Figure) concerning interpolation of data received from macroscale grid into points of mesoscale grid. The diagram shows computation time (in sec, axis Z) on various numbers of processors (from 1 up to 8, axis Y) at the various sizes mesoscale grid (axis X).



Results of interpolation macroscale data into mesoscale grid

CONCLUSION

We have presented a new computational method for the efficient solution of the complex problem of forecasting regional meteorological processes. Our method follows the approach of “unilateral influence” to combine macro- and mesoscale models [3,4]. It gives opportunity to replace the Cauchy problem in the atmospheric model (1) by a boundary-value problem and introduces a specific interpolation method (2) that has a number of advantages:

- the time step in getting macroscale information for regional forecasting can be significantly increased and reach $\tau = 12$ hours [6];
- as opposed to classical numerical methods for solving the equations of mathematical physics, the offered method is deprived of instability problems;

- the accuracy of the offered method has fourth order and is determined by the same order of accuracy of the following constituent methods: smooth filling up of macroscale values into mesoscale grid nodes (3), (4), approximating differential operators by grid ones (6)–(9) and interpolation method (2) for solving the boundary-valued problem based on the approach of «unilateral influence».

The model and method have been implemented in a software package and tested by the Hydrometeorological Center of Ukraine. The comparison with actual wheather cards has shown that the numerical forecasts qualitatively and quantitatively well coordinate with real observed data. The model and method have been successfully applied in regional short- and middle-term weather forecasting for districts of Ukraine.

Results of experiments in parallel implementation of the computational scheme for solving problems in regional meteorological forecasting in Ukraine show its good computational efficiency, scalability and applicability for parallel computation.

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REFERENCES

1. *Pielke R.A. and Carbone R.* Weather impacts, forecasts and policy // Bull. Amer. Meteor. Soc. — 2002. — **83**. — P. 393–403.
2. *Tikhonov A.N., Samarskiy A.A.* The equations of mathematical physics. — Moscow: Science, 1977. — 735 p. (in Russian).
3. *Miyakoda K., Rosati A.* One-way nested grid models: The interface condition and the numerical accuracy // Mon. Weather Review. — 1977. — **105**. — P. 1092–1107.
4. *Phillips N., Shukla J.* On the strategy of combining coarse and fine grid meshes in numerical weather prediction // J. Appl. Meteor. — 1973. — **12**. — P. 763–770.
5. *Prusov V., Doroshenko A.* Modeling and Forecasting Atmospheric Pollution over Region // Annales Univ. Sci. Budapest. — 2003. — **46**. — P. 47–64.
6. *Methods* for efficient solution of problems in modeling and predicting regional atmospheric processes / V. Prusov, A. Doroshenko, S. Prikhodko at al. // Problems in Programming. — 2004. — № 3–4. — P. 556–569 (in Russian).
7. *Dovgyi S.A., Prusov V.A., Kopeika O.B.* Mathematical modeling of man-caused environmental pollution. — Kiev: Nauk. Dumka. — 2000. — 247 p. (in Russian).
8. *Godunov S.K., Riabenkiy V.C.* Difference schemes (introduction to the theory). — Moscow: Science. — 1973. — 400 p. (in Russian).

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