

Conversion of dislocation oscillation waves to spin ones near the OPT temperatures

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Dislocation waves in magnetic crystals near the orientation phase transition temperatures are considered in the frame of the field defect theory. The singularities of the dislocation flows, elastic strains and magnetization occur if the magnetic system is inhomogeneous and the spatial dispersion of the medium is not taken into account. The account for the medium dispersion provides regularity of these parameters and the conversion of the spin wave to the dislocation one.

В рамках полевой теории дефектов рассмотрено распространение дислокационной волны в магнитоупорядоченном кристалле в окрестности температур ориентационных фазовых переходов. Показано наличие сингулярности в выражениях, описывающих колебания компонент тензора дислокационных потоков, упругих напряжений и вектора намагниченности, если магнитная подсистема пространственно неоднородна и не учитывается пространственная дисперсия среды. Учет пространственной дисперсии колебаний намагниченности устраняет эту особенность и приводит к трансформации спиновой волны в дислокационную.

Real crystals contain defects of different nature which influence its wave properties. The dislocation influence on phonon spectra in real crystals is especially notorious. The magnetic structure of ferromagnetics is sensitive to different influences near the temperatures of orientation phase transitions (OPT). The low stability of the magnetic state near the OPT temperature relieves the realization of different non-linear effects Therefore, the interaction between dislocations and magnetic structure of magnetics will be revealed at these temperatures.

Phenomenological description of dislocation ensembles is based on the statistical approach. The statistical effects are not a simple sum of the properties of a number of dislocations. The statistical properties of the dislocation ensembles reveal the wave characteristics thereof. One of these wave effects is the screening of elastic strains, and correct description requires taking into account the dislocation cores.

As the elastic fields of individual dislocations are screened, it is just the behavior of the dislocation core ensemble that becomes the critical factor. When an external action is applied to the crystal, the ensembles of dislocation defects move in the direction of that action, the crystallographic gliding of the individual dislocations within the ensembles causes a non-crystallographic displacement of the ensemble along the action direction [1].

The above model of the dislocation ensemble is described quantitatively by equations similar to the Maxwell ones in the electrodynamics [1, 2]

$$\nabla \times \hat{j}(\mathbf{r},t) = \frac{\partial \hat{\alpha}(\mathbf{r},t)}{\partial t}, \qquad (1)$$

$$S\nabla \times \hat{\alpha}(\mathbf{r},t) = -B\frac{\partial \hat{j}(\mathbf{r},t)}{\partial t} - \hat{\sigma}(\mathbf{r},t). \qquad (2)$$

Here $\hat{j}$ is the defect flow density tensor; $\hat{\alpha}$ is the defect density tensor; $\hat{\sigma}$, the elastic stress tensor; S, a constant having the sense of linear potential energy of the defect; B, a constant characterizing the defect inertial properties. The sign $\times$ denotes a vector product.

Let the system be supplemented by the equation of a continuous medium motion in the form

$$\rho\frac{\partial^2\hat{\sigma}(\mathbf{r},t)}{\partial t^2} - \int\hat{C}(\mathbf{r},t;\mathbf{r_1},t_1) \times \left(\nabla \otimes \nabla\hat{\sigma}(\mathbf{r_1},t_1) + \rho\frac{\partial\hat{j}(\mathbf{r_1},t_1)}{\partial t_1}\right)d\mathbf{r_1}dt_1 = 0, \quad (3)$$

where ρ is the material density; $C(\mathbf{r},t,\mathbf{r_1},t_1)$, the elastic modulus tensor.

In the Eq.(3), let the interaction of the dislocation vibrations with the magnetization ones be taken into account by setting the tensor to be re-normalized at the expense of the magneto-elastic interaction [3]. In materials with strong magnetostriction, the intense interaction between the vibrational waves of dislocations and magnetic moments may cause effects occurring during the propagation of electromagnetic vibrations [4–5]. Note that the re-normalizing additive is in proportion to the dynamic magnetic permittivity tensor.

Eq.(3) is, generally speaking, an integral-differential one. The inhomogeneity of the material where the dislocation wave is propagated is due to that of the magnetic subsystem. This is easy to realize near the OPT using either a temperature gradient or an external magnetic field. The characteristic inhomogeneity size along the X axis are set to exceed considerably the wavelengths in the wave processes under consideration. Then the inhomogeneity will be manifested only in the dependence of the ferromagnetic resonance on the x coordinate. According to Eq.(3), we obtain

$$\hat{\sigma} = -\int \hat{G}(\mathbf{r},t;\mathbf{r_1},t_1)\hat{C}(\mathbf{r_1},t_1;\mathbf{r_2},t_2) \times \frac{\partial\hat{j}(\mathbf{r_2},t_2)}{\partial t_2}d\mathbf{r_1}dt_1d\mathbf{r_2}dt_2. \quad (4)$$

Here, $\hat{G}(\mathbf{r},t,\mathbf{r_1},t_1)$ is the 4^{th} rank tensor Green function of the Eq.(3) where the tensor $\hat{j}(\mathbf{r},t)$ is set to be equal to zero.

To simplify the model, let us consider a plane vibration wave of the screw component $\alpha_{yy} = \alpha$, propagating in the ZX plane (Y axis being the high symmetry one). Then the system (1), (2) takes the form

$$\frac{\partial j_{xy}}{\partial z} - \frac{\partial j_{zy}}{\partial x} = \frac{\partial \alpha}{\partial t}, \quad (5)$$

$$S\frac{\partial\alpha}{\partial z} = B\frac{\partial j_{xy}}{\partial t} + \sigma_{zy}, \quad (6)$$

$$S\frac{\partial\alpha}{\partial x} = -B\frac{\partial j_{xy}}{\partial t} - \sigma_{xy}. \quad (7)$$

In the absence of the magneto-static interaction, the dispersion law for a harmonic dislocation wave has the form

$$\omega^2 = [\rho C_0 + (BC_0 + \rho S)k^2 \pm \sqrt{k^4(\rho S - BC_0)^2 + 2k^2\rho C_0(\rho S + BC_0) + \rho^2 C_0^2}\,]/2B\rho \quad (8)$$

The minus sign at the radical in Eq.(8) corresponds to the acoustic wave while the plus, to the dislocation one. Thus, the dislocation wave spectrum has a gap ω_0,

$$\omega_0 = \sqrt{C_0/B}. \quad (9)$$

Here, $C_0 \equiv C^0{}_{xyxy}$ is the elastic module. Experimental measurements of the gap width and the curvature of the curve (8) at the point $k = 0$ make it possible to determine constants S and B.

Let us consider again the interaction of a time-harmonic dislocation wave with the magnetic subsystem. First, let the medium spatial dispersion be neglected for long dislocation waves. Then it follows from Eq.(4) taking into account that $G_{xyxy} = G_{zyzy} = 1/\omega^2\rho$:

$$\sigma_{xy} = -i\left(\frac{C_0}{\omega} - \frac{4\lambda^2\Omega g}{M(\Omega^2 - \omega^2)\omega}\right)j_{xy}, \quad (10)$$

$$\sigma_{zy} = -i\left(\frac{C_0}{\omega} - \frac{4\lambda^2\Omega g}{M(\Omega^2 - \omega^2)\omega}\right)j_{zy}. \quad (11)$$

Here, ω is the dislocation wave cyclic frequency; $\lambda = \lambda_{xyxy} = \lambda_{zyzy}$, the magneto-elastic constant having the dimensionality of the energy density; M, magnetization of a sample with the easy magnetization axis along the Y one; Ω, the ferromagnetic resonance frequency depending on the x coordinate; g, the gyro-magnet ratio. Substituting (10) and (11) into (6) and (7), we obtain

$$S\frac{\partial\alpha}{\partial z} = i\varepsilon(x)j_{xy}, \quad (12)$$

$$-S\frac{\partial\alpha}{\partial x} = i\varepsilon(x)j_{zy}, \quad (13)$$

where

$$\varepsilon(x) = \omega B - \frac{C_0}{\omega} + \frac{4\lambda^2\Omega g}{M(\Omega^2 - \omega^2)\omega}. \qquad (14)$$

Eliminating the j_{xy} and j_{zy} from Eq.(5) using (12) and (13), we get

$$\frac{S}{\varepsilon(x)}\frac{\partial^2\alpha}{\partial z^2} + S\frac{\partial}{\partial x}\left(\frac{1}{\varepsilon(x)}\frac{\partial\alpha}{\partial x}\right) + \omega\alpha = 0. \qquad (15)$$

Since Ω in (14) depends on x, it is possible to find a point in the magnetics at the specified frequency ω where $\varepsilon(x)$ is zeroed under the sign change. In other words, the frequency Ω attains in this point the value Ω_0 defined as

$$\Omega = \Omega_0 = -\frac{2\lambda^2 g\omega_0}{(\omega^2 - \omega_0^2)C_0M} + \left(\frac{4\lambda^4g^2\omega_0^2}{(\omega^2 - \omega_0^2)^2C_0^2M^2} + \omega^2\right)^{1/2}. \qquad (16)$$

This can be realized at $\lambda \sim 10^8$ erg/cm^3, $\Omega \sim 10^9$ s^{-1} and $\omega_0 \sim \omega \sim \omega - \omega_0 \sim \Omega$.

Let the $\varepsilon(x)$ be zeroed in the point $x = 0$. Then in the vicinity of that point,

$$\varepsilon(x) = -\omega bBx, \quad b > 0. \qquad (17)$$

If, for example, the medium inhomogeneity, that is, the ferromagnetic resonance frequency dependence on x, is due to a temperature gradient, then

$$b = -\frac{4\lambda^2g(\Omega_0^2 + \omega^2)}{M\omega^2B(\Omega_0^2 - \omega^2)^2}\cdot\frac{d\Omega_0}{dT}\cdot\frac{dT}{dx}. \qquad (18)$$

As a rule, $(d\Omega_0/dx) < 0$; thus, to provide the positive b, it is nesessary to put $(dT/dx) > 0$.

Substituting into Eq.(15) the solution

$$\alpha = \alpha_0(x)e^{i(\omega t - kz)}, \qquad (19)$$

and taking (17) into account, we obtain

$$\frac{\partial}{\partial x}\left(\frac{1}{x}\frac{\partial\alpha_0(x)}{\partial x}\right) - \left(\frac{\kappa^2}{x} + \frac{\omega^2bB}{S}\right)\alpha_0(x) = 0. \qquad (20)$$

It is rather difficult to solve the Eq.(20) exactly [7]. For our purposes, let only the part of the solution be written that is connected with the singularity of interest [8].

$$\alpha_0(x) = \alpha_0(1 + \frac{1}{2}\kappa^2x^2\ln\kappa x). \qquad (21)$$

Then it follows from (12), (13) и (21) that

$$j_{xy} = \frac{\kappa\alpha_0S}{Bb\omega x}e^{i(\omega t - kz)}, \qquad (22)$$

$$j_{zy} = i\frac{\kappa^2\alpha_0S\ln\kappa x}{Bb\omega}e^{i(\omega t - kz)}. \qquad (23)$$

Thus, a singularity occurs in components of the dislocation flow tensor. According to (9) and (10), the components of the elastic stress tensor are singular, too. This singularity can be eliminated, for example, due to damping in the magnetization vibration spectrum. The account for damping at the re-normalization of the elastic modules can be realized by substitution the expression $\Omega - i\delta\omega/gM$ for Ω in (10) and (11), where δ is a small damping in the magnetic subsystem [9]. In this case, the function $\varepsilon(x)$ according to (17) is replaced by

$$\varepsilon(x) = -b\omega x + i\omega\nu, \qquad (24)$$

where ν is expressed as

$$\nu = \frac{4\lambda^2\delta(\omega^2 + \Omega_0^2)}{\omega B[(\Omega_0^2 - \omega^2)^2 + \frac{4\omega^2\Omega_0^2\delta^2}{g^2M^2}]^2M^2}. \qquad (25)$$

Let be calculated, for example, the averaged magneto-elastic energy being absorbed near the point $x = 0$ using the formula

$$Q = \frac{i\omega}{2}\int\overline{\left\{\mathrm{Re}\sigma_{xy}\mathrm{Re}\varepsilon_{xy}\right\}}dx. \qquad (26)$$

Here, Re denotes the real part and the upper line, the averaging over the vibration period; ε_{xy} is the component of the elastic strain tensor.

Substituting the σ_{xy} and ε_{xy} values from (10), we obtain:

$$Q = -\frac{S^2\kappa^2\alpha_0^2}{2B^2\omega}\int\frac{\nu dx}{b^2x^2 + \nu^2}. \qquad (27)$$

For small ν values, the integrand in Eq.(27) is the δ function

$$\frac{\nu}{b^2x^2 + \nu^2} = \delta(bx). \qquad (28)$$

Thus, the vibration energy absorption has a resonance character within a narrow layer near the point $x = 0$ where the dislocation wave frequency coincides with the gap in the dislocation vibration spectrum re-normalized due to the magnetostrictive interaction. We obtain at the end:

$$Q = \frac{S^2\kappa^2\alpha_0^2}{2\omega b B^2}. \qquad (29)$$

That is, the absorption rate of the dislocation vibration energy is independent of ν, since the effective width of the area where a substantial absorption occurs is in proportion to ν.

Since the presence of singularities in the dislocation flow dependence on x is due to the interaction of the dislocation vibrations with the magnetization ones, the dislocation waves can build effectively the magnetic moment vibrations. The account for the spatial dispersion in the magnetic subsystem can result in the dislocation wave transformation into the spin ones. In fact, in the absence of dispersion, the effective magnet field caused by elastic strains has the strength

$$H_x = -\frac{\lambda\varepsilon_{xy}}{M} = \frac{2i\lambda\omega\rho G(\omega) j_{xy}}{M}, \qquad (30)$$

and the magnetization vector components are expressed as

$$M_x = \chi_{xx} H_x = \frac{i\lambda\kappa S\alpha_0 g\Omega_0}{(\Omega_0^2 - \omega^2)\omega^2 bx} e^{i(\omega t - kz)}, \qquad (31)$$

$$M_z = \chi_{zx} H_x = -\frac{\lambda\kappa S\alpha_0 g}{(\Omega_0^2 - \omega^2)\omega bx} e^{i(\omega t - kz)}. \qquad (32)$$

Here, χ_{xx} and χ_{zx} are the components of magnetic susceptibility tensor. It is seen that in the point $x = 0$, the magnetization shows also a singularity. To take into account the spin wave dispersion, let the wave number in (12) and (13) be substituted by the

$$k^2 \to -\frac{\partial^2}{\partial x^2}, \qquad (33)$$

where k is the wave vector projection on the X axis. Then the differential equations for the spin waves take the form

$$\beta\frac{d^2 M_x}{dx^2} - bxM_x = \frac{i\lambda\kappa S\alpha_0 g\Omega_0}{B(\Omega_0^2 - \omega^2)\omega^2} e^{i(\omega t - kz)}, \qquad (34)$$

$$\beta\frac{d^2 M_z}{dx^2} - bxM_z = -\frac{\lambda\kappa S\alpha_0 g}{B(\Omega_0^2 - \omega^2)\omega} e^{i(\omega t - kz)}. \qquad (35)$$

In these equations, α is taken in the point $x = 0$,

$$\beta = \frac{2\gamma\Omega_0 g M C_0(\omega^2 - \omega_0^2)}{(\omega^2 - \Omega_0^2)\omega^2\omega_0^2 B} > 0, \qquad (36)$$

and γ is the inhomogeneous exchange parameter.

The solution of Eqs.(34) and (35) describes a spin wave to which the dislocation one propagating to left from the point $x = 0$ and being described as

$$M_x = -\frac{\lambda\alpha_0\kappa S g\Omega_0}{B(\Omega_0^2 - \omega^2)\omega^2 b^{2/3}\beta^{1/3}} \times \qquad (37)$$
$$\times\, e^{i(\omega t - kz)}\int_0^\infty d\nu \exp\left\{i\left(\nu x(b/\beta)^{1/3} + \frac{\nu^3}{3}\right)\right\},$$

$$M_z = -\frac{i\lambda\alpha_0\kappa S g}{B(\Omega_0^2 - \omega^2)\omega b^{2/3}\beta^{1/3}} \times \qquad (38)$$
$$\times\, e^{i(\omega t - kz)}\int_0^\infty d\nu \exp\left\{i\left(\nu x(b/\beta)^{1/3} + \frac{\nu^3}{3}\right)\right\}.$$

At $|x| >> \lambda$ and $x < 0$, M_x and M_y can be estimated using the crossing method:

$$M_x = -i\frac{\Omega_0}{\omega}M_z =$$
$$= -\frac{\lambda\alpha_0\kappa S g\Omega_0\sqrt{\pi}}{B(\omega^2 - \Omega_0^2)\omega b^{3/4}\beta^4 x^{1/4}} \times \qquad (39)$$
$$\times \exp\left\{i\left(\omega t - kz) - \frac{2}{3}\sqrt{\frac{b|x|^3}{\beta}} - \frac{\pi}{4}\right)\right\}.$$

As the obtained dislocation flows and magnetization values are in proportion to κ (the wave vector component on the Z axis, the effect under consideration occurs if the dislocation wave propagates at an angle to the X axis. Let the propagation angle θ to the X axis be estimated for a dislocation wave to the left from the point $x = 0$ at which the dislocation wave transformation into the spin one is maximum. In the geometric optics approximation, the dislocation wavelength is much smaller than the inhomogeneity characteristic size

$$\frac{\left\{(\omega^2 - \omega_0^2)B/S\right\}^{1/2}}{b} >> 1\ , \qquad (40)$$

and the vibration amplitude α decreases exponentially to right from the reflection point x_0 defined by the relation

$$\kappa^2 S + \omega^2 bBx_0 = 0. \qquad (41)$$

The point $x = 0$ is to the right of x_0. The singularity is observable if the point x_0 is near $x = 0$, that is,

$$|kx_0| \sim 1, \qquad (42)$$

where k is the x component of the wave vector.

Far from the reflection point the magneto-static re-normalization is negligible, so the dispersion equation has the form

$$k^2 + \kappa^2 = \frac{B}{S}(\omega^2 - \omega_0^2). \qquad (43)$$

Thus,

$$\kappa^2 = \frac{B}{S}(\omega^2 - \omega_0^2)\sin^2\theta, \qquad (44)$$

Near the point $x = 0$,

$$k^2 + \kappa^2 = -\frac{\omega^2 Bb}{S}x, \qquad (45)$$

and thus,

$$|k| \sim \kappa = \left(\frac{B(\omega^2 - \omega_0^2)}{S}\sin\theta\right)^{1/2}. \qquad (46)$$

Substituting the Eq.(46) into the expression for x_0 following from (41), and then x_0 into (42) and taking into account the inequality (40), we obtain

$$\sin^3\theta \sim \frac{\omega^2 B\sqrt{S}}{(\omega^2 - \omega_0^2)^{3/2}\sqrt{B}} << 1.$$

Thus, the angle θ should be small but nonzero.

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Конверсія спінових хвиль у дислокаційні коливні хвилі поблизу температури ОФП

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Розповсюдження дислокаційної хвилі у магнітовпорядкованому кристалі поблизу температур орієнтаційних фазових переходів розглянуто у рамках польової теорії дефектів. Показано сингулярності у виразах, що описують коливання компонентів тензора дислокаційних потоків, пружних напруг та вектора намагніченості у випадку, коли магнітна підсистема є просторово неоднорідною і просторова дисперсія середовища не береться до уваги. Прийняття до уваги просторової дисперсії коливань намагніченості знімає цю особливість та призводить до трансформації спінової хвилі у дислокаційну.