Functional Materials 15, No.1 (2008) © 2008 — Institute for Single Crystals

Characteristics of electromagnetic and magneto-static surface waves in metal-dielectric-ferrite-left handed waveguide layered structure

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Received October 6, 2007

Some works have been carried out on studying metamaterials with negative electrical permittivity and negative magnetic permeability which called Left-handed materials (LHM), the propagation dispersion characteristics of magnetic waves in waveguide structures containing LHM. In this communication, we investigate the dispersion characteristics of magnetic surface waves in a metal-dielectric-ferrite (YIG)-LHM layered waveguide structure. Several characteristics of the propagation are also obtained especially on the non-reciprocal behavior. It has been noticed that the non-reciprocal behavior of the wave propagation has been obtained in the forward direction for the electromagnetic waves whereas this reciprocal behavior has been found in the forward and backward directions for the magneto-static surface waves. It should be stressed that the addition of the LHM layer to the above waveguide structure has been noticed to dominate the control of the direction of the dispersion curves from forward to backward wave propagation.

Рассматрены материалы с отрицательной диэлектрической проницаемостью и отрицательной магнитной проницаемостью, так называемые "материалы левой руки" (left-handed materials, LHM), исследованы характеристики дисперсии поверхностных магнитных волн в слоистой волноводной системе типа металл-диэлектрик-феррит (ЖИГ)-LHM. Получены некоторые характеристики, касающиеся, в частности, неэквивалентного поведения распространяющихся волн. Отмечен неэквивалентный характер распространения электромагнитных волн в прямом направлении, в то время как для магнитостатических поверхностных волн такой характер обнаружен при распространении как в прямом, так и в обратном направлениях. Следует отметить, что введение слоя LHM в вышеупомянутую волноводную систему играет доминирующую роль в регулировании направления дисперсионных кривых при переходе от прямого к обратному распространению волн.

During the decades of 1950's and 1960's, there was great interest in producing artificial dielectrics with arbitrary values of permittivity by inclusion of metal particles or wires in the natural (host) dielectric medium [1]. There are several advantageous applications of the artificial dielectrics such as microwave/millimeter wave leaky wave antennas [2], where the effective dielectric constants lie between zero and unity. The characteristics of the leaky waves are unique in comparison with the natural leaky wave antennas.

Late the 1990's, attempts were carried out by Pendry et al. [3] to obtain negative permittivity by using metallic wires in the GHz band. Pendry et al. also expected that an effective negative permeability could be achieved in the GHz band by a periodic arrangement of split ring resonators (SRRs) [4]. During the year 2000, depending on Pendry suggestions and ideas on artificially controllable permittivity and permeability of media, Smith et al. [5] have used metal wires and SRRs to attain simultaneously negative permittivity and permeability in the GHz frequency range. This important and interesting progress also depended on the theoretical prediction back in the 1968, by the Russian physicist, V.G. Veselago, who predicted several extraordinary electromagnetic phenomena for materials having simultaneously negative permittivity and permeability [6-8]. These phenomena are a sign change of group velocity, reversals of the Doppler and the Vavilov-Cerenkov effects, negative refraction, perfect lensing and a reversal of radiation pressure to radiation tension. Because these exotic materials were thought not to exist in nature at all, Veselago's ideas were forgotten until the experimental verification was made by Smith et al. which encouraged the present explosive research worldwide. These researches are in fundamental electromagnetic phenomena and practical applications of these interesting materials in various frequency ranges from radio to optical frequencies.

These materials are called metamaterials. "Meta–" is a Greek prefix that means "beyond," Metamaterials refer to artificially designed structures that exhibit unusual electromagnetic properties and responses. Metamaterials are also known as left-handed materials (LHMs), because the directions of the electric field (E), the magnetic field (H), and the wave propagation vector (k) follow the left-hand rule instead of the right-hand rule in ordinary dielectric materials. Due to the left-handedness in such materials, the reversal of the phase and the energy propagations lead to negative phase velocity or negative group velocity. This is attributed, in other words, to backward wave propagations with opposite directions of the phase and the group velocities. LHMs are also referred to as materials with negative refractive index or negative index media, because when electromagnetic waves are incident thereon, the refraction is reversed. Finally, these materials are also known as double-negative media because at a given frequency, these media have simultaneously negative permittivity and permeability.

At the interface between the LHMs and conventional dielectric media [9], surface waves differ in their propagation behavior from that at the interface between two dissimilar conventional dielectric media. For example, slab waveguides with LHMs also have unusual guided dispersion characteristics [10-17]. In this article, we have studied the dispersion characteristics of magnetic surface waves in a metal-dielectric-ferrite (YIG)-LHM layered waveguide structure. A similar investigation had been conducted before [18] but without included LHM layer. Our study includes theoretical analysis based on Maxwell equations in order to investigate the electromagnetic and magneto-static surface waves.

Let an infinite plane-parallel ferrite slab be considered, having magnetization M_0 and of thickness d_2. The slab is bounded by a left-handed material cover and a nonmagnetic dielectric layer of thickness d_1. The dielectric layer is placed on a metal substrate, as shown in Fig. 1. The ferrite slab has infinite extent in the $\hat{y}$ and $\hat{z}$ directions. A static magnetic field is applied in the $\hat{z}$ direction, resulting in an uniform field intensity H_i within the ferrite.

Electromagnetic propagation phenomena are described by Maxwell equations, which are given by:

$$\vec{\nabla} \times \vec{E} = -i\omega\mu_0\mu\vec{H} \tag{1}$$

$$\vec{\nabla} \times \vec{H} = i\omega\varepsilon_0\varepsilon\vec{E} \tag{2}$$

It is assumed that the amplitude of the system response to electromagnetic excitations is sufficiently small and of the form exp(iωt) such that B and H are related by the permeability tensor (19):

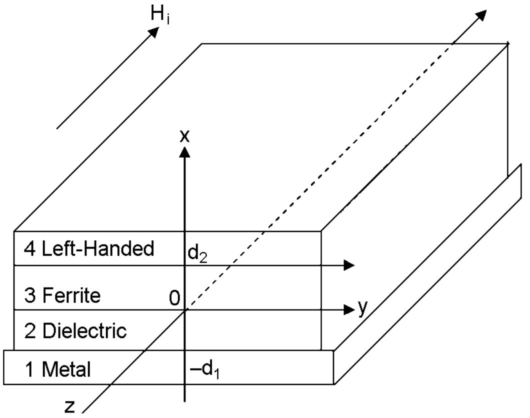

Fig. 1: Electromagnetic surface wave guide geometry containing: (1) left handed material, (2) ferrite slab, (3) dielectric medium, and (4) metal substrate.

$$\|\mu\| = \begin{Vmatrix} \mu_{11} & i\mu_{12} & 0 \\ -i\mu_{12} & \mu_{11} & 0 \\ 0 & 0 & \mu_{33} \end{Vmatrix}, \tag{3}$$

where (neglecting exchange and losses)

$$\mu_{11} = 1 + \frac{\omega_i \omega_m}{\omega_i^2 - \omega^2}$$

$$\mu_{12} = \frac{\omega \omega_m}{\omega_i^2 - \omega^2}$$

$$\omega_m = \mu_0 \gamma M_0$$

$$\omega_i = \mu_0 \gamma H_i$$

ω is operating angular frequency where $\omega = 2\pi f$; γ is the gyromagnetic ratio and f is electromagnetic oscillation frequency.

The form of the response must be chosen to satisfy Maxwell equations throughout the media and the boundary conditions at their interfaces while vanishing at infinity. If it is assumed that the system response propagates in the $\hat{y}$ direction, and that $\partial/\partial_z = 0$.

For the Left-Handed Material Layer: From the above Maxwell's equations (1, 2), we get:

$$\frac{\partial^2 E_z^{(1)}}{\partial x^2} - (k^2 - \frac{\omega^2}{c^2}\mu_1 \varepsilon_1) E_z^{(1)} = 0, \tag{4}$$

where μ_1, ε_1 the permeability and permittivity of the left-handed material; k is the propagation constant, and ω is operating angular frequency.

The solution of Eq. (4) is:

$$E_z^{(1)} = A e^{-\gamma_1 x} e^{i(\omega t - k y)} \tag{5}$$

$$H_x^{(1)} = \frac{k}{\omega \mu_0 \mu_1} A e^{-\gamma_1 x} e^{i(\omega t - k y)} \tag{6}$$

$$H_y^{(1)} = -\frac{\gamma_1}{i \omega \mu_0 \mu_1} A e^{-\gamma_1 x} e^{i(\omega t - k y)}, \tag{7}$$

where $\gamma_1^2 = k^2 - \frac{\omega^2}{c^2}\mu_1 \varepsilon_1$, where $\varepsilon_1 = 1 - \dfrac{\omega_p^2}{\omega^2}$, $\mu_1 = 1 - \dfrac{F\omega^2}{\omega^2 - \omega_r^2}$

ω_p, ω_r and F is chosen to fit approximately to the experimental data [17]: $\omega_p/2\pi = 10GHz$, $\omega_r/2\pi = 4GHz$, and F = 0.56. For this set of parameters, the region in which permeability and permittivity are simultaneously negative is between 4 and 6 GHz.

For the Ferrite Layer: The above Maxwell equations lead to the following differential equation which has the form:

$$\frac{\partial^2 E_z^{(2)}}{\partial x^2} - \gamma_2^2 E_z^{(2)} = 0, \tag{8}$$

where $\gamma_2^2 = k^2 - \dfrac{\omega^2}{c^2}\mu_v \varepsilon_f$

$$\mu_v = \frac{\mu_{11}^2 - \mu_{12}^2}{\mu_{11}}$$

and ε_f is the permittivity of the ferrite slab.

The solution of the equation (8) can be obtained as:

$$E_z^{(2)} = [B \cosh \gamma_2 x + C \sinh \gamma_2 x]\, e^{i(\omega t - k y)} \qquad 9)$$

and

$$H_y = -\frac{1}{i\omega\mu_0 \mu_u \mu_{11}} [B(\mu_{12}\ k \cosh\gamma_2\ x - \mu_{11}\gamma_2 \sinh\gamma_2 x) + \\ + C(\mu_{12}\ k \sinh \gamma_2 x - \mu_{11}\ \gamma_2 \cosh \gamma_2\ x)] \tag{10}$$

For the dielectric medium:

$$E_z = (F e^{\gamma_3 x} + G e^{-\gamma_3 x})\, e^{i(\omega t - k y)}, \tag{11}$$

where $\gamma_3^2 = k^2 - \dfrac{\omega^2}{c^2}\varepsilon_3$;

$$H_y^3 = \frac{1}{i\omega\mu_0}\gamma_3 [F e^{\gamma_3 x} - G e^{-\gamma_3 x}]\, e^{i(\omega t - k y)}. \tag{12}$$

A, B, C, D, F, and G are constants can be determined from the boundary conditions.

Applying the boundary conditions of electric and magnetic field components at $x = -d_1$ and $x = -d_2$, such as:

$$E_z^{(1)} = E_z^{(2)} \quad at\ \ x = d_2$$

$$H_y^{(1)} = H_y^{(2)} \quad at\ \ x = d_2$$

and

$$E_z^{(2)} = E_z^{(3)} \quad at\ \ x = -d_1$$

$$H_y^{(2)} = H_y^{(3)} \quad at\ \ x = -d_1$$

$$E_z^{(3)} = 0 \quad at\ \ x = -d_1$$

we can obtain easily the electromagnetic surface wave dispersion equation, which can be written as:

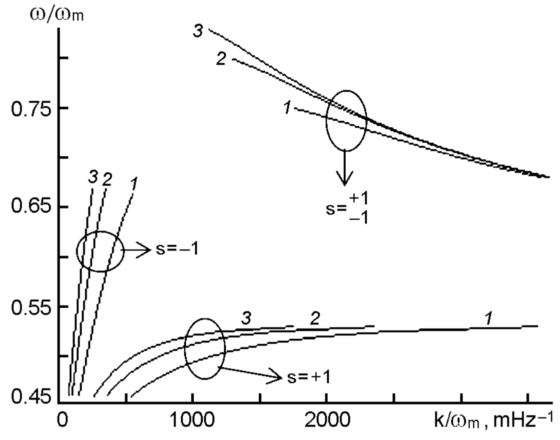

Fig. 2: Dispersion curves for different ferrite layer thickness d_2: (1) 1×10^{-5} m; (2) 1.5×10^{-5} m; (3) 2×10^{-5} m, $d_1 = 4\times10^{-6}$ m, $\varepsilon_2 = 14$, $\varepsilon_3 = 3$, $\frac{\omega_i}{\omega_m} = 0.1124$.

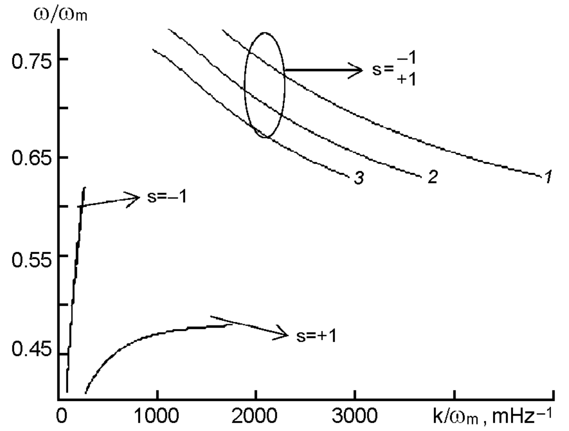

Fig. 3: Dispersion curves for different dielectric layer thickness d_1: (1) 3×10^{-6} m; (2) 4×10^{-6} m; (3) 5×10^{-6} m, $d_2 = 2\times10^{-5}$ m, $\varepsilon_2 = 14$, $\varepsilon_3 = 3$, $\frac{\omega_i}{\omega_m} = 0.1124$.

$$\begin{aligned}&\left(\gamma_2+\frac{\mu_{12}}{\mu_{11}}k-\mu_v\frac{\gamma_1}{\mu_1}\right)\left[\left(\frac{\mu_{12}}{\mu_{11}}k-\gamma_2\right)\tanh(\gamma_3 d_1)+\mu_v\gamma_3\right]\\ &x\,e^{-2\gamma_2 d_2}+\left(\gamma_2-\frac{\mu_{12}}{\mu_{11}}k+\mu_v\frac{\gamma_1}{\mu_1}\right)\times\left[\left(\left(\frac{\mu_{12}}{\mu_{11}}k+\gamma_2\right)\tanh(\gamma_3 d_1)+\mu_v\gamma_3\right)\right]=0\end{aligned} \tag{13}$$

k is defined as :

$k = s\,.\,|\mathrm{k}|$, where s takes +1 or -1 according as the static magnetic field is applied in +y or –y direction respectively.

This equation is non-reciprocal dispersion equation, i.e. $\omega(k)\neq\omega(-k)$.

If $d_1\to-\infty$, $\varepsilon_3=\mu_1=\varepsilon_1=1$, $\gamma_3=\gamma_1$,
then we get:

$$\left(\frac{1}{\mu_{11}}+\mu_v\right)k^2-(\varepsilon_f+\mu_v)k_0^2+2\gamma_1\gamma_2\cosh(\gamma_2 d_2)=0 \tag{14}$$

this is similar to equation (17) in [19].

If $d_1 = 0$, from the above equation (13):

$$\begin{aligned}&\gamma_2\cosh\gamma_2 d_2-\frac{\mu_{12}}{\mu_{11}}k+\mu_v\gamma_1=0\\ &or\ \gamma_1=\frac{k\mu_{12}-\gamma_2\mu_{11}\coth\gamma_2 d}{\left(\mu_{11}^2-\mu_{12}^2\right)},\end{aligned} \tag{15}$$

which is a similar to equation (25) in reference [19].

The dispersion relation [13] has been solved to get the dispersion curves as shown in Figs. 2, 3. It has been noticed in these Figures that the electromagnetic surface waves will propagate if there is a solution of the dispersion relation. Figs. 2, 3 demonstrate that for s = +1, there is a cutoff in the dispersion curves which means that there is no surface waves propagation. This behavior cannot be seen in the case s = -1. The non-reciprocal behavior of the dispersion curves has, however, been noticed for $s = \pm 1$ in the forward propagating waves (positive gradient curves) but cannot be seen in the backward waves (negative gradient curves).

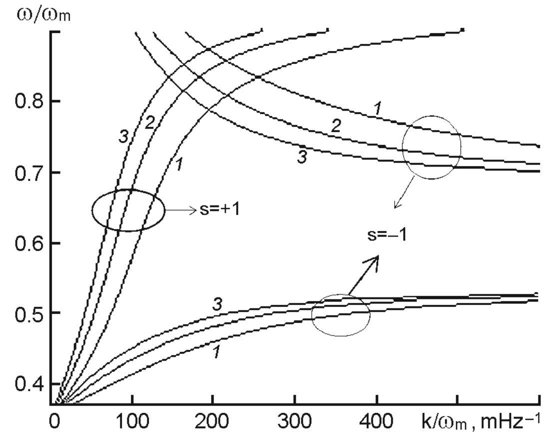

Fig. 4: Dispersion curves for different ferrite layer thickness d_2; (1) 2×10^{-5} m; (2) 3×10^{-5} m; (3) 4×10^{-5} m, $d_1= 6\times10^{-5}$m, $\varepsilon_2=14$, $\varepsilon_3= 3$, $\frac{\omega_i}{\omega_m} = 0.1124$.

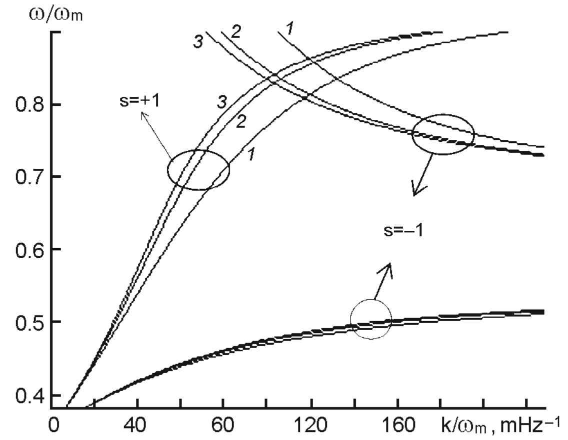

Fig. 5: Dispersion curves for different dielectric layer thickness d_1; (1) 2×10^{-5} m; (2) 6×10^{-5} m; (3) 8×10^{-5} m, $d_2= 6\times10^{-5}$ m, $\varepsilon_2= 14$, $\varepsilon_3= 3$, $\frac{\omega_i}{\omega_m} = 0.1124$.

It has also been shown in Fig.s 2-3 that by varying the normalized frequency $\frac{\omega_i}{\omega_m}$, the dispersion curves can change their direction from forward to backward one.

Fig. 2 shows that the dispersion curves are sensitive to the change of the ferrite layer thickness for both the forward and backward waves whereas in Fig. 3, these dispersion curves are insensitive to the thickness change of the dielectric layer for forward waves only. The dispersion curves of the backward waves in Fig. 3 are, however, more sensitive to the thickness variation than those shown in Fig. 2.

Moreover, for the forward waves, the LHM layer behaves as a metal (Right Handed Material – RH) where $\varepsilon_1< 0$ and $\mu_1>0$. On the other hand, for the backward waves, the LHM layer has negative electrical permittivity ($\varepsilon_1< 0$) and negative magnetic permeability ($\mu_1<0$), i.e. it behaves as an LHM. Therefore, the LHM behavior seems to control the direction of the propagated waves from forward to backward direction. The dispersion relation is normally solved if the normalized frequency $\frac{\omega}{\omega_m}$ has values in the range 0-1 where μ_v satisfies the condition for the solution.

Magneto-Static Surface Waves: Propagation of magneto-static surface waves through a composite structure containing ferrite layer has been studied theoretically and experimentally [18, 19]. In the short wavelength region (large k), where the phase velocity of the response is much lower than the intrinsic velocity of the ferrite medium:

$k^2 \cong \gamma_2$, and $k^2 \cong \gamma_3$.

In this the magneto-static limit where $\nabla \times \bar{H} \cong 0$ (20) the dispersion relation (13) reduces to the following form:

$$e^{-2kd_2} = \frac{\left[\mu_l-(\mu_{11}+\mu_{12}\, s)\right]\left[(\mu_{11}-\mu_{12}\, s) - \tanh kd_1\right]}{\left[(\mu_{11}+\mu_{12}\, s) + \tanh kd_1\right]\left[-\mu_l-(\mu_{11}-\mu_{12}\, s)\right]}, \tag{16}$$

By solving the above magneto-static dispersion relation (16), the results presented in Figs. 4, 5 can be obtained.

It is seen from Figs. 4, 5 that for $s = +1$ we have only the forward dispersion curves which are not observed in the case of the electromagnetic surface waves shown in Figs. 2-3. It is also seen that when $s = -1$, the dispersion curves in Fig. 4 are more sensitive to the thickness variation of the ferrite layer than the curves in Fig. 5 to the thickness variation of the dielectric layer.

Thus, the dispersion characteristics of magnetic surface waves in a metal-dielectric-ferrite (YIG)-LHM layered waveguide structure have been investigated. We found that LHM can control the direction of the curves from forward to backward wave propagation. In the case of the electromagnetic waves, the non-reciprocal behavior has been obtained in the forward propagated waves only. The non-reciprocal behavior has been also found in forward and backward directions for the case of the magneto-static surface waves. These interesting results could be used in design some future microwave technology devices.

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Характеристики електромагнітних та магнітостатичних поверхневих хвиль у шаруватій хвилевідній системі метал-діелектрик-ферит-матеріал лівої руки

А.Х.Ель-Асталь, М.С.Хамада, М.М.Шабат

Розглянуто матеріали з негативною діелектричною проникністю та негативною магнітною проникністю, так звані «матеріалами лівої руки» (left-handed materials, LHM). Досліджено характеристики дисперсії поверхневих магнітних хвиль у шаруватій хвилевідній системі типу метал-діелектрик-ферит (ЗІГ)-LHM. Одержано також деякі характеристики, які стосуються, зокрема, нееквівалентної поведінки хвиль, що поширюються. Відзначено нееквівалентний характер поширення електромагнітних хвиль у прямому напрямі, в той час як для магнітостатичних поверхневих хвиль такий характер виявлено при поширенні як у прямому, так і у зворотному напрямах. Слід зазначити, що введення шару LHM у вищезгадану хвилевідну систему відіграє домінантну роль у регулюванні напряму дисперсійних кривих при переході від прямого до зворотного поширення хвиль.