DISSIPATIVE GENERATION REGIME OF A SYSTEM OF STATIONARY OSCILLATORS

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A system of independent oscillators is considered. The interaction of the oscillators occurs through the field of the wave. The initial phases of the oscillators are random. The synchronization of the oscillators by an external field of large amplitude is discussed. The effect of nonlinearity due to relativistic effects is taken into account. It is shown that in the regime of synchronization of oscillators by a strong field the nonlinearity does not change the nature of energy exchange between the wave and the oscillators. In the generation regime, energy is exchanged between the oscillators and the growing field. At low loss levels, the process of increasing the amplitude of the field is prolonged. The efficiency of energy exchange between oscillators and the field reaches 75%. With an increase of the loss level, the process of oscillators' synchronization changes its character and accelerates, but the amplitude of the field is small. The characteristic time of emission of the system is comparable with the characteristic growth time of the field.

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INTRODUCTION

Interest to the dissipative instabilities has been related to the necessity of getting energy output from the system where oscillations are generated. It is always possible to introduce a parameter $\theta = \delta_D / \gamma_{\max}$ that is equal to the ratio of the energy absorption decrement of the system in the absence of an active element δ_D (a beam of charged particles, an external field, an active medium, etc.) to the maximum oscillation generation increment with a frequency ω in the system with the active element in the absence of any loss of HF energy ($\mathrm{Im}\,\omega = \gamma_{\max} = \gamma |_{\delta_D = 0}$). In generators of the TWT type, in the nonlinear regime, the phenomenon of anomalously large energy losses of the beam particles was observed [1, 2]. For the θ, in this case the expression $\theta = (\delta_D / \omega)(\omega / \omega_b)^{2/3}$ is valid, where $\omega_b = (4\pi e^2 n_{b0} / m)^{1/2}$ is the plasma frequency of the beam, and e, m, n_{b0} are the charge, mass and density of the electrons in the beam. The increment of the dissipative instability when $\theta > 1$ is equal to $\mathrm{Im}\,\omega = \omega_b (\omega / \delta_D)^{1/2} / \sqrt{2}$. One could verify that the energy of perturbations in the system is negative, that is, the presence of perturbations leads to a decrease of the total energy of the system (consisting of the medium and the beam). In this case, the dissipative processes do not lead to the appearance of an instability threshold [3]. The largest energy flux from the system was achieved at $\theta \approx 3$ in a single-mode regime [1, 2] and at $\theta \le 1$ in a multimode regime [4] (a lower absorption level is observed here due to the presence of a multitude of modes in the excited spectrum, each of which undergoes absorption). Below we consider the dissipative mode of generation of an electromagnetic wave in a system of oscillators with fixed centers [5, 6].

1. THE SYSTEM OF OSCILLATORS WITH THE FIXED CENTERS

Let the frequency of the wave and the frequency of the oscillators coincide and are equal to ω. The wave vector of the oscillations is $\vec{k} = (0,0,k)$, $\vec{E} = (E,0,0)$, $\vec{B} = (0,E,0)$ are the components of the field and $E = |E| \cdot \exp\{-i\omega t + ikz + i\varphi\}$. Oscillators are arranged along the OZ axis in an amount of N at the wavelength $2\pi / k$. The mass of the oscillator is m, its charge is $-e$, frequency is ω. The initial oscillation amplitude of the oscillator is equal to a_0. We assume that the oscillator moves only in the direction of the OX axis. In this case, the influence of the magnetic field of the wave on the dynamics of the oscillator can be neglected [5, 6]. For the extended systems, or in the case of a small group velocity of the excited oscillations, the energy can accumulate around oscillators even when there is energy loss due to radiation.

Below we will assume that the system is rather extended, and the group velocity of the wave is moderate. These conditions can be met with appropriate effective permittivity ($c_{eff} = v_g = k_0 c^2 / \omega_0 \varepsilon_0$) or near the cutoff frequency of the waveguide, similarly to how it is realized in gyrotrons. Here we neglect the reflection effects on the boundary of the system. The effective decrement of absorption in this case is $\delta_D = 2c_{eff} / b$ and $\theta = \delta / \gamma_1 = 2c_{eff} / b \cdot \gamma_1$. The equations of motion for oscillators take the following form

$$\frac{d}{d\tau} A_j = E_+ \cos\{\phi_+ + 2\pi Z_j - \psi_j\} + E_- \cos\{\phi_- - 2\pi Z_j - \psi_j\}, \quad (1)$$

$$A_j [\frac{d}{d\tau} \psi_J - \Delta_J] = E_+ \sin\{\phi_+ + 2\pi Z_j - \psi_j\} + E_- \sin\{\phi_- - 2\pi Z_j - \psi_j\}. \quad (2)$$

For the fields $E_\pm$ propagating in both directions:

$$\frac{d}{d\tau} E_\pm + \theta E_\pm = -\frac{1}{N} \sum_{i=1}^{N} A_j \cos\{\varphi_+ \pm 2\pi Z_j - \psi_j\} \quad (3)$$

$$E_{\pm}(\frac{d\varphi_{\pm}}{d\tau}-\Delta\varphi)=\frac{1}{N}\sum_{i=1}^{N}A_j\,\sin\{\varphi_{\pm}\pm 2\pi Z_j-\psi_j\}\,, \quad (4)$$

where $\mathrm{E}=eE/\omega m\gamma a_0$, $\tau=\gamma t$, $A_i=|x_i|/a_0$, $\gamma^2=\pi e^2 n_0/m$, $kz_i=Z_i\subset(0,2\pi)$.

In the non-relativistic case $\Delta_i=0$, and the presence of relativism leads to the nonlinearity of the oscillators, $\Delta_i=\alpha\cdot(A_i^2-A_{i0}^2)$, $\alpha=3\omega(k\cdot a_0)^2/4$. With the substitutions $\psi_j\to-\psi_j$, $\varphi\to-\varphi$, and $Z_j\to-Z_j$, $\alpha\to-\alpha$, the system is invariant, that is, the sign of α does not change the dynamics of the process, except for the direction of the phase change of the wave and the particles. One can see that for the waves with different polarization (where the particles move only in the direction of the electric field vector of the wave), the same equations can be used. Here particles density per unit length is $n_0=M/b$, where M is the total number of particles in the beam, and each simulated particle contains M/N real particles; $\pi e^2 M/2mc=\gamma_D$. The energy density of the radiation at the ends of the system can be estimated as:

$$w=\left(m\omega_0^2 a_0^2 n_0/2\right)\cdot W, \quad (5)$$

where

$$W=\{\mathrm{E}_+^2+\mathrm{E}_-^2\}. \quad (6)$$

The density of the energy flux from the system can also be estimated from the relationship $p=c\cdot w$. The integral field is formed due to the initial perturbation or occurs when the RF energy is accumulated in a sufficiently extended beam. In this case, the beam particles do not directly interact with each other and interact only with the field of the wave. Generally speaking, such field is usually considered in the problems of generating and amplifying induced radiation. For small θ, that is, for an extended beam, this field accumulates in its volume. In addition, we restrict our study to considering only a wave propagating in the positive direction (writing equations similarly to [4])

$$\frac{\partial \mathrm{E}}{\partial\tau}+\theta\cdot\mathrm{E}=\frac{1}{2N}\sum_{i=1}^{N}A_i\cdot\mathrm{Sin}(\psi_i-Z_i-\varphi), \quad (7)$$

$$\mathrm{E}(\frac{\partial\varphi}{\partial\tau}-\Delta\varphi)=-\frac{1}{2N}\sum_{i=1}^{N}A_i\cdot\mathrm{Cos}(\psi_i-Z_i-\varphi). \quad (8)$$

Motion equations of the oscillators are:

$$\frac{\partial A_i}{\partial\tau}=\frac{\mathrm{E}}{2}\mathrm{Sin}(\varphi-\psi_i+Z_i), \quad (9)$$

$$\frac{\partial\psi_i}{\partial\tau}-\Delta_i=-\frac{\mathrm{E}}{2A_i}\mathrm{Cos}(\varphi-\psi_i+Z_i). \quad (10)$$

The conservation law of energy follows from the equations (10) and (12):

$$\mathrm{E}^2+\frac{1}{N}\sum_{i=1}^{N}A_i^2+\theta\int_0^{\tau}\mathrm{E}^2(\tau')\cdot d\tau'=const, \quad (11)$$

It can also be shown that with some simplifications, the system of equations that describes the excitation of electromagnetic waves by an electron beam moving in a constant magnetic field in a metal waveguide also reduces to the system (7) - (11) considered above.

2. RESULTS OF NUMERICAL SIMULATION OF THE SYSTEM (7) - (11)

Analyzing the results of numerical simulation of the system (7) - (11), note the following. When $\theta<<1$ and $\Delta_i=\alpha\cdot(A_i^2-A_{i0}^2)$ generation is possible only if $-\alpha<\Delta\varphi<-\frac{1}{2}\alpha$. The value of the parameter $\alpha\neq 0$ basically affects the speed of the processes. When $\theta<<1$ the phase of the field φ increases in jumps (Fig. 1), the envelope of the amplitude increases exponentially. Here we use the following initial conditions and parameters: $A_j\big|_{t=0}\equiv A_{j0}=1$, $\varphi\big|_{t=0}=\psi_j\big|_{t=0}=0$, $\alpha=0.1$, $E\big|_{t=0}=E_0=0.01$, $\theta=0.001-40$, $\Delta_\varphi=0;\alpha$.

Depending on the value of the parameter, there are two distinct modes. In the first, practically non-dissipative (reactive) mode, when $0.001<\theta<0.05$ the field, oscillating, grows to approximately to the value of 0.7, as can be seen from Fig. 1, and the total energy ouput is about 50%.

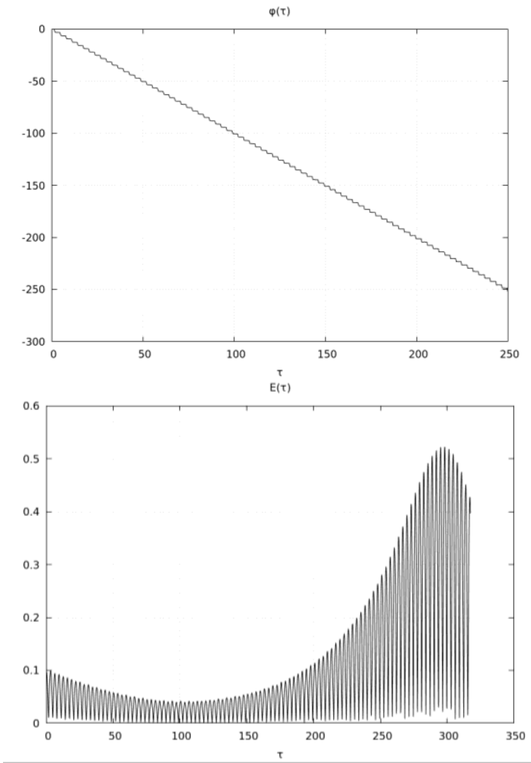

Fig. 1. The dependence of the phase (top figure) and amplitude (bottom figure) of the field on time at $\alpha=0.1$; $E_0=0.1$; $\Delta\varphi=-\alpha$; $\theta=0.02$

When $\theta>0.05$ the second mode of instability is realized. In this case, the maximum achievable amplitude of the field decreases by an order of magnitude. In the range $0.05<\theta<0.5$, the wave energy output from the oscillator system is negligibly small, but starting from $\theta>0.5$ it increases noticeably with the increase of θ and reaches 70% approximately when $\theta=20$, although it takes rather long time. The rate of energy output $\theta E_{\max}^2$ has two approximately equal maximums: at $\theta=0.02$ and at $\theta=20$ (Fig. 2).

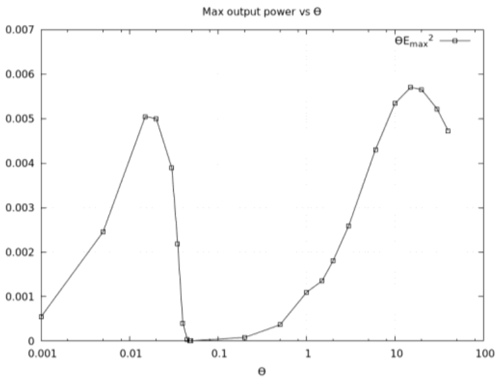

Fig. 2. The maximum achievable rate of energy output $\theta E_{\max}^2$ upon θ; $\alpha = 0.1$; $E_0 = 0.1$; $\Delta\varphi = -\alpha$

The presence of two waves E_+ and E_- does not change anything fundamental in the behavior of the system. Using an external "seed" field allows one to accelerate the achievement of the field maximum, without changing the nature of the process. Thus, the system of excited oscillators, as noted earlier in [6], is extremely sensitive to phase detuning of the electromagnetic wave and oscillators. The most interesting for the output of energy was a slighly dissipative regime, although the efficiency of the energy output from the system in the case of large levels of θ is quite comparable with it. Obtained results make it necessary to study more closely the nonlinear regimes of such system, which is a direct simplified analog of both cyclotron instabilities in waveguides and gyrotron modes of generation of electromagnetic oscillations.

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ДИССИПАТИВНЫЙ РЕЖИМ ГЕНЕРАЦИИ СИСТЕМЫ НЕПОДВИЖНЫХ ОСЦИЛЛЯТОРОВ

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Рассмотрена система независимых осцилляторов. Взаимодействие осцилляторов происходит через поле волны. Начальные значения фаз осцилляторов являются случайными. Обсуждается синхронизация осцилляторов внешним полем большой амплитуды. Проведен учет влияния нелинейности за счет релятивистских эффектов. Показано, что нелинейность в режиме синхронизации осцилляторов сильным полем не нарушает характер обмена энергией между волной и системой осцилляторов. В режиме генерации наблюдается обмен энергией между осцилляторами и растущим полем. При малых уровнях потерь процесс роста амплитуды поля затягивается. Эффективность обмена энергией между осцилляторами и полем достигает 75%. При увеличении уровня потерь процесс синхронизации осцилляторов меняет характер и ускоряется, однако амплитуда поля невелика. Характерное время излучения системы сравнимо с характерным временем роста поля.

ДИСИПАТИВНИЙ РЕЖИМ ГЕНЕРАЦІЇ СИСТЕМИ СТАЦІОНАРНИХ ОСЦИЛЯТОРІВ

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Розглянуто систему незалежних осциляторів. Взаємодія осциляторів відбувається за допомогою поля хвилі. Початкові фази осциляторів обираються випадково. Обговорюється синхронізація осциляторів зовнішнім полем великої амплітуди. Враховано вплив нелінійності за рахунок релятивістських ефектів. Показано, що нелінійність у режимі синхронізації осциляторів сильним полем не порушує характер обміну енергією між хвилею та системою осциляторів. У режимі генерації спостерігається обмін енергією між осциляторами та зростаючим полем. За умов малих рівнів енергетичних втрат процес зростання амплітуди поля затягується. Ефективність обміну енергією між осциляторами та полем сягає 75%. Коли рівень енергетичних втрат зростає, процес синхронізації осциляторів змінює характер та прискорюється, проте амплітуда поля при цьому мала. Характерний час випромінювання системи приблизно дорівнює характерному часу зростання поля.