DEUTERON-DEUTERON NUCLEOSYNTHESIS CONSIDERED WITH ACCOUNTING OF CENTRIFUGAL AND COULOMB BARRIERS

I.V. Tkachenko[1], S.V. Trubnikov[2], V.I. Tkachenko[1,3]
[1]National Science Center "Kharkov Institute of Physics and Technology", Kharkov, Ukraine
E-mail: tkachenko@kipt.kharkov.ua;
[2]Kharkiv State Polytechnic College, Kharkov, Ukraine
E-mail: sergiy.v.trubnikov@gmail.com;
[3]V.N. Karazin Kharkov National University, Kharkov, Ukraine

The basic analysis of nuclear reactions in the deuteron-deuteron (*dd*) system at low energies is carried out. For the case of nuclear *dd*-fusion special attention is paid to the difference in the height of the Coulomb barriers, which arises from different values of the deuteron radius. It is declared that a centrifugal barrier greatly hinders the nuclear reaction at low energies: in 19 out of 20 allowed transitions there is a strong suppression of nuclear fusion. Only in one single transition (S → S) is there no centrifugal suppression, but in the *dd*-system this transition is not dominant. Attention is drawn to the prospects (despite the smallness of the cross section) of considering radiation fusion in view of the significant energy release in a single act of reaction.

PACS: 52.20.Hv

INTRODUCTION

Deuteron-deuteron (*dd*) scattering, like any nuclear-nuclear scattering, has in principle a multi-channel nature. A unique feature of the deuteron is the smallness of its binding energy. Therefore, even at sufficiently low energies, several different reaction channels (both exo- and endoenergetic) are opened in *dd*-scattering. Among them are the following processes, proceeding, naturally, with different probabilities:

Coulomb scattering:

$$d+d \rightarrow d+d\,.$$

Because of the absence of excited states in the deuteron, there is no so-called reaction of Coulomb excitation in this process.

Elastic nuclear scattering:

$$d+d \rightarrow d+d\,.$$

Complete or partial fission:

$$d+d \rightarrow p+p+n+n\,,\quad d+d \rightarrow d+n+p\,.$$

Nuclear fusion:

$$d+d \rightarrow p+{}^3H\,,\quad d+d \rightarrow n+{}^3He\,.$$

Radiation fusion:

$$d+d \rightarrow {}^4He+\gamma\,.$$

Below the reactions of nuclear fusion with both unpolarized and polarized deuterons will be generally considered. Particular attention will be paid to differences in the height of the Coulomb barrier arising when different permissible values of the deuteron radius are chosen, as well as the role of the centrifugal barrier. Previously, in our opinion, these issues were not adequately covered in the current literature.

For the same reason questions connected with the possible role of the deuteron polarizability in the *dd* interaction and the prospects of radiative capture in the general complex of questions of "slow" controlled thermonuclear synthesis were also added to the review procedure.

GENERAL REMARKS

The present work arose as a reaction to the ongoing discussion of the pilot project PolFusion [1]. In this project it is proposed to study the interaction of two beams of polarized deuterons at low energies with all possible combinations of the mutual orientation of the spins of the colliding nuclei.

Charged products of nuclear *dd*-fusion (p, ${}^3He$, ${}^3H$) will be detected by a system of detectors with a geometry close to the full solid angle. The dependence of the differential and total cross sections of the *dd*-fusion on the spin-spin correlations will be measured.

It is assumed that the results obtained during the implementation of the project will be useful in developing of the realistic schemes of controlled thermonuclear fusion using polarized nuclei. The scheme of opposing polarized *dd* beams is good because the polarization and energy of deuterons can be varied and thereby isolate and study in an almost pure form individual effects in the *dd* interaction, which can not be done in a large volume of a heated *dd* plasma confined by an external magnetic field, where all effects will be mixed. Below we discuss some of these effects.

COULOMB BARRIER

Nuclear *dd*-synthesis begins when deuterons approach the distance of the action of nuclear forces $r \le r_n \approx 1,7-2\,fm$. To reach this it is first of all necessary to overcome the Coulomb repulsion of two extended positively charged particles with unit charges e. For the *dd* interaction, the Coulomb barrier is:

$$B_{Coulomb} = \frac{e^2}{2R_d}\,,$$

where R_d – deutron's radius.

Full (not spontaneous i.e. tunneling) nuclear fusion can begin only when the sum of the kinetic energies of two deuterons exceeds the height of the Coulomb barrier:

$$E_{kin}(d_1)+E_{kin}(d_2) > B_{Coulomb}\,.$$

However, what value of the deuteron radius should be substituted into this formula? One can specify multiple values of deuteron radii.

1. The first estimate of the dimensions of the deuteron appeared at the very beginning of the development of the

nucleus theory [2] and was based on the form of the wave function of the S-state of the deuteron at the boundary of the action of nuclear forces (see, for example, [http://rspa.royalsocietypublishing.org/content/148/863/146]):

$$\psi_d \sim \frac{e^{-\alpha r}}{r}, \quad (1)$$

where $\alpha^2 = M\varepsilon / \hbar^2$, M is the nucleon mass, ε is the binding energy of the deuteron. From the relation (1) we obtain the value of the parameter characterizing the spatial dimensions of the deuteron:

$$R_d = \frac{1}{\sqrt{M\varepsilon}} = 4.3\,fm. \quad (2)$$

This value can be conditionally called the nuclear (or hadron) radius of the deuteron. The value of (2) has long been widely accepted and widely used in the scientific literature, although such an estimate of the Coulomb barrier is not indisputable.

2. The quadrupole moment of a deuteron is completely determined by the presence of the tensor component of nuclear forces. Therefore the radius of action of the tensor forces [2] can be regarded as the tensor radius of the deuteron:

$$R_{dT} = 3\,fm. \quad (3)$$

3. Analysis of electromagnetic processes involving the deuteron gives other methods for determining the radius of the deuteron.

Elastic scattering of electrons by deuterons at very small values of the transferred momenta leads to the following value of the charge distribution radius in the deuteron:

$$R_{d(\mathrm{ch})} \equiv < r_d^{\,2} >_{ch}^{\frac{1}{2}} = 2.12 \pm 0.05\,fm. \quad (4)$$

More precise results can be obtained by studying the Lamb shift in deuterium (see, for example, the discussion in [2]).

Even more accurate results were obtained as a result of laser spectroscopy of atoms of ordinary ($e^- d$) and muonic ($\mu^- d$) deuterium [3]:

$$R_d = 2.1256\,fm \text{ for } (e^- d)\text{-atom}, \quad (5)$$

$$R_d = 2.1424\,fm \text{ for } (\mu^- d)\text{-atom}. \quad (6)$$

The values of the three charge radii (2 - 6) practically coincide for the purposes of this paper, but in principle a slight difference (5) and (6) makes us recall the "intrigue" regarding the proton radius that arose in 2013 about the ($\mu - e$)-universality?

4. In [4], while calculating the permeability coefficients of the potential barrier, deuterons were assumed to be "hard spheres" with a radius:

$$R_d = 7\,fm.$$

It is unlikely that such assumptions can be called realistic: the origin and nature of such a large one R_d is not explained in any way.

The Coulomb barriers calculated for the "minimum" and "maximum" deuteron radii take the following values.

1. $R_d = 2.12\,fm$.

In this case $B_{Coulomb} \simeq 360 keV = 3.6 \cdot 10^9 K$.

2. $R_d = 4.3\,fm$

it gives $B_{Coulomb} \simeq 170 keV = 1.7 \cdot 10^9 K$.

A comparison of the two Coulomb barrier values leads to the following conclusions:

1. The height of the Coulomb barrier can vary more than twofold, depending on the deuteron radius used.

2. The value $R_d = 2.12\,fm$ is preferable for the following reason: in the "organization" of the barrier, not all of the deuteron "as a whole" participates, but only its charged part. Therefore, the use of the charge radius of the deuteron in the calculation of the $B_{Coulomb}$ is more justified.

3. In any case, the Coulomb barrier is quite high for modern experimental facilities on controlled fusion studies. At the temperature of the *dd*-plasma $T \sim 10^7 ... 10^8 K$, nucleosynthesis is possible only as a sub-barrier [5] transition or due to reactions on the "tails" of the Maxwellian distribution.

THE CENTRIFUGAL BARRIER

Let's suppose that we managed to increase the energy of deuterons in a sufficiently large volume, and exceed the height of the Coulomb barrier. Does this mean that all deuterons with energies $E_1 + E_2 > B_{Coloumb}$ will enter into the reactions of a full-fledged *dd* synthesis?

No, it does not mean! The *dd*-fusion will undoubtedly occur with central *dd*-collisions. But such clashes are not necessarily dominant.

In noncentral collisions, when the orbital angular momentum l of the relative motion of two deuterons is different from zero $l \geq 1$, when two deuterons approach each other by the distance of action of nuclear forces r, the centrifugal B_{cf} barrier appears:

$$B_{cf} = \frac{\hbar \cdot l(l+1)}{2\mu_{dd} r^2}, \quad (7)$$

where m_N is the mass of the nucleon, $\mu_{dd} = \frac{m_d m_d}{m_d + m_d}$ is the reduced mass. For the reduced mass in our problem, we can take an approximate value: $\mu_{dd} = m_N$.

The centrifugal barrier as the suppressing factor of the synthesis reaction in most cases arises in the final state during the expansion of the (pt) and (n^3He) pairs, even if in the initial state the *dd* interaction was of a central character. In the final states, the reduced masses of reaction products take on values $\mu_{pt} \approx \mu_{n^3He} \approx \frac{3}{4} m_N$. To estimate the height of the centrifugal barrier, we choose an allowable value of the impact parameter $r = r_{\max} = b$, which is most favorable for the onset of *dd*-fusion. As such, we can take the value: $b = 2R_d + r_{nucl}$, where r_{nucl} is the radius of action of nuclear forces. For the $r_{nucl} \approx 1.6...2\,fm$ and $R_d = 4.3\,fm$ produces $b \approx 10\,fm$, which gives the centrifugal barrier estimate:

$$B_{cf}(dd) \cong l(l+1) \times 200\; keV.$$

In other words, even for lower partial waves and "minimizing" the B_{cf} (7) parameters R_d and r_{nucl} the centrifugal barrier is very high:

$$B_{cf}(dd)\Big|_{l=1} \cong 400keV \approx 4\cdot 10^9 K,$$
$$B_{cf}(dd)\Big|_{l=2} \cong 1.2MeV. \quad (8)$$

The centrifugal barrier is even higher when the reaction products, i. e. (pt) – and (n^3He) -pairs emerge from the nuclear interaction area r_{nucl} with nonzero orbital angular momenta. A sufficient increase in the barrier B_{cf}(pt), for example, will occur due to small (in comparison with the radius of the deuteron R_d) proton and tritium radii ($R_p \approx 0.9\,fm$, $R_t \approx 2\,fm$ respectively). In this case we have a very high barrier:

$$B_{cf}(pt) \cong l(l+1)\times 1.2MeV. \quad (9)$$

Obviously, in the case of a noncentral approach of two deuterons when the impact parameter approaches the radius of action of the nuclear forces, the emerging centrifugal barrier reaches a considerable height and greatly complicates the *dd* fusion.

The allowed transitions are determined by the most general laws of physics of fundamental interactions. In our case, there are three such patterns that can not be subjected to any revision:

1. Bose statistics for the system of two identical bosons in the initial state. The requirement of symmetry of the wave function for permutation of all coordinates leads to the relation: $(-1)^{S+l} = +1$, where $S-$ is the total spin, and l is the orbital angular momentum of the *dd* system. Such correlation between the values of the spin and the orbital angular momentum will greatly reduce the number of allowed transitions.

2. The law of conservation of spatial parity in nuclear interactions means the equality of parities (or oddities) of the orbital moments of the initial $l_{initial} = l_i$(DD) and final $l_{final} = l_f(p\,^3H)$ states that: $(-1)l_i = (-1)l_f$, or $l_i = l_f + 2n$, where n is an integer number.

3. The law of conservation of the total angular momentum: $\vec{J} = \vec{S} + \vec{l}$. This law in the theory of nuclear reactions at low energies has a great "practical" sense. It follows from this that at low energies reactions are possible only for a certain values of the orbital angular momentum, which does not exceed a certain small number.

These three laws define allowed multipole transitions in nuclear two-particle *dd* synthesis like:

$$^{2S+1}l_J \rightarrow {}^{2S'+1}l'_J. \quad (10)$$

Full multipole analysis is beyond the scope of this article, but it is necessary to point out that under (10) out of 20 possible transitions of the *dd*-system with $S(dd) = 0,1,2$ and $l(dd) = 0,1,2,3$ initial states there is only one "pure" transition $S \rightarrow S$ not suppressed by a centrifugal barrier at any energies. But in the *dd*-system such "pure" transition is not dominant one.

POLARIZABILITY

The question of the effect of the deuteron polarizability on the *dd*-fusion is comparatively poorly studied.

The polarizability of any extended microobject (ie, nuclei and hadrons) is, in principle, the same fundamental characteristic as mass, charge, spin, etc. In the first approximation, for example, for a nucleon, $\alpha-$ electric and $\beta-$ magnetic polarizabilities can be determined [6] as coefficients in the expression for the effective energy of the nucleon interaction with external electric and magnetic fields:

$$E = -\frac{1}{2}\alpha\vec{E}^2 - \frac{1}{2}\beta\vec{H}^2. \quad (11)$$

In other words, a charged object with internal structure placed in an electric field undergoes deformation i.e. in the system appears an additional (except for the Coulomb) long-range action, due to the electric polarizability of the particle. The potential of the polarization interaction is attractive. At relatively large distances, the polarization potential decreases as $1/r^4$, but on the boundary of the nuclear forces distance its role may turn out to be significant.

A convincing theoretical analysis of the polarizability role in nuclear reactions is complicated by the fact that the experimental determination of polarizability is a rather difficult task. Nevertheless, it is clear that in such a loosely coupled system as the deuteron, the polarizability must be large and in principle can affect the magnitude of the Coulomb barrier and hence the rate of nuclear fusion reactions.

For comparison let's give some estimates. The electric polarizability of the deuteron $\alpha(d) = 0.70 \pm 0.05\,fm^3$ [7], while for a proton this value is almost three orders of magnitude smaller $\alpha(p) = (12.10 \pm 0.9)\cdot 10^{-4}\,fm^3$.

In a few literature on this topic were expressed directly opposite points of view. Thus, the estimates given in [7] show that the effects of polarizability in the reactions $pp \rightarrow de^+\nu_e$, $pd \rightarrow \gamma He^3$, $td \rightarrow nHe^4$ are very small. The reaction of *dd*-fusion was not considered. And in [8] it is asserted that taking into account the polarization interaction greatly increases (by several orders of magnitude) the cross section for nucleosynthesis reactions.

In the case of continuing work on nuclear *dd* synthesis, the question of the role of the polarizability of deuterons requires additional study.

RADIATION FUSION

The cross sections of the electro- and photonuclear processes at the energies representing the interest to fusion ($10^8 \ldots 10^9$ K) are approximately two orders of magnitude smaller than the cross sections of purely nuclear interactions. Nevertheless, these processes, in our opinion, deserve a separate discussion.

Radiation *dd*-fusion:

$$d + d \rightarrow {}^4He + \gamma \quad (12)$$

is characterized by a very high energy release: the energy of the γ-quantum produced in such an exothermic reaction is $E_\gamma \approx 24\,MeV$. Therefore, in cases where nuclear sub-barrier synthesis is suppressed (for example, in a quintet state $S(dd) = 2$), the contribution of radiation synthesis becomes significant. The reaction $^2H(d,\gamma)^4He$ is discussed in many works (see, for example, [9, 10]), but mainly from the point of view of studying the structure of the ground state of the nucleus.

In conclusion, we note that another "remarkable property" of radiation synthesis, in addition to high energy release, is the complete absence of neutrons and any radioactive reaction products.

CONCLUSIONS

The height of the Coulomb barrier in deuteron-deuteron nucleosynthesis reactions depends significantly on the assumed value of the deuteron radius. The commonly used value for the deuteron radius of *4.3 fm*, from our point of view, is not entirely justified, since it represents the radius of the hadronic matter distribution in the deuteron, and only the radius of the electric charge distribution equal to *2.12 fm*, takes part in the formation of the Coulomb barrier. The use of this radius increases the height of the Coulomb barrier in two times.

For nonzero values of the orbital angular momenta of the *dd*-system the heights of the centrifugal barriers far exceed the height of the Coulomb barrier and significantly hinder the above-barrier synthesis process. The estimated heights of the centrifugal barriers are very impressive and, undoubtedly, should be taken into account in more detail in the scope of problems and prospects of *dd*-fusion. Of great interest also would be the experimental studies of the above estimates.

The deuteron is a much more "loose" system than a proton or a neutron. Therefore, in the theoretical calculations of deuteron-deuteron fusion, it is necessary to take into account the polarizability of the deuteron. Existing estimates of this effect are in sharp contradiction. The contribution of polarizability to *dd*-synthesis requires additional studies.

Radiation deuteron-deuteron fusion may be of great interest in view of the huge energy release and the absence of radioactive components in reaction products.

All the above consideration was based only on the fundamental principles of quantum theory. Therefore, the conclusions drawn by us are common to all controlled fusion installations and facilities, regardless of the plasma creation methods or confinement types.

REFERENCES

1. G. Ciullo, R. Engels, M. Büscher, A. Vasilyev // *Nuclear Fusion with Polarized Fuel, Springer Proceedings in Physics*. 2016, p. 154.
2. William Rarita and Julian Schwinger // *Phys. Rev.* 1941, v. 59, Iss. 7, p 556.
3. R. Pohl, F. Nez, L.M. Fernandes, et al. Laser spectroscopy of muonic deuterium // *Science*. 2016, 353(6300):669-73.
4. B.P. Ad'yasevich, D.E. Fomenko. Analiz rezultatov issledovaniya reaktsii D(d,p)T s polyarizovannyimi deytronami // *Yadernaya fizika*. 1969, v. 9, № 2, p. 283-291 (in Russian).
5. L.D. Landau, E.M. Lifshitz. Quantum Mechanics (Volume 3 of A Course of Theoretical Physics). Pergamon Press, 1965.
6. A. Klein // *Phys. Rev*. 1955, v. 99, № 2, p. 998.
7. V.P. Levashev. *On The Role Of Polarization Effects*: Preprint. Academy of Sciences of Ukraine, Institute of theoretical physics, 1998, p. 34.
8. V.B. Belyaev. *Lektsii po teorii malochastichnyih sistem.* M.: "Energoatomizdat", 1986, 128 p.
9. G. Blüge, H.J. Assenbaum, and K. Langanke. Potential model analysis of low energy 2H(d,γ) 4He fusion data // *Phys. Rev*. 1987, v. 36, iss. 1, p. 21.
10. J. Piekarewicz and S.E. Koonin. Importance of the deuteron quadrupole moment in 2H(d,γ)4He // *Phys. Rev*. 1987, v. 36, iss. 3, p. 875.

Article received 30.10.2017

ДЕЙТРОН-ДЕЙТРОННЫЙ НУКЛЕОСИНТЕЗ С УЧЕТОМ КУЛОНОВСКОГО И ЦЕНТРОБЕЖНОГО БАРЬЕРОВ

И.В. Ткаченко, С.В. Трубников, В.И. Ткаченко

Проведен базовый анализ ядерных реакций в системе дейтрон-дейтрон (*dd*) при низких энергиях. Для случая ядерного *dd*-синтеза особое внимание уделяется разности в высоте кулоновских барьеров, которая возникает из разных значений радиуса дейтрона. Указывается, что центробежный барьер значительно затрудняет ядерную реакцию при низких энергиях: в 19 из 20 разрешенных переходов наблюдается сильное подавление ядерного синтеза. Только в одном единственном переходе (S→S) нет центробежного подавления, но в *dd*-системе этот переход не является доминирующим. Обращается внимание на перспективы (несмотря на малость сечения) учета радиационного синтеза за счет значительного энерговыделения за один акт реакции.

ДЕЙТРОН-ДЕЙТРОНИЙ НУКЛЕОСИНТЕЗ ІЗ УРАХУВАННЯМ КУЛОНІВСЬКОГО ТА ВІДЦЕНТРОВОГО БАР'ЄРІВ

І.В. Ткаченко, С.В. Трубников, В.І. Ткаченко

Проведено базовий аналіз ядерних реакцій в системі дейтрон-дейтрон (dd) при низьких енергіях. Для випадку ядерного dd-синтезу особлива увага приділяється різниці в висоті кулонівських бар'єрів, яка виникає з різних значень радіуса дейтрона. Стверджується, що відцентровий бар'єр значно ускладнює ядерну реакцію при низьких енергіях: у 19 з 20 дозволених переходів спостерігається сильне придушення ядерного синтезу. Тільки в одному єдиному переході (S → S) немає відцентрового придушення, але в dd-системі цей перехід не є домінуючим. Увага звертається на перспективи (незважаючи на малість перетину) обліку радіаційного синтезу за рахунок значного енерговиділення за один акт реакції.