# ELECTRON FLOW STABILITY IN THE GAS FILLED DIODE

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For the electron flow in the short-circuited diode filled with gas, with taking into account the braking force proportional to velocity, the stationary modes and their linear perturbations are considered. The equation for increment of perturbation amplitude increase is obtained. The ranges of braking coefficient values are found, in which existence of stable or unstable stationary modes is possible.

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## 1. INTRODUCTION

In many types of electron sources, there are parts in which an electron multiplication is practically absent and the electron motion is mainly determined by space charge, similarly to one in diode. Electron flow stability in the short-circuited diode was considered in the paper [1]. In it, the equation for the increments of perturbation development was obtained and the dependence of the increments on the flow parameters was built. The results of the paper [1] also were presented in the monograph [2]. An external field accelerating the electrons, as a rule, strengthens the flow stability [3]. In the model considered in the paper [1], electron motion is completely determined with electrostatic forces. In the present work, collisions are taken into account through effective braking force proportional to electron velocity. In the section 2, the considered model is described and the equations for the parameters of stationary modes and for the increments of perturbation development are obtained. In the section 3, the dependence of the main instability increment on the parameters of stationary mode is studied.

## 2. MODEL AND SOLUTION

Let us consider one-dimensional electron flow under the electrostatic forces and the braking force proportional to electron velocity with the ratio $\beta_0$ of relevant acceleration to velocity ($\beta_0 > 0$). To write the dimensionless equations, let us denote by $e_0$ the elementary charge ($e_0 > 0$), by $m_0$ electron mass, by $\varepsilon_0$ electric constant, by $j_0$ current density in stationary mode and let us take the following units: the diode gap width $z_0$ for length, the entrance velocity $v_0$ for velocity, the ratios $t_0 = z_0/v_0$, $n_0 = j_0/(e_0 v_0)$, and $E_0 = (m_0 v_0^2)/(e_0 z_0)$ for time, electron density, and field strength, respectively. It is assumed that $z = 0$ for entrance, so, $z = 1$ for exit. The equations in the dimensionless Euler variables have the form

$$\partial_t n + \partial_z (nv) = 0, \tag{1}$$

$$\partial_t v + v \partial_z v = -E - \beta v,$$

$$\partial_z E = -qn, \tag{2}$$

where $\beta = (\beta_0 z_0)/v_0$, $q = e_0^2 n_0 z_0^2 (\varepsilon_0 m_0 v_0^2)^{-1}$, the quantities $v$, $n$, and $E$ are dependent on the variables $(z, t)$, $\partial$ is partial derivative, its index indicates the variable, with respect to which the derivative is taken. Parameter $q$ is proportional to the entrance electron current. An operation mode of the diode may be effectively controlled by its value. It is expedient to use Lagrange variables to simplify the equations solving. Let $z_e(\tau, t)$ and $v_e(\tau, t)$ are coordinate and velocity at the time $t$ of the electron which has come in gap at the time $\tau$ ($\tau < t$). Let $n_e(\tau, t)$ and $E_e(\tau, t)$ are electron density and field strength in the point $z = z_e(\tau, t)$ at the time $t$. It is assumed that during the considered stage of the process all electrons are moving in positive $z$ direction and $z_e(\tau, t)$ monotonously decreases with $\tau$ increase. So, at this stage, the relative disposition of electrons in flow is not changed, electrons do not outrun one another, though the distance between them may be changed. An electron motion in Lagrange variables is described with the equations

$$\partial_t (n_e \partial_\tau z_e) = 0, \tag{3}$$

$$\partial_t z_e = v_e, \tag{4}$$

$$\partial_t v_e = -E_e - \beta v_e, \tag{5}$$

in which the quantities $z_e$, $v_e$, $n_e$, and $E_e$ are dependent on the variables $(\tau, t)$. The equation (3) may be obtained from (1) and (4) with taking into account the equalities

$$\partial_t (n_e \partial_\tau z_e) = (\partial_t n + v \partial_z n) \partial_\tau z_e + \\ + n \partial_z v \partial_\tau z_e = \partial_\tau z_e [\partial_t n + \partial_z (nv)],$$

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in which it is assumed that the partial derivatives are concerned to the dependence on $(\tau, t)$ for the quantities with the index 'e' and to the dependence on $(z, t)$ for the quantities without indexes, and the equality $z = z_e$ is held. In the same assumptions, from (2) and (3), one can get the equalities

$$\begin{gathered}\partial_\tau E_e = \partial_z E \partial_\tau z_e = -q n_e \partial_\tau z_e = \\ = -q[n_e(\tau, t)\partial_\tau z_e(\tau, t)]_{t=\tau}.\end{gathered} \tag{6}$$

From (4) and (6), taking into account that $z_e(t, t) = 0$, $v_e(t, t) = 1$, $n_e(t, t) = 1$, one can obtain the equalities

$$[\partial_\tau z_e(\tau, t) + v_e(\tau, t)]_{t=\tau} = \partial_\tau z_e(\tau, \tau) = 0,$$

$$[\partial_\tau z_e(\tau, t)]_{t=\tau} = -v_e(\tau, \tau) = -1,$$

$$\partial_\tau E_e(\tau, t) = q. \tag{7}$$

Integration of (7) gives the equality

$$E_e(\tau, t) = E_{e0}(t) + (\tau - t)q, \tag{8}$$

where $E_{e0}(t) = E_e(t, t)$. The possibility to obtain the explicit expression (8) for the field strength is connected with the considered stage of the process, at which relative disposition of electrons in flow is not changed, electrons do not outrun one another, though the distance between them may be changed. For $m = 1, 2, 3, \ldots$, let us define the functions

$$e_m(x) = (-1)^m \left[e^{-x} - \sum_{k=0}^{m-1} (-x)^k/k!\right].$$

The integration of (5) and (4), with the field strength from (8), gives the equalities

$$\begin{gathered}v_e(\tau, t) = -\int_\tau^t d\xi e^{\beta(\xi - t)} E_{e0}(\xi) + \\ + e^{\beta(\tau - t)} + q\beta^{-2} e_2(\beta t - \beta\tau),\end{gathered}$$

$$\begin{gathered}z_e(\tau, t) = -\int_\tau^t d\xi \beta^{-1} e_1(\beta t - \beta\xi) E_{e0}(\xi) + \\ + \beta^{-1} e_1(\beta t - \beta\tau) + q\beta^{-3} e_3(\beta t - \beta\tau).\end{gathered} \tag{9}$$

For the electron, which goes out from the gap at time $t$, let us denote by $T(t)$ the time during which it moves through the gap. That is, it has come in the gap at the time $t - T(t)$ and the equality

$$z_e(t - T(t), t) = 1 \tag{10}$$

should be held. For the field strength the condition

$$\int_t^{t - T(t)} d\tau E_e(\tau, t) \partial_\tau z_e(\tau, t) = -V(t) \tag{11}$$

should be imposed. In it $V(t)$ is applied voltage. From (11) and (10) taking into account (8) and (9), one can get the equations

$$\begin{gathered}E_{e0}(t) = -V(t) - q^2\beta^{-4} e_4(\beta T(t)) + \\ + q[T(t) - \beta^{-2} e_2(\beta T(t))] + \\ + q\int_0^{T(t)} d\xi \beta^{-1} e_1(\beta\xi)[T(t) - \xi] E_{e0}(t - \xi),\end{gathered} \tag{12}$$

$$\begin{gathered}-\int_0^{T(t)} d\xi \beta^{-1} e_1(\beta\xi) E_{e0}(t - \xi) + \\ + \beta^{-1} e_1(\beta T(t)) + q\beta^{-3} e_3(\beta T(t)) = 1.\end{gathered} \tag{13}$$

Diode can operate in stationary mode under stationary external conditions. In the case of short-circuited diode ($V(t) = 0$), for the quantities $E_{e0}$ and $T$ independent on $t$, the equations (12) and (13) give the equalities

$$\begin{gathered}E_{e0}\beta e_2(\beta T) = \\ = q e_3(\beta T) + \beta^2 e_1(\beta T) - \beta^3,\end{gathered}$$

$$\begin{gathered}E_{e0}\beta[\beta^3 - q e_3(\beta T)] = \\ = q\beta^2[\beta^2 T - e_2(\beta T)] + q^2 e_4(\beta T).\end{gathered}$$

Excluding $E_{e0}$ from them, one can obtain the equality

$$\begin{gathered}q^2\beta^{-4}[e_3^2(\beta T) - e_2(\beta T) e_4(\beta T)] + \\ + q\beta^{-2}\{e_2(\beta T)[\beta^2 T - e_2(\beta T)] + \\ + e_3(\beta T)[e_1(\beta T) - 2\beta]\} + \\ + \beta[\beta - e_1(\beta T)] = 0,\end{gathered}$$

which gives $q$ for the given $\beta$ and $T$.

The stationary mode may be unstable. Let us consider development of small perturbation caused by the short-time non-zero applied voltage $V'(t)$. Denoting the perturbations with prime, from (12) and (13), in linear approximation, one can obtain the equations

$$\begin{gathered}E'_{e0}(t) + V'(t) = \\ = q\beta^{-1}\int_0^T d\xi e_1(\beta\xi)(T - \xi) E'_{e0}(t - \xi),\end{gathered} \tag{14}$$

$$T'(t) = (v_{e1}\beta)^{-1}\int_0^T d\xi e_1(\beta\xi) E'_{e0}(t - \xi), \tag{15}$$

where $v_{e1}$ is the value of exit velocity (that is, $v_e(t - T(t), t)$) in the stationary mode. Assuming absence of perturbations at $t < 0$, let us apply Laplace transformation to the equations (14) and (15). Denoting the transforms with tilde, according to the example $\widetilde{f}(\kappa) = \int_0^\infty dt e^{-\kappa t} f'(t)$, and defining the functions

$$\begin{gathered}F_E(b, x) = b x^{-1}(b + x)^{-1} + \\ + (b + x)^{-2} e_1(b + x) - x^{-2} e_1(x),\end{gathered}$$

$$\begin{gathered}F_T(b, x) = (b + x)^{-1} \times \\ \times [x^{-1} e_1(x) - e^{-x} b^{-1} e_1(b)],\end{gathered}$$

$$D(\kappa) = q T^2 \beta^{-1} F_E(\beta T, \kappa T) - 1, \tag{16}$$

one can obtain the equations

$$\widetilde{E}_{e0}(\kappa) = [D(\kappa)]^{-1}\widetilde{V}(\kappa),$$

$$\widetilde{T}(\kappa) = v_{e1}^{-1}T^2F_T(\beta T, \kappa T)\widetilde{E}_{e0}(\kappa),$$

and the equation for the increment $\kappa$ of self-consistent perturbation development may be written in the form $D(\kappa) = 0$. Instability of the stationary mode is characterized by positiveness of real part of the increment.

## 3. STABLE AND UNSTABLE MODES

In the cases of the braking force presence ($\beta > 0$) and absence ($\beta = 0$), the characteristics of the short-circuited diode are essentially different. The transitions between one-stream and two-stream modes is controlled by the value of the parameter $q$. In the case $\beta = 0$, for $q < 8/9$, there is one-stream mode, with symmetric distribution of electron density, velocity, field strength and potential with respect to the middle of the gap. For small $q$ the difference between the values of potential in the gap and its boundary value is small. If $q$ increases, but remains less than 16/9, then one-stream mode remains stable, electron velocity in the point of zero field strength $v_s$ remains greater than half of the entrance velocity ($v_s > 1/2$ in the dimensionless units), and maximum of electron density (in the same point) remains less than double density at entrance. If $q$ becomes greater than 16/9, then the stationary one-stream mode disappears, as the increase of the electron charge in the gap decelerates electrons more and leads to the further charge increase. As a result, inside the gap, virtual cathode (the point with zero field strength and zero electron velocity) is formed, and some part of electron flow is rejected from it. But if the parameter $q$ is decreased after formation of virtual cathode (for example, due to entrance electron current decrease) then for $q\in(8/9, 16/9)$ the flow mode remains two-stream. Although existence of the symmetric one-stream mode with the minimum velocity value smaller than half of the entrance velocity does not contradict to the stationary equations, the linear analysis of the non-stationary equations carried out in [1] shows the instability of such mode. When $q$ become less than 8/9 two-stream mode disappears, and the space charge, the value of which is excessive for one-stream mode, goes away from the gap and forms the current pulse. In the Fig.1, the correspondence between the value of dimensionless velocity $v_s$ in the point of zero field strength and the value of parameter $q$ is shown for the different braking coefficient $\beta$ values. In [1], for $\beta = 0$, it is shown that the points to right from the curve maximum, with greater $v_s$ values, correspond to the stable one-stream modes, whereas the points to left correspond to the unstable ones.

Appearance of braking force makes the distributions of electron velocity and density, field strength and potential non-symmetric with respect to the middle of the gap for any entrance electron density. In the Fig.1 the curves with greater $\beta$ in the interval $\beta\in(0, 1)$ give smaller $q$ values for the same $v_s$ values, as both the space charge field and the braking force decelerate electrons. Also, in the case $\beta > 0$ the arbitrary small $1 - v_s$ values are impossible even for arbitrary small $q$ values (that is, the velocity in the point of zero field strength cannot approach the entrance velocity value), and the greater the $\beta$ value, the smaller the limit value $v_{s0}$ of $v_s$ at $q\to 0$. For $\beta$ near to 0, the limit value $v_{sq0}$ of the derivative $\partial_q v_s$ at $q\to 0$ is negative, whereas for $\beta$ near to 1, it is positive, and for the limit transition $\{\beta < 1, \beta\to 1\}$ one can get $v_{s0}\to\exp(-1)$, $v_{sq0}\to +\infty$. The equality $v_{sq0} = 0$ takes place at $\beta$ near to 0.974589. For the smaller $\beta$ values at any $v_s$ value (from zero to maximum possible for the given $\beta$), there is only one corresponding $q$ value, whereas for greater $\beta$ values the interval of $v_s$ values appears (near to the maximum possible $v_s$ value), in which for any $v_s$ there are two values of $q$.

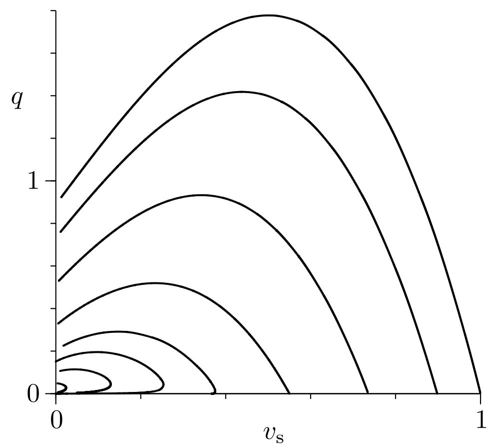


***Fig.1.*** *Dimensionless entrance current $q$ versus the value $v_s$ of flow velocity in the point of zero field strength, for different values of braking coefficient $\beta$: 0 (upper curve), 0.2, 0.5, 0.8, 1, 1.1, 1.2, 1.30685*

If $\beta > 1$ then the stationary modes with too small $q$ values are impossible, as without aid of the space charge field, under the action of the braking force only, according to the equation $v\partial_z v = -\beta v$, electron has to stop at the point $z = 1/\beta$, which is situated inside the gap (as $\beta > 1$), and such stopping contradicts to the mode stationarity. The smallest $q$ value corresponds to zero value of $v_s$. The quantity $v_s$ increases monotonously up to its maximum with increase of $q$ from its minimum value.

Stability or instability of the stationary mode is determined by negativeness or positiveness of real part of the increment $\kappa$, which is the root of the equation $D(\kappa) = 0$ with $D(\kappa)$ defined in (16). The results of study are somewhat similar to ones obtained in [1] for the case $\beta = 0$. In the Fig.1, for $\beta\in(0, 1)$, the parts of curves, which go out from the maximum to the left, and come to the line $v_s = 0$, correspond to the unstable stationary modes, and the parts of curves, which go out from the maximum to the right, and come to the line $q = 0$, correspond to the stable stationary modes. Similarly, in the case when the difference $\beta - 1$ is positive, but sufficiently small, the parts of curves, which go out from the maximum to the left, and come to the line $v_s = 0$ with the greater of the two possible values of $q$, correspond to the unstable stationary modes, and the parts of curves, which first go out from the maximum to the right, but then turn clockwise and come to the line $v_s = 0$ with the smaller of the two possible values of $q$, correspond to the stable stationary modes.

It is natural that the point of $q$ maximum on the curve of correspondence between $q$ and some characteristic of the stationary mode divides the curve on the parts related to the stable and unstable modes. For the given $\beta$, the parameter $q$ remains the unique one in the dimensionless equations, but in the considered system there are two possible stationary modes for some values of $q$ and there are characteristic quantities, which are different for the different modes. The quantity $v_s$ is one of them. The stationary distributions of $v$, $n$, and $E$ are the functions of the Euler variable $z$ and they are dependent on $v_s$ as on parameter. Even for the point of the $q$ maximum on the curve $q$ versus $v_s$, the derivatives of these functions with respect to $v_s$ are not zero ones, whereas the derivative of $q$ with respect to $v_s$ is equal to zero there. And so, the mentioned derivatives of the functions form the nonzero solution of the linear equations for the Laplace transforms of the not stationary perturbations (dependent on the Euler variable $z$), which corresponds to zero increment. That is, the point of $q$ extremum on the curve $q$ versus $v_s$ corresponds to zero increment for any $\beta$, and this point divides the curve on the parts connected with the stable and unstable stationary modes.

But at some $\beta$ value (near to 1.30685) the point of $q$ maximum (with $q$ value near to $4.87{\cdot}10^{-2}$) comes to the line $v_s = 0$, and the mentioned part of the curve, corresponding to the unstable stationary mode, disappears. In that case all one-stream modes possible for greater $\beta$ values are stable. For such $\beta$ values the curve on the plot $q$ versus $v_s$ gives two values of $q$ for $v_s = 0$, and for any $q$ between these values there is one value of $v_s$. But if $\beta$ becomes equal to some another number (near to 1.36111) then both points of the curve at the line $v_s = 0$ meet each other at the $q$ value near to $1.33{\cdot}10^{-2}$. Then one-stream modes with another $q$ values or with nonzero $v_s$ value become impossible. For the greater $\beta$ values one-stream modes are impossible at all.

At the large $q$ values the cause of impossibility of one-stream mode existence lies in too large decelerating force of the space charge field in the part of the gap nearer to entrance, as it is, in particular, in the case $\beta = 0$. At the small $q$ values and sufficiently large $\beta$, the cause of impossibility of one-stream mode existence is insufficiently large accelerating force of the space charge field in the part of the gap nearer to exit, so that such force cannot overcome the braking force and it is incapable to push all the flow through this part of the gap. In the two-stream mode only the part of input flow passes the whole gap, and the electrostatic field may be capable to push some part of input flow through the whole gap if the space charge of the rejected part of flow is large enough.

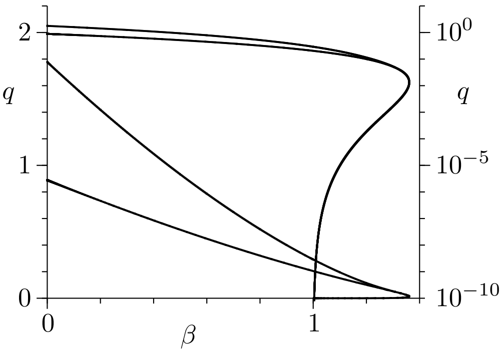


***Fig.2.*** *Boundaries of $q$ for one-stream and two-stream modes versus $\beta$ (8/9 and 16/9 at $\beta = 0$), in linear and logarithmic scales*

In the Fig.2, the dependence of critical values of $q$ (corresponding to the boundaries of one-stream and two-stream modes existence) on the $\beta$ value is shown. The upper curve is determined up to the $\beta$ value near to 1.30685. It gives one value of $q$ for any $\beta$ and its point for the maximum $\beta$ value also belongs to the the upper part of the lower curve. The lower curve is determined up to the $\beta$ value near to 1.36111. It gives one value of $q$ for $\beta < 1$ and two ones for $\beta > 1$. Between them the two-stream mode cannot exist. For $q$ values greater than ones on the upper curve or less than ones on the lower part of the lower curve the one-stream mode cannot exist. Between the upper curve and the upper part of the lower curve the one-stream mode is unstable. Really it cannot exist, and it is replaced with two-stream mode.

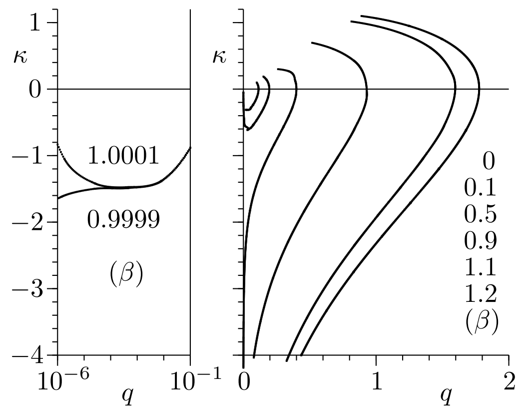


***Fig.3.*** *Increment $\kappa$ versus $q$ for the written $\beta$ values increased from the right curve to left*

In the Fig.3, the dependence of $\kappa$ on $q$ at the fixed $\beta$ values is shown for the main self-consistent perturbation. It develops monotonously and its increment $\kappa$ is real. This increment is positive for the unstable modes and negative for the stable ones. For any $\beta$

in interval $(0, 1)$, the stationary modes with small $q$ are characterized by negative $\kappa$ with great absolute values, and $\kappa \to -\infty$ when $q \to 0$. With $\beta$ increase the dependence of $\kappa$ on $q$ changes essentially when $\beta$ passes the value 1. Namely, for $\beta$ near to 1, at small $q$, in the case $\beta < 1$, the curve $\kappa$ versus $q$ goes to $-\infty$ with $q$ decrease, whereas in the case $\beta > 1$ it goes to zero.

### 4. CONCLUSIONS

The electron flow in the short-circuited diode is considered taking into account the braking force proportional to velocity. Explicit solution of the equations is obtained with the usage of Lagrange variables, that gives comparatively simple expression for electric field, in the processes, in which relative disposition of electrons in flow is not changed, electrons do not outrun one another, though the distance between them may be changed. Appearing of the braking force and increase of braking coefficient leads to changes of the input current intervals, in which stationary modes are stable or unstable. For small braking coefficient, the flows with sufficiently small input current are stable, in some interval of input current the stable and unstable one-stream modes may exist, and for sufficiently large input current one-stream modes are impossible. Also, the one-stream modes are impossible in the case when input current is very small and the braking coefficient is so large, that in absence of the electric forces electron stops inside the gap. But the possible modes with not very small input current are stable. The unstable one-stream modes disappear at some sufficiently large value of braking coefficient. And for the braking coefficients values greater then some still greater threshold value, one-stream modes are impossible et all.

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### УСТОЙЧИВОСТЬ ПОТОКА ЭЛЕКТРОНОВ В ДИОДЕ, ЗАПОЛНЕННОМ ГАЗОМ


***А. Пащенко, В. Остроушко***



Для потока электронов в короткозамкнутом диоде, заполненном газом, с учетом тормозящей силы, пропорциональной скорости, рассмотрены стационарные режимы и их линейные возмущения. Получено уравнение для инкремента увеличения амплитуд возмущений. Найдены диапазоны значений коэффициента торможения, в которых возможно существование устойчивых либо неустойчивых стационарных режимов.


### СТІЙКІСТЬ ПОТОКУ ЕЛЕКТРОНІВ У ДІОДІ, ЗАПОВНЕНОМУ ГАЗОМ


***А. Пащенко, В. Остроушко***



Для потоку електронів у короткозамкненому діоді, заповненому газом, з урахуванням гальмівної сили, пропорційної до швидкості, розглянуто стаціонарні режими та їхні лінійні збурення. Отримане рівняння для інкременту збільшення амплітуд збурень. Знайдено діапазони значень коефіцієнту гальмування, у яких можливе існування стійких або нестійких стаціонарних режимів.