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NUCLEAR REACTOR ON CYLINDRICAL STANDING BURNING WAVE WITH AN EXTERNAL NEGATIVE REACTIVITY FEEDBACK

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Neutron kinetics of nuclear burning wave in moving incompressible neutron-multiplying medium in the presence of nuclear reactions is developed. A cylindrical reactor is considered, where the burning wave moves radially from the reactor axis, and fuel moves towards the axis. It was shown that when feeding such a system with ^{238}U standing wave of nuclear burning might exist. A comparison of theoretical results with simulation data is made using MCNPX code.

INTRODUCTION

Concept of the traveling wave nuclear reactor (TWR) is one of the brilliant ideas of 20-th century. It suggests using depleted uranium (or thorium) as fuel and promises to supply inexhaustible source of energy worldwide. This idea was proposed by S.M. Feinberg, realized theoretically by L.P. Feoktistov [2] and developed in many publications (see bibliography in [3]), in which several ways of its practical implementation were suggested. One of the most promising designs (TWR) is a fast reactor with negative reactivity feedback, which is able to work in maneuverable mode. Mathematical modeling of such a reactor was made in [4] using MCNPX code.

In the most of published works, various cases of linear traveling burning waves in stationary neutron-multiplying media were considered. However, the practical implementation of this idea assumed to be in the form of standing waves [5] in the reactor of the cylindrical symmetry with periodic shuffling fuel elements [6]. This requires to develop the theory of nuclear burning wave in moving medium. In this article, the theory of nuclear reactor on cylindrical standing burning wave is developed. Computer simulation of cylindrical standing burning wave in slow moving neutron-multiplying medium is performed. Possibility of using depleted uranium as a nuclear fuel in reactors with the cylindrical geometry is shown.

This article develops neutron kinetics of the burning wave moving in incompressible neutron-multiplying media, taking into account nuclear reactions with fissile isotopes A cylindrical reactor was considered where the burning wave moves radially from the reactor axis, and fuel moves towards the axis. It's shown that when such system is fed isotope ^{238}U from the periphery, a standing burning wave may exist. Non-fissile isotope ^{238}U is converted to fissionable ^{239}Pu in such wave. Mathematical simulation of such a reactor was carried out using MCNPX code [1], and comparison of theoretical results simulation data was made.

1. NEUTRON KINETICS EQUATION IN MOVING NEUTRON-MULTIPLYING MEDIUM

Let us consider the process of nuclear burning in incompressible uranium-base medium, which moves with constant velocity v along the *x*-axis of fixed reference, *y*, *z*, in which the wave profile and reactor control organs are stationary.

The simplest description of neutron kinetics and burnout of nuclear fuel can be obtained using the coordinate system *x '*, *y '*, *z '*, in which the fuel is stationary:

$$\frac{1}{\mathrm{v}}\frac{\partial \Phi}{\partial t} = \hat{D}\Phi + (\nu\Sigma_f - \Sigma_a)\Phi + S\,, \qquad (1)$$

$$\frac{\partial n_8}{\partial t} = -n_8\sigma_{a8}\Phi\,, \qquad \frac{\partial \tilde{n}_9}{\partial t} = \sigma_{89}\, n_8\Phi - \frac{\tilde{n}_9}{\tau_{89}}\,,$$

$$\frac{\partial n_9}{\partial t} = \frac{\tilde{n}_9}{\tau_{89}} - \sigma_{a9}\, n_9\Phi\,, \quad \frac{\partial n_c}{\partial t} = 2\sigma_{f9}\, n_9\Phi\,, \qquad (2)$$

where $\Phi(x', t)$ is neutron flow; v is the speed of neutrons; $n_8(x', t)$ is the concentration of ^{238}U; $\tilde{n}_9(x', t)$ is the concentration of ^{239}Np; $n_9(x', t)$ is the concentration of ^{239}Pu; $n_c(x', t)$ is the concentration of fission products; $\hat{D}$ – neutron transport operator; $\Sigma_f = \sigma_{f9}\, n_9$ – macroscopic cross-section of fission and $\Sigma_a = \sigma_{a8}n_8 + \sigma_{a9}n_9 + \sigma_c n_c$ – macroscopic neutron absorption cross section; *S* – term describing the feedbacks and reactor control organs; σ_{89} – transmutation cross-section of ^{238}U to ^{239}Pu; τ_{89} – time of the decay in chain $^{239}U \rightarrow {}^{239}Np \rightarrow {}^{239}Pu$; σ_{f9} – fission cross-section of ^{239}Pu; ν – the number of fission neutrons; σ_{a8} and σ_{a9} – neutron absorption cross-sections for nuclei ^{238}U and ^{239}Pu; σ_c is neutron absorption cross-section for fission products; *Q* – nuclei ^{239}Pu fission energy release.

Boundary conditions have to be added to Eqs. (1) and (2):

$\Phi(\infty, t) = 0$, $\Phi(-\infty, t) = 0$, $n_9(\infty, t) = 0$, $\tilde{n}_9(\infty, t) = 0$,

$n_c(\infty, t) = 0$, $n_8(\infty, t) = n_8(0)$. (3)

Now, instead of the x coordinate let us introduce new variable $\varphi(x) = \frac{\sigma_a}{V}\int_x^\infty \Phi(x')dx'$, which is proportional to fluency $F(x)$ and ranges from 0 to a maximum value of $\chi = \sigma_a F_{\max}$. Let us choose the function $S(x)$ describing the negative feedback on reactivity ρ as: $S = -\rho\nu\Sigma_f\Psi$. After these changes Eqs. (1) and (2) become:

$$\frac{D\sigma_a^2}{2V^2}\frac{d^2\Psi^2}{d\phi^2} + \nu(1-\rho)\Sigma_f - \Sigma_a = 0, \quad (4)$$

$$\frac{dn_8}{d\varphi} = -n_8, \quad \frac{dn_9}{d\varphi} = n_8\sigma_{89}/\sigma_a - n_9,$$

$$\frac{dn_c}{d\varphi} = 2n_9\sigma_f/\sigma_a, \quad P = AVQ/\sigma_a\int\Sigma_f d\phi. \quad (5)$$

$$\frac{D\sigma_a}{2n_0V^2}\frac{d^2\Psi^2}{d\varphi^2} = 2\beta + (1-2\beta)e^{-\varphi} + \left(\frac{\sigma_{89}}{\sigma_a} - 2\beta\right)\varphi e^{-\varphi} - \frac{\sigma_f\sigma_{89}}{\sigma_a^2}\nu(1-\rho)\varphi e^{-\varphi}, \quad (6)$$

where $\beta = \frac{\sigma_c\sigma_f\sigma_{89}}{\sigma_a^3}$ – a characteristic parameter of the system.

Solving Eq. (6) we obtain an expression for the density of a neutron flux $\Psi(\varphi)$:

$$\Psi^2(\phi) = \frac{2n_0V^2}{D\sigma_a}[\beta\phi^2 + (1-2\beta-q)\phi +$$

$$+(1-2\beta-2q)(e^{-\phi}-1) - qe^{-\phi}\phi]. \quad (6a)$$

Finally, we have an implicit function of fluency $\varphi(x)$ on x:

$$\int_C^\varphi \frac{d\varphi'}{\sqrt{f(\varphi')}} = \sqrt{\frac{2\sigma_a n_0}{D}}x. \quad (6b)$$

Constant of integration C determines the position of the wave maximum in space.

Spatial profiles of standing burning wave $\Psi(x)$ for several values of χ are shown in Fig. 1.

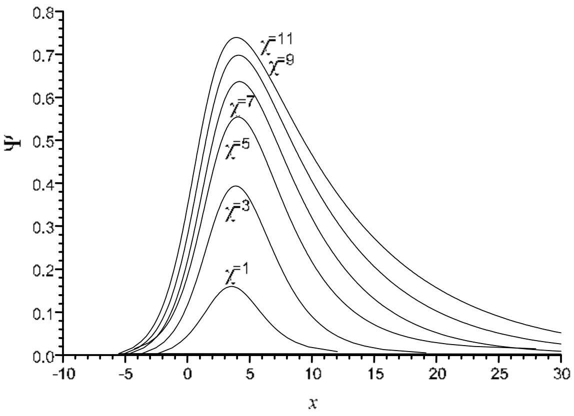

Fig. 1. Function of neutron flux $\Psi(x)$ for several values of χ

The boundary conditions (3) for the functions $\Psi(\varphi)$, $n_8(\varphi)$; $\tilde{n}_9(\varphi)$; $n_9(\varphi)$ will look as follows:

$\Psi(0) = 0$, $\Psi(\chi) = 0$, $n_9(0) = 0$, $n_c(0) = 0$, $n_8(0) = n_0$

and Eq. (5) takes the following form:

2. REACTOR ON STANDING WAVE IN AN INFINITE CYLINDRICAL BURNING MEDIUM

Let us consider a cylindrical reactor in which nuclear burning spreads radially from the axis. ^{238}U fuel (which we consider incompressible) moves continuously from infinity toward the axis, and material is removed from the system near the axis point. Speed of movement C depends on the radius according to the law: $V(r) = V_R R/r$, where V_R is fuel movement speed at distance R. Neutron kinetics in moving media can be described by Eqs. (1) and (2) in which $\frac{\partial}{\partial t}$ should be replaced by $\frac{\partial}{\partial t} - V_R\frac{R}{r}\frac{\partial}{\partial r}$ with the following boundary conditions:

$\Psi'(0, t) = 0$, $\Psi(\infty, t) = 0$, $n_9(\infty, t) = 0$,

$\tilde{n}_9(\infty, t) = 0$, $n_c(\infty, t) = 0$, $n_8(\infty, t) = n_0$. (7)

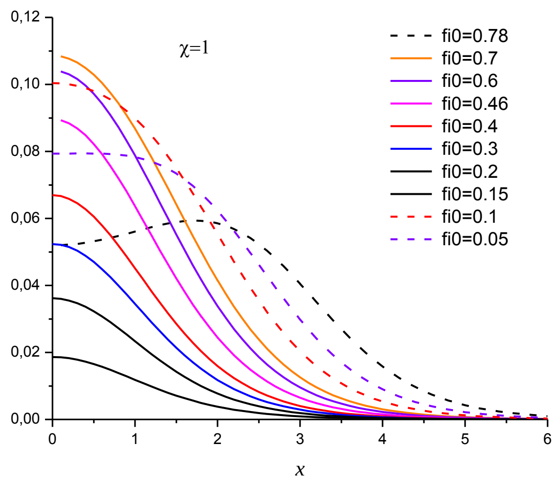

Fig. 2. Function of $\Psi(r)$ for a series of values φ_0

We will look for the stationary solution of Eqs. (1) and (2):

$$-\frac{V(r)}{\text{v}}\frac{\partial\Psi}{\partial r} = -\frac{D}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\Psi}{\partial r}\right) + (\nu\Sigma_f - \Sigma_a)\Psi + S, \quad (8)$$

$$V(r)\frac{\partial n_8}{\partial r} = -n_8\sigma_a\Psi; \quad V(r)\frac{\partial n_c}{\partial r} = 2\sigma_{f9}n_9\Psi,$$

$$V(r)\frac{\partial n_9}{\partial r} = (\sigma_{89}n_8 - \sigma_a n_9)\Psi. \quad (9)$$

Instead of the r-coordinate, we enter variable $\varphi(r)=\sigma_a\int_r^\infty \frac{\Psi(r')}{V(r')}dr'=\frac{\sigma_a}{V_R R}\int_r^\infty \Psi(r')r'dr'$ then Eq. (8) becomes:

$$\frac{D\sigma_a}{2n_0V_R^2}\frac{d}{d\phi}\left(\frac{r^2(\phi)}{R^2}\frac{d\Psi^2}{d\phi}\right)=2\beta+(1-2\beta)e^{-\phi}+$$
$$\left(\frac{\sigma_{89}}{\sigma_a}-2\beta\right)\phi e^{-\phi}-\frac{\sigma_f\sigma_{89}}{\sigma_a^2}\nu(1-\rho)\phi e^{-\phi}. \qquad (10)$$

Boundary conditions (7) for the functions $\Psi(\varphi)$, $n_8(\varphi)$, $n_9(\varphi)$ become:

$\Psi'(0)=0$, $\Psi(\infty)=0$, $\varphi(0)=\varphi_0$, $\varphi(\infty)=0$,

$n_9(0)=0$, $n_c(0)=0$, $n_8(0)=n_0$, (11)

where $\varphi_0(r)=\frac{\sigma_a}{V_R R}\int_0^\infty \Psi(r')r'dr'$ – is the maximum neutron fluency.

Solving Eq. (10) with boundary condition (11) gives the needed function $\varphi(r)$.

Solving Eq. (10) with boundary condition (11) gives us the needed function. Fig. 2 shows radial profiles of standing burning waves for $\chi=1$ and series of values of φ_0 ranging from 0.05 to 0.65.

3. CALCULATION THE SYSTEM EXCESS REACTIVITY

Integrating Eq. (10) by φ, we get:

$$\frac{D\sigma_a}{2n_0V_R^2}\frac{r^2(\phi)}{R^2}\frac{d\Psi^2}{d\phi}=2\beta\phi-(1-2\beta)(e^{-\phi}-1)+$$
$$+\left[\frac{\sigma_{89}}{\sigma_a}-2\beta-\frac{\sigma_f\sigma_{89}}{\sigma_a^2}\nu(1-\rho)\right][1-(1+\phi)e^{-\phi}]. \qquad (12)$$

Substituting φ for φ_0 in Eq. (10) and given that $\Psi'(0)=0$, we obtain expressions for the excess reactivity ρ which need to be compensated using external negative feedback:

$$\rho=\frac{\sigma_a^2}{\sigma_f\sigma_{89}\nu}(c-q_w), \qquad (13)$$

$$q_w=\frac{2\beta\phi_0-(1-2\beta)(e^{-\phi_0}-1)}{1-(1+\phi_0)e^{-\phi_0}}. \qquad (14)$$

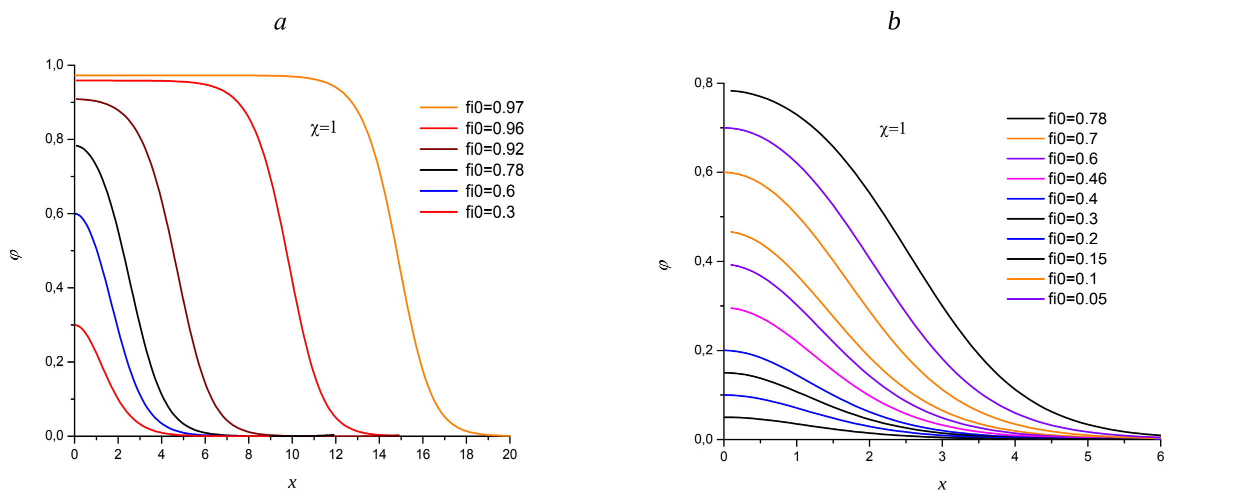

Fig. 3. Radial profiles of standing burning waves at $\chi=1$. Functions of $\varphi(r)$ for $\varphi_0=0.3\ldots0.97$ (a) and $\varphi_0=0.05\ldots0.78$ (b)

Simplifying Eq. (12) we get

$$\frac{D\sigma_a}{2n_0V_R^2}\frac{r^2(\phi)}{R^2}\frac{d\Psi^2}{d\phi}=f_1(\phi), \qquad (15)$$

where

$f_1(\varphi)=2\beta\varphi-(1-2\beta-q_w)(e^{-\varphi}-1)+q_w\varphi e^{-\varphi}$,

$f_1(\varphi_0)=0$ and $f_1(0)=0$.

Solving Eq. (15) numerically, we find radial profiles of neutron fluence (Fig. 3) and profiles of neutron flux in standing burning wave for $\chi=1$ (Fig. 4).

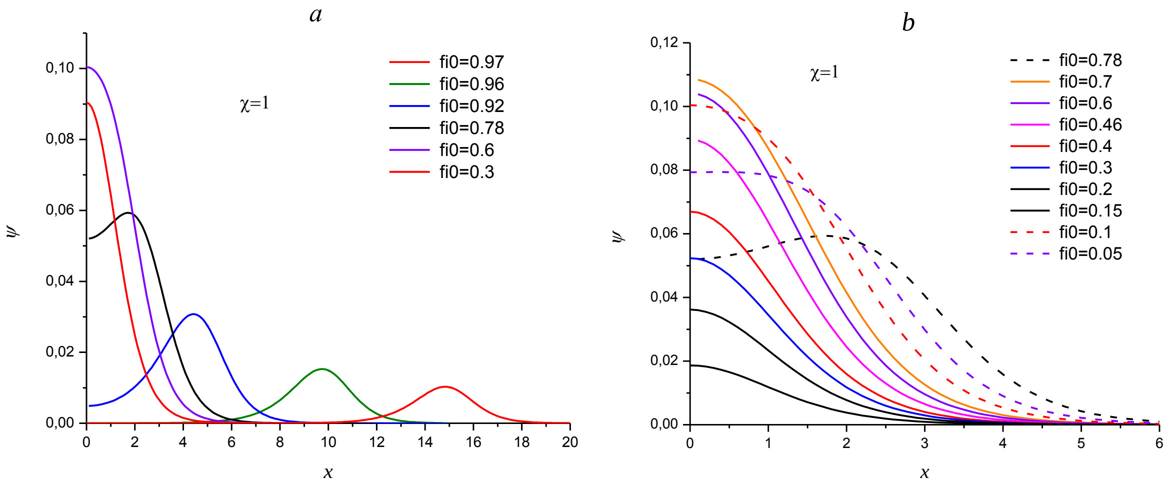

Fig. 4. Radial profiles of standing burning waves for $\chi = 1$.
Functions of $\Psi(r)$ for φ_0= 0.3….0.97 (a) and φ_0= 0.05 … 0.78 (b)

Knowing functions $\varphi(r)$ and $\Psi(r)$, we find concentration profiles for plutonium $n_9(r) = \sigma_{89} / \sigma_a n_0\ \varphi(r)\, e^{-\varphi(r)}$, uranium $n_8(r) = n_0 e^{-\varphi(r)}$ and the fission products $n_c(r) = \sigma_f \sigma_{89} / \sigma_a^2 n_0 [1-(1+\varphi(r))e^{-\varphi(r)}]$. Also we get the expression for the power:

$$P = H\int Q\Sigma_f \Phi dA = 2\pi\, HQV_R R n_0 \sigma_{89}\sigma_f \,/\, \sigma_a^2 (1-e^{-\phi_0} - \phi_0 e^{-\phi_0}), \tag{16}$$

where H is the height of the reactor and φ_0 is thefluence in fuel in the center of reactor core. Fig. 5 shows radial dependence $r\Psi(r)$ on radius of standing waves. It can be noted that at large distances the wave amplitude decreases according to $1/r$, while the width of the wave is independent of r.

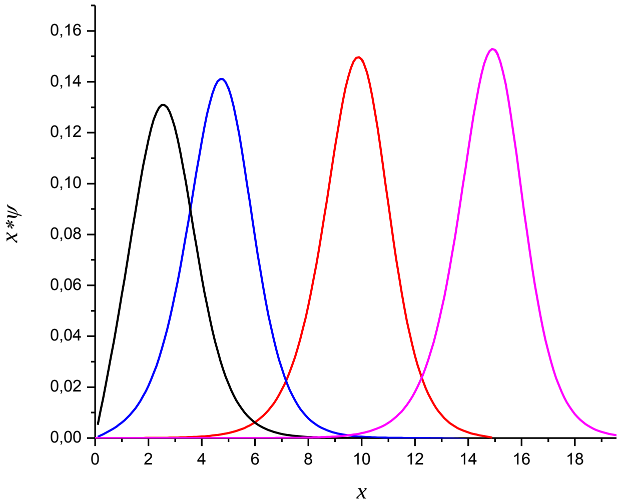

Fig. 5. Radial dependence $r\Psi(r)$ on radius of standing burning waves at $\chi = 1$ for φ_0 = 0.78, 0.92, 0.93, and 0.97

It follows that the total power of the wave does not depend on its location. Condition for the existence of the standing cylindrical burning waves is determined by the availability of excess reactivity $\rho > 0$ and is expressed by the ratio $c \geq q_w$. As a result we have a state diagram of the standing burning wave nuclear reactor (Fig. 6).

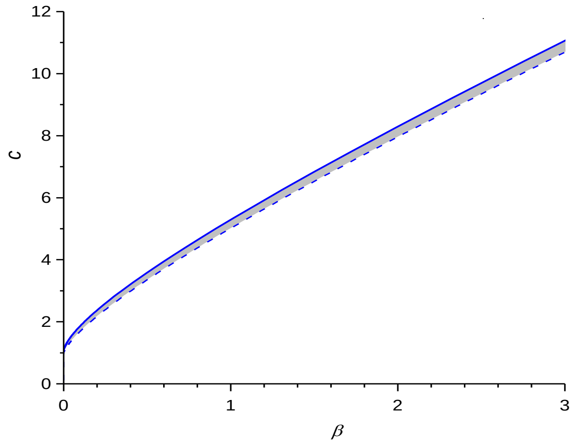

Fig. 6. State diagram ofstanding burning wavereactor, where the solid line represents the lower bound of the sustainability of standing wave $q(\beta)$, and the dotted line shows lowestboundary of stability of coupled standing waves $c_{mm}(\beta)$

4. COMPUTER SIMULATION OF CYLINDRICAL STANDING WAVE REACTOR

Computer model of "Traveling Wave Reactor" is a cylinder of 20 cm height and the radius of 2 m, filled with uranium dioxide based fuel. At the ends of the cylinder mirror boundary conditions were chosen and lateral surface was considered open.

In the beginning, conditions for existence of the cylindrical burning wave were achieved. In the center pane of the active zone theignitorwas located containing enriched uranium, and the system was set to reachcritical mode. Due to fast neutron irradiation ^{238}U isotope becomes ^{239}Pu:^{238}U + n = $^{239}U \rightarrow {}^{239}Np \rightarrow {}^{239}Pu$. When concentration of ^{239}Pu in the fuel reaches high level, cylindrical burning wave appears; it breaks away from the vent area and continues to move to the edges of the active zone for 60 years. In this model the burning wave speed is about 1 cm/year at 250 MW power. Fig. 7 illustrates the movement of the traveling burning wave.

Computer simulation of the standing nuclear burning wave was carried out in the following manner: the reactor was divided by systems of coaxial cylinders into circular sections of the same thickness (5 cm); at regular time intervals (cycle of burnout – 100 days) the spent fuel was removed from the central section. The removed fuel was replaced with the fuel from the following section in the same amount. The profile of wave and fuel movement speed were obtained from calculations of traveling wave.

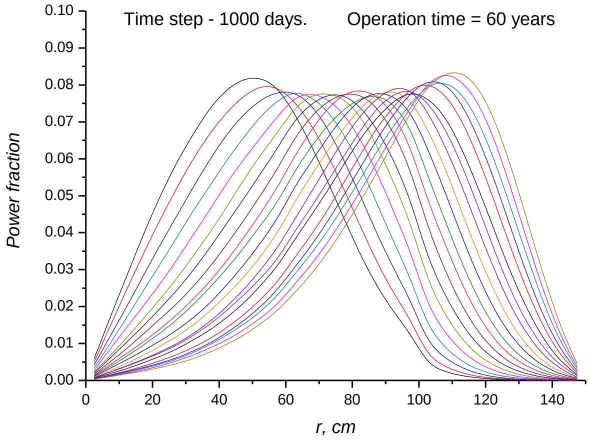

Fig. 7. Travelling-wave profile in the set mode

Using MCNPX code,the fuel burnout calculations were made with fuel shuffle intervalof 100 days. With the correctselection of reactor powerlevel the burning wave profile should remain unchanged.

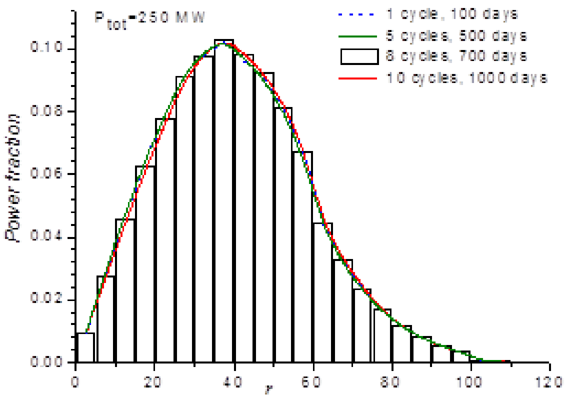

Fig. 8. Standing wave profiles for different points in time

Fig. 8 shows the burning wave profiles through 100, 500, 700, and 1000 days of work. Matching profiles in Fig. 8 indicate that in this computer simulation the standing wave mode was achieved.

In a reactor, one can achieve a continuous set of standing burning wave profiles that are different in widths (Fig. 9).

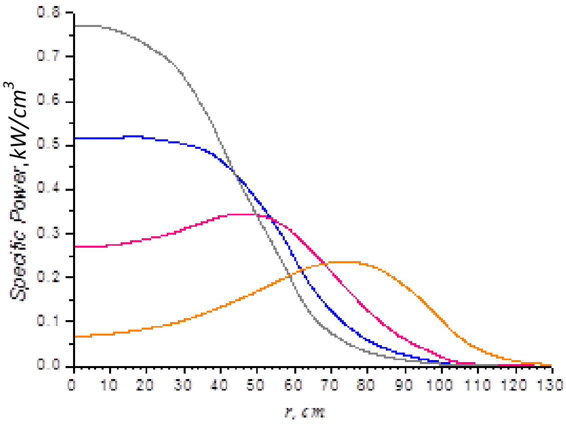

Fig. 9. Profiles of standing waves under different initial conditions

Comparison of Figs. 2 and 9 shows qualitative agreement of simulation data with the theoretical results.

Concentration profiles ^{239}Pu and ^{238}U in the standing waveare shown in Figs. 10 and 11. It follows from the provided data that depleted uranium loaded to the reactor can burn out to 60% while the discharged fuel still contains 6% of the unburned isotope ^{239}Pu.

Computer simulation of the active zone of UO_2, containing coolant (8% Na) and construction materials (9% Fe) was performed. Fig. 12 shows that the standing burning wave can exist in the active zone with the specified amount of sodium and iron.

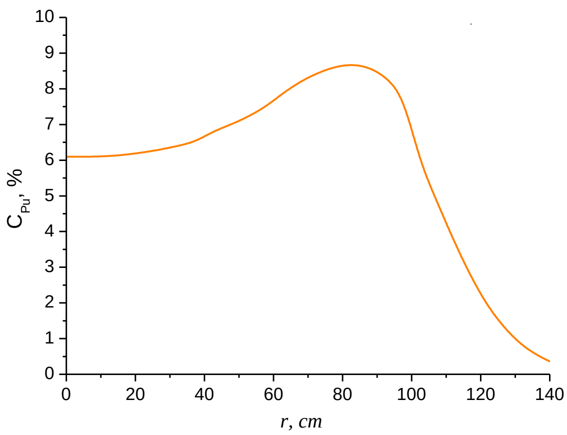

Fig. 10. Profile concentrations of plutonium

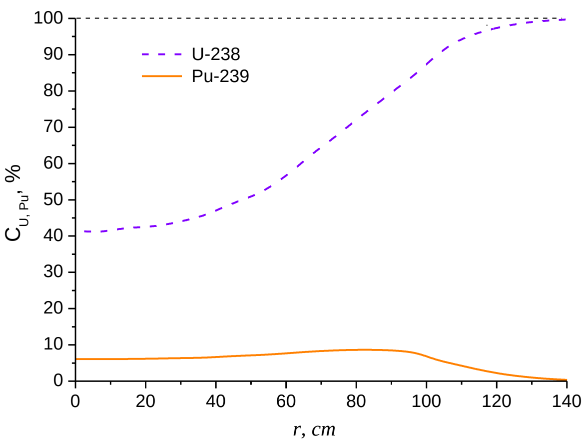

Fig. 11. Profiles of concentrations of uranium and plutonium

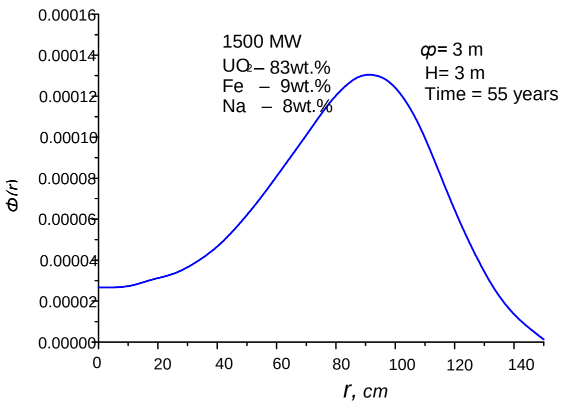

Fig. 12. Profiles of neutron flux

CONCLUSIONS

• Nuclear burning wave can exist not only in one-dimensional geometry, but in systems with cylindrical and spherical symmetries as well. We considered cylindrical reactor in which nuclear burning wave spreads radially from the axis.

• Phenomenological theory of cylindrical nuclear burning wave was developed.

• Possibility of the existence of a standing cylindrical nuclear burning wave was shown in a reactor where ^{238}U fuel is continuously entering the peripheral region, it is then moved toward the axis where is removed.

• State diagram of such a reactor was drawn and the boundaries of the sustainability of its work were defined. Simple formulas for neutron flux, power density and concentration of uranium and plutonium in the standing burning wave were obtained.

• Standing wave nuclear burning reactor has its advantages in comparison with a thermal-neutron reactor. It uses non-fissile isotope of ^{238}U as its fuel; standing wave has a small radiation load on the vessel of the reactor (see Fig. 12); spent fuel can be reused in thermal-neutron reactors.

• Exploitation term is not limited by the time of wave propagation; the reactor can operate in variable mode [3].

• Mathematical modeling of reactor on cylindrical standing burning wave was carried out. Analytical results are in agreement with the numerical results obtained using MCNPX code.

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РЕАКТОР НА ЦИЛИНДРИЧЕСКОЙ СТОЯЧЕЙ ВОЛНЕ ЯДЕРНОГО ГОРЕНИЯ С ВНЕШНЕЙ НЕГАТИВНОЙ ОБРАТНОЙ СВЯЗЬЮ ПО РЕАКТИВНОСТИ

Ю.Я. Лелеко, В.В. Ганн, А.В. Ганн

Развита нейтронная кинетика волны ядерного горения в движущейся несжимаемой нейтроно-размножающей среде при наличии ядерных реакций. Рассмотрен цилиндрический реактор, в котором волна ядерного горения двигается радиально от оси, а топливо – к оси реактора. Показано, что при подпитке такой системы ^{238}U в ней может существовать стоячая волна ядерного горения. Проведено сопоставление теоретических результатов с данными численного моделирования такого реактора с использованием кода MCNPX.

РЕАКТОР НА ЦИЛІНДРИЧНІЙ СТОЯЧІЙ ХВИЛІ ЯДЕРНОГО ГОРІННЯ З ЗОВНІШНІМ НЕГАТИВНИМ ЗВОРОТНІМ ЗВ'ЯЗКОМ ЗА РЕАКТИВНОСТЮ

Ю.Я. Лелеко, В.В. Ганн, А.В. Ганн

Розвинена нейтронна кінетика хвилі ядерного горіння в нейтронорозмножуючому середовищі, котре не стискається та є рухомим при наявності ядерних реакцій. Було розглянуто циліндричний реактор, в якому хвиля ядерного горіння рухається радіально від осі, а паливо – до осі реактора. Показано, що при підживленні такої системи ^{238}U у ній може існувати стояча хвиля ядерного горіння. Проведено порівняння теоретичних результатів з даними чисельного моделювання такого реактора з використанням коду MCNPX.