

## Cascade model of X-ray $M$ emission for the atoms of heavy metals

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Cascade model of X-ray  $M$  emission for the atoms of heavy metals is proposed that allows one to take into account the main channels of vacancy transfer from  $L$  to  $M$  electronic subshells, which are responsible for the generation of double-vacancy ( $M_{4,5}N$  and  $M_{4,5}O$ ) and triple-vacancy ( $M_{4,5}N^2$ ,  $M_{4,5}NO$ , and  $M_{4,5}O^2$ ) states. The model allows to separately calculate the contributions of  $L$  and  $M$  subshells in the emission cross sections of  $M_5N$  and  $M_4N$  satellites, as well as  $M\alpha_{1,2}$  lines along with  $M_5O$  satellites and  $M\beta$  line with  $M_4O$  satellites. Comparison of relative emission cross sections of the main components of  $M\alpha$  and  $M\beta$  atomic spectra of Au with the experimental relative intensities of these components, excited by  $K\alpha_{1,2}$  radiation of Cu and Mo, indicates the correctness of the cascade model used.

Предложена каскадная модель рентгеновской  $M$  эмиссии для атомов тяжелых металлов, позволяющая учесть основные каналы переноса вакансий из  $L$  в  $M$  электронные подоболочки, в результате которых генерируются двухвакансионные ( $M_{4,5}N$  и  $M_{4,5}O$ ) и трехвакансионные ( $M_{4,5}N^2$ ,  $M_{4,5}NO$  и  $M_{4,5}O^2$ ) состояния. Модель позволяет отдельно рассчитать вклады  $L$  и  $M$  подоболочек в сечения излучения  $M_5N$  и  $M_4N$  спутников, а также  $M\alpha_{1,2}$  линий совместно с  $M_5O$  спутниками и  $M\beta$  линии с  $M_4O$  спутниками. Сравнение относительных сечений излучения основных компонентов  $M\alpha$  и  $M\beta$  спектров атомов Au с экспериментальными относительными интенсивностями этих компонент при возбуждении  $K\alpha_{1,2}$  излучением Cu и Mo свидетельствует о корректности использованной каскадной модели.

### 1. Introduction

Thin films of heavy metals, in particular, Au, are widely used in modern microelectronic devices. Particular important for these films is the control of the films physical state which is mainly defined by the state of electronic subsystem of such structures. Among effective diagnostic methods of the state of electronic subsystem of the metal films, X-ray emission spectroscopy methods of  $K$  and  $L$  bands are worth to be noted. These methods allow to obtain data of electronic states density in particular, near the Fermi level [1–3]. However, X-ray  $M$  spectroscopy was not used for in-

vestigation of thin films of heavy metals. At the same time, in order to develop the new materials for protection from a radiation as well as to develop X-ray emitters with certain properties it should properly take into account the X-ray of  $M$  series, which in the case of elements with an atomic number  $Z > 70$  is already enough intensive.

The X-ray  $M$  emission spectrum contains three most intense groups of lines emitted upon radiative filling of the initial  $M_5$ ,  $M_4$ , and  $M_3$  vacancies by electrons from  $N$  subshells, i.e.,  $M\alpha_{1,2}$ ,  $M\beta$ , and  $M\gamma$  lines, respectively. It is important to note that, for heavy elements with  $Z > 70$ , the diagram

lines  $M\alpha_{1,2}$  (the  $M_5-N_{6,7}$  transitions),  $M\beta$  ( $M_4-N_6$ ), and  $M\gamma$  ( $M_3-N_5$ ) are accompanied by the groups of rather intense high energy  $M_{4,5}N$  and  $M_{4,5}O$  satellites related to the listed one-electron transitions in the presence of one or several additional vacancies in  $N$  or  $O$  subshells [4]. The main mechanism by which these double-vacancy ( $M_iN$  and  $M_iO$ ) and triple-vacancy ( $M_iN^2$ ,  $M_iNO$ , and  $M_iO^2$ ) states ( $i = 3-5$ ) are generated is related to Coster-Kronig (CK) transitions  $M_j-M_iN$  and  $M_j-M_iO$  ( $i > j$ ) because their yields for  $M$  subshells of atoms with  $Z > 70$  are significant and their sums for each of  $M_1$ ,  $M_2$  and  $M_3$  subshell of, e.g., Au, reach 0.93, 0.90, and 0.83, respectively [5]. In a number of studies, such relative integrated intensities of  $M\alpha_{1,2}$ ,  $M\beta$ , and  $M\gamma$  lines of the heavy elements were measured experimentally under various excitation conditions, for example, at photoabsorption (elements studied are Au, Pb, Th, U [5], and Bi [6]), bombardment by  $\alpha$ -particles (Hf-Th) [7, 8], electron (Au, Bi) [9] and proton impacts (Hg, Th, U) [10, 11], and bombardment by ions of F, C, Li (Au) [12]. The obtained values of relative intensities were usually compared with calculated emission cross sections of  $M\alpha_{1,2}$ ,  $M\beta$ , and  $M\gamma$  lines, in which the total yields of the CK transitions and their cascades were taken into account, but the contributions of the CK transitions  $M_j-M_iN$  and  $M_j-M_iO$  were not separated. In addition, the relative intensities of  $M_{4,5}N$  and  $M_{4,5}O$  satellites in these papers are not separately determined. In its turn, the use of relative intensities of separated high energy satellites allows one to extend the number of equations describing the intensity of X-ray  $M$  emission, thus increasing the informative value of such a description. Similarly, in the case of spectra of  $L$  series of metals W, Re, Os, Ir, Pt, the consideration of the relative intensities of separated  $L_3M$  satellites of  $L\alpha_{1,2}$  and  $L\beta_{2,15}$  lines has allowed us to determine the partial width of  $L_1$  level connected with the CK transitions  $L_1-L_3M_5$  [13].

It should be noted that the proposed models of X-ray  $M$  emission [14, 15] are restricted to the absorbed photon energies below the ionization threshold of  $L$  subshells, which corresponds to the allowance for the generation and migration of vacancies only in  $M$  subshells. At the same time, a calculation of the emission cross sections of the high energy satellites of  $M$  series at the energies of ionizing particles (photons,

electrons, and ions) above the ionization threshold of  $L$  subshells would be useful for investigating both X-ray emission and ionization cross sections of  $M$  subshells of the heavy elements in a rather wide energy range of ionizing particles.

Therefore in the present paper the cascade model of X-ray  $M$  emission which takes into account the basic channels of migration of vacancies from  $L$  to  $M$  electronic subshells has been proposed that allows calculating the emission cross sections of the main components of the  $M\alpha$  and  $M\beta$  atomic spectra of the heavy metals with  $Z > 70$ .

## 2. The equations describing X-ray $M$ emission

We assume that the energy of a projectile is sufficient for ionization of all  $M$  and  $L$  subshells and consider the possible processes resulted in formation of the states which are initial for the group of high energy  $M_5N$  satellites separated from the  $M\alpha$  spectrum. In addition to two-vacancy  $M_5N_i$  states, we will take into account three-vacancy  $M_5N_iN_j$  and  $M_5N_iO_j$  states, since their radiative decay also gives rise to the high energy satellites. Let the primary vacancy appears in  $M_1$  subshell. Then the following processes are possible: (a) CK transitions  $M_1-M_5N_i$ , whose yield is  $f_{15N}$ ; (b) cascades of two consecutive CK transitions  $M_1-M_jN_i$ ,  $M_j-M_5N_k$  ( $j = 2-4$ ), the yield is  $f_{1jN}f_{j5N}$ ; (c) cascades of two consecutive CK transitions  $M_1-M_jN_i$ ,  $M_j-M_5O_k$  ( $j = 2-4$ ), the yield is  $f_{1jN}N f_{j5O}$ , and  $M_1-M_jO_i$ ,  $M_j-M_5N_k$  ( $j = 2-4$ ), the yield is  $f_{1jO} f_{j5N}$ ; (d) cascades of CK transitions  $M_1-M_jN_i$  with the following radiative transitions  $M_j-M_5$  (the yield  $\omega_{j5}$ ), cascades yield is  $f_{1jN} \omega_{j5}$ ;

(e) cascades of radiative transitions  $M_1-M_j$  with the following CK transitions  $M_j-M_5N_i$ , the yield is  $\omega_{1j} f_{j5N}$ ; (f) the shake-off (SO) process with additional ionization of the one of  $N_i$  subshells (the probability  $P_{M1}^{(N)}$ ) and the following CK transitions  $M_1-M_5N_i$ ,  $M_1-M_5O_i$  (the yield of the last ones  $f_{15O}$ ), and radiative transition  $M_1-M_5$ , cascades yields are  $P_{M1}^{(N)}f_{15N}$ ,  $P_{M1}^{(N)} f_{15O}$ , and  $P_{M1}^{(N)} \omega_{15}$ , respectively; (g) SO process with additional ionization of the one of  $O_i$  subshells (the probability  $P_{M1}^{(O)}$ ) and the following CK transitions  $M_1-M_5N_i$ , the yield is  $P_{M1}^{(O)} f_{15N}$ . The total yield of the processes, as a result of which a vacancy from  $M_1$  subshell migrates into  $M_5$  subshell with for-

mation of the additional vacancies  $N$ ,  $N^2$ , and  $NO$ , is following

$$\begin{aligned}
F_{M1} = & f_{15N} + f_{12N}(f_{25O} + f_{25N} + \omega_{25}) + \\
& + f_{13N}(f_{35O} + f_{35N} + \omega_{35}) + \\
& + f_{14N}(f_{45O} + f_{45N} + \omega_{45}) + \\
& + f_{12O}f_{25N} + f_{13O}f_{35N} + f_{14O}f_{45N} + \\
& + \omega_{12}f_{25N} + \omega_{13}f_{35N} + \omega_{14}f_{45N} + \\
& + P_{M1}^{(N)}(\omega_{15} + f_{15N} + f_{15O}) + P_{M1}^{(O)}f_{15N}.
\end{aligned} \quad (1)$$

Analogous equations can also be written for the yields of the processes of vacancy migration from  $M_2$ ,  $M_3$ , and  $M_4$  subshells to  $M_5$  subshell ( $F_{Mi}$ ). In the case of the direct ionization of  $M_5$  subshell,  $M_5N$  satellites can be formed only due to the SO process with emission of  $N$  electron (the probability  $P_{M5}^{(N)}$ ).

Then, in  $M$  emission model, it is necessary to take into account the processes of vacancy transfer from  $L$  to the  $M$  subshells. In particular, under the absorbed photon energies  $E = 17.479$  keV (Mo  $K\alpha_{1,2}$  lines) the role of these processes can be significant because, in this case, the photoionization cross sections of  $L$  subshells ( $\sigma_{Li}$ ) are 5–10-fold larger than that of  $M$  subshells ( $\sigma_{Mi}$ ) (for example, for Au under this photon energy, we have  $\sigma_{M1} = 1401$  b,  $\sigma_{M5} = 877$  b,  $\sigma_{L1} = 5712$  b, and  $\sigma_{L3} = 12958$  b [16]). Let the primary vacancy appears in  $L_1$  subshell. Then, the creation of the initial states for  $M_5N$  satellites can occur via the following channels: (a) Auger transitions  $L_1-M_5N$ , whose yield is  $a_{L1M5N}$ ; (b) cascades of initial Auger transitions  $L_1-M_iN$ , subsequent CK transitions  $M_i-M_5N$ ,  $M_i-M_5O$ , and radiative transitions  $M_i-M_5$  ( $i = 1-4$ ), the yield is  $a_{L1MiN}(f_{i5O} + f_{i5N} + \omega_{Mi5})$ ; (c) cascades of the initial Auger transitions  $L_1-M_iM_5$ , subsequent Auger transitions  $M_i-NN$ ,  $M_i-NO$ , and radiative transitions  $M_i-N$  ( $i = 1-5$ ), the yield is  $a_{L1MiM5}(a_{MiNO} + a_{MiN} + \omega_{MiN})$ ; (d) cascades of the initial Auger transitions  $L_1-M_iO$  and subsequent CK transitions  $M_i-M_5N$  ( $i = 1-4$ ), the yield is  $a_{L1MiO}f_{i5N}$ ; (e) cascades of initial radiative transitions  $L_1-M_i$  and subsequent CK transitions  $M_i-M_5N$  ( $i = 1-4$ ), the yield is  $\omega_{L1Mi}f_{i5N}$ ; (f) the SO process  $L_1-L_1N$  (the probability  $P_{L1}^{(N)}$ ) with subsequent Auger transitions  $L_1-M_5N$ ,  $L_1-M_5O$  and radiative transitions  $L_1-M_5$ , the yield is  $P_{L1}^{(N)}(\omega_{L1M5} + a_{L1M5O} + a_{L1M5N})$ ; (g) SO process  $L_1-L_1M_5$  (the probability  $P_{L1}^{(M5)}$ ) with the sub-

sequent Auger transitions  $L_1-N_iN_j$ ,  $L_1-N_iO_k$  and radiative transitions  $L_1-N$ , the yield is  $P_{L1}^{(M5)}(\omega_{L1N} + a_{L1NO} + a_{L1NN})$ ; (h) SO process  $L_1-L_1O$  (the probability  $P_{L1}^{(O)}$ ) with the subsequent Auger transitions  $L_1-M_5N$ , the yield is  $P_{L1}^{(O)}a_{L1M5N}$ ; (i) cascades of the initial CK transitions  $L_1-L_3M_5$ , subsequent Auger transitions  $L_3-NN$ ,  $L_3-NO$ , and radiative transitions  $L_3-N$ , the yield is  $f_{L1L3M5}(a_{L3NO} + a_{L3NN} + \omega_{L3N})$ ; (j) cascades of the initial CK transitions  $L_1-L_3N$ , subsequent Auger transitions  $L_3-M_5N$ ,  $L_3-M_5O$ , and radiative transitions  $L_3-M_5$ , the yield is  $f_{L1L3N}(a_{L3M5O} + a_{L3M5N} + \omega_{L3M5})$ ; (k) cascades of the initial CK transitions  $L_1-L_2N$ , subsequent Auger transitions  $L_2-M_5N$ ,  $L_2-M_5O$ , and radiative transitions  $L_2-M_5$ , the yield is  $f_{L1L2N}(a_{L2M5O} + a_{L2M5N} + \omega_{L2M5})$ ; (l) cascades of the initial CK transitions  $L_1-L_3O$ , subsequent Auger transitions  $L_3-M_5N$ , the yield is  $f_{L1L3O}a_{L3M5N}$ ; (m) cascades of the initial CK transitions  $L_1-L_2O$  and subsequent Auger transitions  $L_2-M_5N$ , the yield is  $f_{L1L2O}a_{L2M5N}$ .

Note that Auger transitions  $L_1-M_iM_5$  and CK transitions  $L_1-L_3M_5$  are accompanied by the formation of double vacancy states  $M_iM_5$ ,  $L_3M_5$ , whose subsequent decay may be related to the filling of  $M_5$  vacancy in the presence of  $M_i$  or  $L_3$  vacancies; these processes eliminate the  $M_iM_5$ ,  $L_3M_5$  states from the subsequent cascades of  $M_5N$  satellites formation. They can be taken into account using the rearrangement coefficients  $R_{i5} = \Gamma_{M5}/(\Gamma_{Mi} + \Gamma_{M5})$  and  $R_{L3M5} = \Gamma_{M5}/(\Gamma_{L3} + \Gamma_{M5})$ ,  $i = 1-5$ . Then, the total yield of the processes responsible for the vacancy migration from  $L_1$  to  $M_5$  subshell with the formation of additional  $N$ ,  $N^2$ , and  $NO$  vacancies is equal to

$$\begin{aligned}
F_{L1} = & a_{L1M5N} + P_{L1}^{(N)}(\omega_{L1M5} + a_{L1M5O} + a_{L1M5N}) + \\
& + P_{L1}^{(M5)}(\omega_{L1N} + a_{L1NO} + a_{L1NN}) + \\
& + P_{L1}^{(O)}a_{L1M5N} + \sum_{i=1}^4 a_{L1Mi5N}(f_{i5O} + f_{i5N} + \omega_{Mi5}) + \\
& + \sum_{i=1}^5 a_{L1MiM5}(1 - R_{i5})(a_{MiNO} + a_{MiNN} + \omega_{MiN}) + \\
& + \sum_{i=1}^4 a_{L1MiO}f_{i5N} + \sum_{i=1}^4 \omega_{L1Mi}f_{i5N} + \\
& + f_{L1L3M5}(1 - R_{L3M5})(a_{L3NO} + a_{L3NN} + \omega_{L3N}) +
\end{aligned}$$

$$\begin{aligned}
 & f_{L1L3N}(a_{L3M5O} + a_{L3M5N} + \omega_{L3M5}) + \\
 & \quad + f_{L1L3O}a_{L3M5N} + \\
 & + f_{L1L2N}(a_{L2M5O} + a_{L2M5N} + \omega_{L2M5}) + \\
 & \quad + f_{L1L2O}a_{L2M5N}.
 \end{aligned} \tag{2}$$

A similar expression can also be written for the yields of the processes of vacancy migration from  $L_2$  and  $L_3$  subshells to  $M_5$  subshell ( $F_{L2}$  and  $F_{L3}$ ). Using the migration coefficients of vacancies in  $M_i$  subshells ( $F_{Mi}$ ), we can present the emission cross section of  $M_5N$  satellites with an allowance for the ionization of  $L_i$  subshells in the form

$$\begin{aligned}
 \sigma_{\alpha S} = & \frac{k_{\alpha}\Gamma_{\alpha}^R}{\Gamma_{M5}}(\sigma_{M5}P_{M5}^{(N)}) + \\
 & + \sum_{i=1}^4 \sigma_{Mi}F_{Mi} + \sum_{i=1}^3 \sigma_{Li}F_{Li}(1 - R_{\alpha}),
 \end{aligned} \tag{3}$$

where  $\Gamma_{\alpha}^R$  is the portion of  $M_5$  level width that corresponds to the radiative transition  $M_5-N_{6,7}$ . We may assume that the radiative transition width  $\Gamma_{\alpha}^R$  does not change with the appearance of an additional vacancy [17]. The coefficient  $k_{\alpha} = 13/14$  takes into account the decrease in the total probability of the radiative transition  $M_5-N_{6,7}$  in the presence of  $N_{6,7}$  vacancy. The rearrangement coefficient  $R_{\alpha}$  determines the relative number of atoms in which  $N_{6,7}$  vacancy decays in the presence of  $M_5$  vacancy,  $R_{\alpha} = \Gamma_{N6,7}/(\Gamma_{N6,7} + \Gamma_{M5})$ , where  $\Gamma_{M5}$  and  $\Gamma_{N6,7}$  are the total widths of  $M_5$  and  $N_{6,7}$  levels. This process converts  $M_5N_{6,7}$  and  $M_5N_{6,7}O$  states into the  $M_5O$ ,  $M_5O^2$  states, which are initial for  $M_5O$  satellites.

Reasoning in the similar way, we can obtain the equation for the yield of the processes of migration of  $M_1$  vacancy into  $M_5$  subshell ( $G_{M1}$ ), and the equation for the yield of the processes of migration of  $L_1$  vacancy into  $M_5$  subshell ( $G_{L1}$ ), which are accompanied with the formation of two-vacancy  $M_5O$  and three-vacancy  $M_5O^2$  states initial for  $M_5O$  satellites

$$\begin{aligned}
 G_{M1} = & f_{15O} + \omega_{15} + \\
 & + f_{12O}(f_{25O} + \omega_{25})f_{13O}(f_{35O} + \omega_{35})f_{14O}(f_{45O} + \omega_{45}) + \\
 & + \omega_{12}f_{25O} + \omega_{13}f_{35O} + \omega_{14}f_{45O} + \\
 & + P_{M1}^{(O)}(\omega_{15} + f_{15O}),
 \end{aligned} \tag{4}$$

$$\begin{aligned}
 G_{L1} = & a_{L1M5O} + P_{L1}^{(O)}(a_{L1M5O} + \omega_{L1M5}) + \\
 & + P_{L1}^{(M5)}\omega_{L1O} + \sum_{i=1}^4 a_{L1MiO}(f_{i5O} + \omega_{Mi5}) + \\
 & + \sum_{i=1}^5 a_{L1MiM5}(1 - R_{i5})\omega_{MiO} + \\
 & + f_{L1L3M5}(1 - R_{L3M5})\omega_{L3O} + \\
 & + f_{L1L3O}(\omega_{L3M5} + a_{L3M5O}) + \\
 & + f_{L1L2O}(\omega_{L2M5} + a_{L2M5O}).
 \end{aligned} \tag{5}$$

The equations for the yields of such processes of vacancy migration from  $M_2$ ,  $M_3$ ,  $M_4$  subshells into  $M_5$  subshell ( $G_{Mi}$ ) and the equations for the yields of such processes of vacancy migration from  $L_2$ ,  $L_3$  subshells into  $M_5$  subshell ( $G_{Li}$ ) are similar to that presented above. Then the emission cross section of  $M\alpha_{1,2}$  lines together with  $M_5O$  satellites is equal to

$$\begin{aligned}
 \sigma_{\alpha} = & \\
 = & \frac{\Gamma_{\alpha}^R}{\Gamma_{M5}}(\sigma_{M5}(1 - P_{M5}^{(N)}) + \sum_{i=1}^4 \sigma_{Mi}G_{Mi} + \sum_{i=1}^3 \sigma_{Li}G_{Li}) + \\
 & + \sigma_{\alpha S} \frac{R_{\alpha}}{1 - R_{\alpha}},
 \end{aligned} \tag{6}$$

We can also easily take into account the contributions of  $M$  and  $L$  subshells to the emission cross sections of  $M_4N$  satellites and  $M\beta$  line together with  $M_4O$  satellites. The corresponding emission cross sections are equal to

$$\begin{aligned}
 \sigma_{\beta S} = & \frac{k_{\beta}\Gamma_{\alpha}^R}{\Gamma_{M4}}(\sigma_{M4}P_{M4}^{(N)}) + \sum_{i=1}^3 \sigma_{Mi}H_{Mi} + \\
 & + \sum_{i=1}^3 \sigma_{Li}H_{Li}(1 - R_{\beta}),
 \end{aligned} \tag{7}$$

$$\begin{aligned}
 \sigma_{\beta} = & \\
 = & \frac{\Gamma_{\beta}^R}{\Gamma_{M4}}(\sigma_{M4}(1 - P_{M4}^{(N)}) + \sum_{i=1}^3 \sigma_{Mi}K_{Mi} + \sum_{i=1}^3 \sigma_{Li}K_{Li}) + \\
 & + \sigma_{\beta S} \frac{R_{\beta}}{1 - R_{\beta}},
 \end{aligned} \tag{8}$$

where  $H_{Mi}$ ,  $K_{Mi}$ , and  $H_{Li}$ ,  $K_{Li}$  are the yields of the processes of migration of  $M_i$  and  $L_i$  vacancies into the  $M_4$  subshell,  $R_{\beta} =$

Table 1. Contributions of  $M$  and  $L$  subshells to the emission cross sections of  $M_5N$  satellites of Au (normalized to  $\sigma_{\alpha S}$ )

$\sigma_{\alpha S}(M_1)$	$\sigma_{\alpha S}(M_2)$	$\sigma_{\alpha S}(M_3)$	$\sigma_{\alpha S}(M_4)$	$\sigma_{\alpha S}(M_5)$	$\sigma_{\alpha S}(L_1)$	$\sigma_{\alpha S}(L_2)$	$\sigma_{\alpha S}(L_3)$	$\sigma_{\alpha S}(M)$	$\sigma_{\alpha S}(L)$
0.10	0.04	0.22	$6 \cdot 10^{-5}$	$2 \cdot 10^{-3}$	0.08	0.15	0.41	0.36	0.64

Table 2. Contributions of  $M$  and  $L$  subshells to the emission cross sections of  $M\alpha_{1,2}$  lines with  $M_5O$  satellites of Au (normalized to  $\sigma_{\alpha}$ )

$\sigma_{\alpha}(M_1)$	$\sigma_{\alpha}(M_2)$	$\sigma_{\alpha}(M_3)$	$\sigma_{\alpha}(M_4)$	$\sigma_{\alpha}(M_5)$	$\sigma_{\alpha}(L_1)$	$\sigma_{\alpha}(L_2)$	$\sigma_{\alpha}(L_3)$	$\sigma_{\alpha}(R)$
$6 \cdot 10^{-4}$	$3 \cdot 10^{-3}$	0.05	0.01	0.23	0.01	0.04	0.15	0.51

Table 3. Contributions of  $M$  and  $L$  subshells to the emission cross sections of  $M_4N$  satellites of Au (normalized to  $\sigma_{\beta S}$ )

$\sigma_{\beta S}(M_1)$	$\sigma_{\beta S}(M_2)$	$\sigma_{\beta S}(M_3)$	$\sigma_{\beta S}(M_4)$	$\sigma_{\beta S}(L_1)$	$\sigma_{\beta S}(L_2)$	$\sigma_{\beta S}(L_3)$	$\sigma_{\beta S}(M)$	$\sigma_{\beta S}(L)$
0.08	0.28	0.06	$3 \cdot 10^{-3}$	0.11	0.27	0.20	0.42	0.58

Table 4. Contributions of  $M$  and  $L$  subshells to the emission cross sections of  $M\beta$  line with  $M_4O$  satellites of Au (normalized to  $\sigma_{\beta}$ )

$\sigma_{\beta}(M_1)$	$\sigma_{\beta}(M_2)$	$\sigma_{\beta}(M_3)$	$\sigma_{\beta}(M_4)$	$\sigma_{\beta}(L_1)$	$\sigma_{\beta}(L_2)$	$\sigma_{\beta}(L_3)$	$\sigma_{\beta}(R)$
$7 \cdot 10^{-4}$	0.02	$4 \cdot 10^{-5}$	0.28	0.02	0.20	0.04	0.44

$\Gamma_{N6,7}/(\Gamma_{N6,7} + \Gamma_{M4})$ ,  $\Gamma_{\beta}^R$  is the width of  $M_4-N_6$  radiative transition, and  $k_{\beta} = 5/6$ .

Emission cross sections (3), (6), (7), (8) of Au were calculated for photon energies in the range of  $E = 5-30$  keV. We used the photoionization cross sections from [16], the yields of CK transitions  $f_{ijN}$  and  $f_{ijO}$  from [18], the yields of Auger and CK transitions from  $L$  subshells  $a_{LjMiM5}$ ,  $a_{LjMiN}$ ,  $a_{LjMiO}$ ,  $a_{LjNO}$ ,  $f_{LiLjM}$ ,  $f_{LiLjN}$ ,  $f_{LiLjO}$  from [19], the fluorescence yields  $\omega_{ij}$  and the radiative transition widths  $\Gamma_{\beta}^R$ , and  $\Gamma_{\alpha}^R$  from [20], the level widths  $\Gamma_{M4}$ ,  $\Gamma_{M5}$  and  $\Gamma_{N6,7}$  from [21], and the probabilities of SO processes from [22] ( $P_{Li}^{(N)} = 0.03$ ,  $P_{Li}^{(O)} = 0.16$ ,  $P_{Li}^{(M5)} = 10^{-3}$ ,  $P_{Mi}^{(N)} = 0.20$ ,  $P_{Mi}^{(O)} = 0.14$ ).

### 3. Results and discussion

Let us estimate the contributions of individual  $M$  and  $L$  subshells to the emission cross sections  $\sigma_{\alpha S}$ ,  $\sigma_{\alpha}$  and  $\sigma_{\beta S}$ ,  $\sigma_{\beta}$  of Au in the case of excitation by photons with the energy  $E = 17.479$  keV ( $M\alpha_{1,2}$  lines). The normalized values of these partial cross sections are determined by individual terms in formulas (3), (6), (7), (8) and are listed in Tables 1–4. Table 1 shows that the main processes of generation of the initial states of  $M_5N$  satellites of Au are Auger and radiative transitions from  $L_3$  subshell. Their contribution to the excitation of  $M_5N$  satellites is 41 %, and the total contribution of

$L$  subshells is 64 %. At the same time, the contribution of  $L$  and  $M$  subshells to the excitation of the  $M\alpha_{1,2}$  lines together with  $M_5O$  satellites (Table 2) does not exceed 49 %. In this case, the main process is the rearrangement with the cross section  $\sigma_{\alpha}(R) = \sigma_{\alpha S}R_{\alpha}/(1 - \alpha)$ , which results in the converting the initial states of  $M_5N$  satellites into the initial states of  $M_5O$  satellites. The contribution of these processes is 51 %. Table 3 shows that the main processes of generation of the initial states of  $M_4N$  satellites of Au are Auger, Coster-Kronig and radiative transitions from the  $L_2$  and  $M_2$  subshells (27 % and 28 %, respectively). At the same time, the contribution of  $L$  subshells to the excitation of the  $M\beta$  line together with  $M_4O$  satellites (Table 4) does not exceed 26 %, and the contribution of  $M$  subshells is almost the same (30 %). In this case, the main process is the rearrangement with the cross section  $\sigma_{\beta}(R) = \sigma_{\beta S}R_{\beta}/(1 - R_{\beta})$ , which results in the converting the initial states of  $M_4N$  satellites into the initial states of  $M_4O$  satellites ( $\sigma_{\beta}(R) = 44$  %).

Table 5 presents the ratios of the emission cross sections of  $M_5N$  satellites to the emission cross sections of  $M\alpha_{1,2}$  lines together with  $M_5O$  satellites  $\chi = \sigma_{\alpha S}/\sigma_{\alpha}$ , as well as the ratios of the emission cross sections of  $M_4N$  satellites to the emission cross sections of  $M\beta$  line together with  $M_4O$  satellites  $\gamma = \sigma_{\beta S}/\sigma_{\beta}$  of Au excited by  $K\alpha_{1,2}$  ra-

Table 5. Ratios of the emission cross sections of components of  $M\alpha$  and  $M\beta$  spectra of Au

Energy of absorbed photons, keV	$\chi$		$\gamma$		$\eta$	
	calculation	experiment	calculation	experiment	calculation	experiment
8.048 Cu $K\alpha_{1,2}$	0.72	0.69±0.07	0.58	0.53±0.05	0.68	0.67±0.05
17.479 Mo $K\alpha_{1,2}$	1.56	1.48±0.15	1.28	1.24±0.12	0.59	0.57±0.04

diation of Cu and Mo anodes. In addition, Table 5 presents the ratios of the emission cross sections of the total profile of  $M\beta$  and  $M\alpha$  lines (including  $M_4N$  and  $M_5N$  satellites, respectively)  $\eta = \sigma_{\beta\Sigma} / \sigma_{\alpha\Sigma}$ . The calcu-

lated values  $\chi$ ,  $\gamma$ , and  $\eta$  were compared with experimentally determined relative intensities of the corresponding components of Au  $M$  spectra (Table 5). The  $M\alpha$  and  $M\beta$  fluorescence spectra of Au were recorded with a DRS-2 X-ray spectrograph in the first order of reflection from the (10 $\bar{1}1$ ) planes of a Johann-bent quartz single crystal. Due to a strong self-absorption, the  $M\gamma$  spectrum was not recorded. The chemical purity of samples was higher than 99.9 %. The spectra were excited by the monochromatized  $K\alpha_{1,2}$  radiation of BSV-29 X-ray tubes with Cu and Mo anodes. The  $K\alpha_{1,2}$  lines were separated from the primary polychromatic spectra by a focusing graphite monochromator in the first order of reflection from the (002) planes. The tubes operated with the voltage  $U = 30$  kV and the current  $I = 30$  mA. It is important that the photons energy of the Mo  $K\alpha_{1,2}$  radiation is sufficiently high to ionize  $L$  subshells of Au atoms (the photons energies of Cu and Mo  $K\alpha_{1,2}$  lines are 8.048 keV and 17.479 keV, respectively, and the ionization potentials of  $L$  subshells of Au are  $E_{L1} = 14.353$  keV,  $E_{L2} = 13.734$  keV and  $E_{L3} = 11.919$  keV [23]). More detailed experimental technique of obtaining and processing  $M\alpha$  and  $M\beta$  fluorescence spectra of Au is presented in our previous paper [24].

Table 5 shows that the calculation reflects experimentally revealed increasing the relative intensities of  $M_{4,5}N$  satellites of Au (by a factor of 2.1 for  $\chi$  and 2.3 for  $\gamma$ ) during the transition from  $K\alpha_{1,2}$  excitation of Cu ( $L$  subshells are not ionized) to  $K\alpha_{1,2}$  excitation of Mo. This increase of the values of  $\chi$  and  $\gamma$  is being caused by appearance of the powerful channels of vacancy migration from  $L$  subshells via excitation of Au  $M\alpha$  and  $M\beta$  spectra by  $K\alpha_{1,2}$  radiation of Mo. It

is important that the calculated and experimentally determined values of  $\chi$ ,  $\gamma$ , and  $\eta$  of Au coincide well with each other, which proves the correctness of the proposed cascade model of X-ray  $M$  emission.

#### 4. Conclusions

Thus, the cascade model of X-ray  $M$  emission for the heavy metals atoms is proposed that allows one to take into account the main channels of vacancy transfer from  $L$  to  $M$  subshells, which are responsible for the generation of double-vacancy ( $M_{4,5}N$  and  $M_{4,5}O$ ) and triple-vacancy ( $M_{4,5}N^2$ ,  $M_{4,5}NO$  and  $M_{4,5}O^2$ ) states. Comparison of the relative emission cross sections of  $M_5N$  and  $M_4N$  satellites of Au with the experimental relative intensities of these satellites, excited by  $K\alpha_{1,2}$  radiation of Cu and Mo anodes, indicates the correctness of the represented cascade model. The proposed cascade model of X-ray  $M$  emission can be also used for determination of the characteristic parameters of electronic subsystem of thin films of heavy metals with  $Z > 70$ , in particular ionization cross sections of electronic shells and probabilities of autoionization processes.

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## Каскадна модель рентгенівської $M$ емісії для атомів важких металів

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Запропоновано каскадну модель рентгенівської  $M$  емісії для атомів важких металів, що дозволяє врахувати основні канали переносу вакансій з  $L$  в  $M$  електронні підоболонки, в результаті яких генеруються двовакансійні ( $M_{4,5}N$  і  $M_{4,5}O$ ) та трихвакансійні ( $M_{4,5}N^2$ ,  $M_{4,5}NO$  і  $M_{4,5}O^2$ ) стани. Модель дозволяє окремо розрахувати внески  $L$  і  $M$  підоболонки до перерізів випромінювання  $M_5N$  і  $M_4N$  сателітів, а також  $M\alpha_{1,2}$  лінії разом з  $M_5O$  сателітами та  $M\beta$  лінії з  $M_4O$  сателітами. Порівняння відносних перерізів випромінювання основних компонентів  $M\alpha$  і  $M\beta$  спектрів атомів Au з експериментальними відносними інтенсивностями цих компонентів при збудженні  $K\alpha_{1,2}$  випромінюванням Cu і Mo свідчить про коректність використаної каскадної моделі.