# On spectra of microwave oscillations of the 0-degree domain wall in a cubic ferromagnetic

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#### Received December 24, 2005

A cubic ferromagnetic with uniaxial anisotropy induced along the [111] axis is considered that contains a defect as a slab-like magnetic inclusion. It is assumed that a magnetic inhomogeneity is localized on that inclusion, the model representation of the inhomogeneity corresponding to the magnetization distribution in a zero-degree domain wall. The microwave excitations of the 0-degree domain wall have been investigated numerically and the features of their spectra have been considered depending on the sample material parameters and the defect characteristics.

Рассматривается кубический ферромагнетик с наведенной вдоль оси [111] одноосной анизотропией, содержащий дефект в виде пластинчатого магнитного включения. Предполагается, что на нем локализована магнитная неоднородность, модельное представление которой соответствует распределению намагниченности в 0-градусной доменной границе. Численно исследованы микроволновые возбуждения 0-градусной доменной границы, рассмотрены особенности их спектров в зависимости от материальных параметров образца и характеристик дефекта.

A theoretical examination of microwave excitations of the domain wall (DW) with adequately accounting for the field created by magnetic moments is quite a nontrivial problem that is mainly solved by numerical methods. Such an analysis performed in [1-3] for the 180degree DWs of the Bloch type has demonstrated that the spectrum of their excitations, in addition to the two low-frequency translations modes and one unidirectional Gilinski branch [4], features a number of other peculiarities (the availability of additional high-frequency branches, the root peculiarity of the translational mode, etc.), which are largely dependent on both the DW topology and the anisotropy type. At the same time, the analysis of the domain structure of cubic ferromagnets with induced uniaxial anisotropy has shown that in near the spin-reorientational phase transition, there appear solutions of the Landau-Lifshits equations to which 0-degree

DWs correspond [6]. Those represent magnetic inhomogeneities, which separate the areas of the ferromagnet with the same direction of the magnetization vector M in domains. It follows from the calculations [6] (which qualitatively agree with the experimental data [7]) that those areas are localized in the defect areas and act as a new phase nuclei in the processes of the magnetic spin reorientation from one state to another. Their dynamic properties remain practically uninvestigated; in this connection, it is of interest to examine the microwave excitation spectrum of the 0-degree DW in a cubic ferromagnet with uniaxial anisotropy induced along the [111] axis (a (111) slab) with defects present therein.

The total energy of such a magnet shall be considered under account for the exchange interaction, the dipole-dipole interaction, the presence of a combined anisotropy, as well as of the defect. It will have the form:

$$\begin{split} E &= \int\limits_{V} \left\{ A[(\nabla \theta)^2 + \sin^2 \theta (\nabla \phi)^2] + \right. \\ &+ K_u \sin^2 \theta - \frac{1}{2} M_s (\boldsymbol{m} \boldsymbol{H}^d) + \\ &+ K_1 \left[ \frac{1}{4} \sin^4 \theta + \frac{1}{3} \cos^4 \theta + \frac{\sqrt{2}}{3} \sin^3 \theta \cos \theta \cos 3 \phi \right] dV, \end{split}$$

where  $\theta$  and  $\phi$  are the polar and the azimuthal angles of the magnetization vector  $\mathbf{M}$ ; A,  $K_u$ ,  $K_1$  are the constants of the exchange interaction, of the uniaxial anisotropy and the cubic anisotropy, respectively;  $M_s$  is saturation magnetization; m, a unit magnetization vector  $(\mathbf{m} = \mathbf{M}/M_s)$ ; V, the magnet volume;  $\mathbf{H}^d$ , the demagnetizing field which can be obtained from the equations of magnetostatics

$$div(\mathbf{H}^d + 4\pi \mathbf{M}) = 0$$
,  $rot \mathbf{H}^d = 0$ . (2)

Here, the coordinate system is selected in such a way that ox [112], oy [110], oz [111].

A slab-like magnetic inclusion [8] is considered as a defect that stabilizes the structure of the 0-degree domain wall. This inclusion is the area of the magnet where the material parameters R have the values different from those in the matrix, i.e. they depend on the coordinate y as

$$R(y) = \begin{cases} R + \Delta R, & |y| \le \frac{l}{2}, \\ R, & |y| \ge \frac{l}{2}, \end{cases}$$
 (3)

where l is the defect size,  $R = \{A, K_u, K_1\}$ ,  $\Delta R = \{\Delta A, \Delta K_u, \Delta K_1\}$  are the values of the parameter R leap in the defect area.

The spin-wave excitation of a DW in a ferromagnet are described by Landau-Lifshits equations of magnetodynamics for the magnetization vector

$$\frac{M_s d\varphi}{\gamma dt} \sin\theta = \frac{\delta E}{\delta \theta},$$

$$-\frac{M_s \partial \theta}{\gamma \partial t} \sin\theta = \frac{\delta E}{\delta \omega}$$

and magnetostatics ones of the form (2).

To investigate small deviations of magnetic moments in the DW from their equilibrium values defined by the exact solution of the Landau-Lifshits equations for the static case, we shall linearized the system (4) with due account taken of (2), substituting  $\mathbf{H}^{\mathrm{d}}$  as:  $\mathbf{H}^{\mathrm{d}} = M_s \nabla \psi$ , where  $\psi$  is the magnetostatic potential. The magnetization distribution in the 0-degree DW in the absence

of defects and small perturbations is determined by the expressions [5, 6]

$$tg\theta_0(y) = \frac{1}{ach(\xi) - c}, \quad \varphi_0 = 0,\pi,$$
 (5)

where  $\xi = by/\Delta_0$ ,  $\Delta_0 = \sqrt{A/K_u}$ , a, b, c are variational parameters. Harmonic deviations of spins from the equilibrium distribution (5) will be identified in the form a small addition to the principal solution

$$\begin{split} \theta &= \theta_0(y) + \theta(y) \sin(\omega t - k_x x - k_z z), \\ \varphi &= \varphi_0(y) + \varphi(y) \cos(\omega t - k_x x - k_z z), \\ \Psi &= \psi(y) \sin(\omega t - k_x x - k_z z). \end{split}$$
 (6)

In this case, the linearized system of equations (2), (4) will be reduced to the following form:

$$\frac{\partial^{2}\theta}{\partial y^{2}} = \frac{f(\theta_{0}) + k^{2}}{\delta A} \theta(y) -$$

$$-\frac{\omega \sin\theta_{0}}{\delta A} \varphi(y) + \frac{k_{x} \cos\theta_{0} - k_{z} \sin\theta_{0}}{Q\delta A} \psi(y),$$

$$\frac{\partial^{2}\varphi}{\partial y^{2}} = -\frac{\omega}{\sin\theta_{0}\delta A} + \frac{g(\theta_{0}) + k^{2}}{\delta A} \varphi(y) -$$

$$-\frac{1}{Q\sin\theta_{0}\delta A} \frac{\partial \psi}{\partial y} - 2 \cot g\theta_{0} \frac{\partial \varphi}{\partial y} \frac{\partial \theta_{0}}{\partial y},$$

$$\frac{\partial^{2}\psi}{\partial y^{2}} = \theta(y)(k_{z} \sin\theta_{0} - k_{x} \cos\theta_{0}) -$$

$$-\sin\theta_{0} \frac{\partial \varphi}{\partial y} + \psi(y)k^{2}.$$

$$(7)$$

Here,

$$\begin{split} f(\theta_0) &= \delta K_u (\cos^2\!\theta_0 - \sin^2\!\theta_0) + \\ &+ \left. 2 \sin^2\!\theta_0 \cos^2\!\theta_0 - \frac{1}{2} \sin^4\!\theta_0 - \frac{2}{3} \cos^4\!\theta_0 \right) + \\ &+ \left. 2 \sin^2\!\theta_0 \cos^2\!\theta + \sqrt{2} \sin\!\theta_0 \cos^3\!\theta_0 - \right. \\ &- \left. \sqrt{2} \sin^3\!\theta_0 \!\cos\!\theta_0 \right), \\ g(\theta_0) &= \frac{3\sqrt{2}}{2} \mathbf{æ} \, \delta K_1 \!\sin\!\theta_0 \!\cos\!\theta_0. \end{split}$$

The boundary conditions will be taken as

$$\lim_{u\to\pm\infty} (\theta, \varphi, \psi) = 0.$$
 (9)

Besides, normalizations have been introduced

$$\theta = \theta \Delta_0, \quad \varphi = \varphi \Delta_0, \quad x_i = x_i / \Delta_0, \quad k_i = k_i \Delta_0 10$$

$$\omega = \frac{\omega M_s}{2K_u \gamma}, \quad \psi = \frac{\psi}{4\pi M_s}, \quad Q = \frac{|K_u|}{2\pi M_s^2},$$

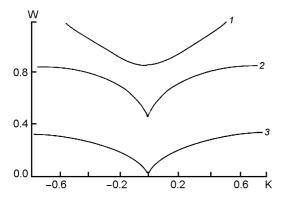


Fig. 1. The spectrum of microwave oscillations of the 0-deg. DW; here,  $\Delta A = 0.5 \, \text{A}$ ,  $\Delta K_1 = 0.5 K_1$ ,  $\Delta K_u = -K_u$ , l = 4, Q = 1,  $\kappa = 0.5$ . Curve 1 is the boundary of the solid spectrum zone; 2, the Gilinski branch; 3, the translational mode.  $\mathbf{æ} = \mathbf{æ}_2$ .

$$\mathbf{æ} = K_1/K_u$$
,  $\delta R = 1 + \Delta R/R$ .

The problem of finding the DW oscillation spectrum can be formulated as follows: in the three-dimensional space with coordinates  $(k_x, k_z, \omega)$ , where  $\mathbf{k}$  is the wave vector, which defines the propagation direction of the oscillations, whereas  $\omega$  defines their frequency, it is necessary to find a set of points where the system of equations (2) and (5) has a solution which decreases as one moves from the DW plane.

The problem was numerically investigated, the calculations involving two steps. First of all, the equilibrium values of the 0-degree DW parameters were determined by solving the variational problem for minimizing the full energy E with due account taken of (3), where the law of changing magnetization in the 0-degree DW in the form of (5) was taken as a trial function. The second step involved the solution of the system (7) with due account taken of the obtained values of the variational parameters a, b, c.

The numerical analysis results of the spectra of 0-degree DWs are substantially different from the properties of a DW of other topologies described in several works [1-4]. This is related to a special structure of the 0-degree DW, since a symmetrical function of magnetic moments distribution in the transition layer corresponds thereto:  $\mathbf{m}_0(-y) = \mathbf{m}_0(y)$ . It is evident from the results obtained that the excitation spectra of the 0-degree DW, including the Gilinski branch, are symmetrical with respect to the frequency axis, whereas in the case of the

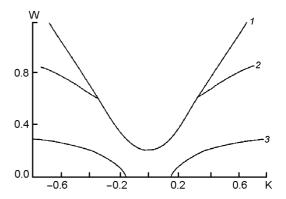


Fig. 2. The spectrum of microwave oscillations of the 0-deg. DW; the material parameters and notations are the same as in Fig. 1 (except for  $\mathbf{æ}$ ;  $\mathbf{æ} = \mathbf{8}$ ).  $\mathbf{æ} = \mathbf{æ}_2$ .

180-degree DW, the latter is asymmetrical [1-4]. The investigation of the spectrum type dependence on the values of the parameter  $\kappa$  has shown the existence of an area of the wave number values, where oscillations of the 0-degree DW localized on its surface are forbidden (Figs. 1,2). As seen from the Figures, as the parameter K increases, at its certain threshold value, the spectrum branches coresponding to the DW oscillations localized on its surface start to "move apart". As they do so, only voluminous spin waves can be excited in the area of the smaller values of the wave number. As æ increases, there takes place an expansion of the zone forbidden for other type excitations except for voluminous spin waves, as well as a movement of its boundary towards lower frequencies. This change in the spectra shape can be related to a loss of the 0-degree DW stability with respect to microwave oscillations with a small value of the wave number as k increases. Figs. 3 and 4 show the kind of the spectrum frequencies as a function of the defect linear dimensions. When analyzing the results thus obtained, one can easily identify a tendency to the expansion of the zone forbidden for spin-wave excitations localized on the DW surface, as the defects linear dimensions increase. As this takes place, the corresponding branches tend to shift towards larger wave number values.

Thus, the spin waves have been analyzed that are localized on the 0-degree DW in a cubic ferromagnet with the uniaxial anisotropy induced along the [111] axis which possesses a defect in the form of a slab-like magnetic inclusion. The analysis has shown that in addition to the properties in com-

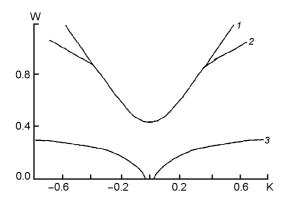


Fig. 3. The spectrum of microwave oscillations of the 0-deg. DW; the material parameters and notations are the same as in Fig. 1 (except for  $\mathbf{x}$ , l;  $\mathbf{x} = 6$ , l = 2).  $\mathbf{x} = \mathbf{x}_{2k}$ 

mon with the DW of other topology (the presence of the same set of branches), the spectra which are characteristic of the 0-degree DW possess a number of differences, these being connected with a peculiarity of distribution therein of magnetic moments. This results, first of all, in the symmetry of all branches of the spectrum (including the Gilinski branches which remain asymmetric in other cases). Besides, a phenomenon has been discovered of the appearance of the 0-degree DW instability zone with respect to the oscillations with small wave numbers, this zone expanding with the increase of the parameter æ and depending on the energy value of the defect and its linear dimensions.

The paper was supported by Grant of the Ministry of Education and Science of the Russian Federation No.56608.

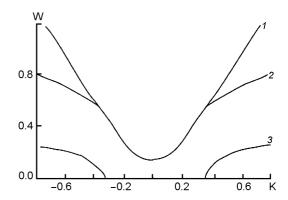


Fig. 4. The spectrum of microwave oscillations of the 0-deg. DW; the material parameters and notations are the same as in Fig. 1 (except for  $\mathbf{x}$ , l;  $\mathbf{x} = 6$ , l = 6).  $\mathbf{x} = \mathbf{x}$ .

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# Про спектри мікрохвильових коливань 0-градусної доменної межі у кубічному феромагнетику

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Розглядається кубічний феромагнетик з наведеною вздовж осі [111] одновісною анізотропією, що містить дефект у формі пластинчастого магнітного включення. Припускається, що на ньому локалізована магнітная неоднорідність, модельне представлення якої відповідає розподілу намагніченості у 0-градусній доменній межі. Виконано числове дослідження мікрохвильових збуджень 0-градусної доменної межі, розглянуто особливості їхніх спектрів залежно від матеріальних параметрів зразка та характеристик дефекту.