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Processes of inhomogeneous magnetization reversal in real crystals

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The model concept of magnetic inhomogeneities being generated in the crystal defect region isconsidered using the magnetization distribution corresponding to the 0 degree domain wall. This solution originated in the idealized model of an unrestricted cubic ferromagnetic (without taking into account the crystal defects) in the spin reorientation region makes it possible to study not only the phase transition kinetics but also the magnetization reversal processes in real crystals.

Рассматривается модельное представление магнитных неоднородностей, зарождающихся в области дефектов кристалла, с помощью распределения намагниченности, соответствующего 0-градусной доменной границе. Показано, что данное решение, возникающее в идеализированной модели неограниченного кубического ферромагнетика (без учета дефектов в кристалле) в области спиновой переориентации, позволяет изучить не только кинетику фазового перехода, но и процессы перемагничивания реальных кристаллов.

The properties of magnetic materials are known to be affected substantially by presence of various defects in those materials. Magnetic inhomogeneities originate on the defects, adapt to the defect structure and play a critical part in the sample magnetization reversal processes from one state to another [1, 2]. A theoretical consideration of phase transition of the spin reorientation type for finite defect-containing ferromagnets has shown [3] that for the description of the fluctuation mechanism of nucleation (most fully realized in the situation of question), magnetic inhomogeneities of the 0-degree domain wall (0°-DW) type can be used as a model representation of the new phase nuclei being "condensed" on the defects. The qualitative agreement of the results obtained with experimental data [2] enables one to apply the mentioned approximation to describe the processes of magnetization and magnetization reversal in real crystals.

Let us consider the influence of the external magnetic field $\boldsymbol{H}$ ($\boldsymbol{H} \parallel [111]$) on nucleation processes on a defect in a cubic ferromagnet with a uniaxial anisotropy induced along [111]. The materials with such a combined anisotropy include the majority of epitaxially grown ferrite-garnet crystals, magnetic semiconductors of the $CdCr_2Se_4$ type [1], some intermetallic compounds [4], etc. The energy of such a magnet, with due account for the exchange and Zeeman interactions, induced uniaxial and cubic anisotropies and magnetostatic energy of the spatial charges localized in the DW, will be written as [3]:

$$E_0 = L_x D \int_{-\infty}^{\infty} \left\{ A\left[\left(\frac{d\theta}{dy}\right)^2 + \sin^2\theta\left(\frac{d\varphi}{dy}\right)^2\right] + K_u \sin^2\theta + K_1\left(\frac{\sin^4\theta}{4} + \frac{\cos^4\theta - 1}{3} + \right.\right. \tag{1}$$

$$\left.\left. + \frac{\sqrt{2}}{3}\sin^3\theta\cos\theta\cos 3(\varphi - \varphi_0)\right) - HM_s(\cos\theta - 1) + 2\pi M_s^2(\sin\theta\sin\varphi - \sin\theta_m\cos\theta_m)^2 \right\} dy,$$

where θ, φ are the polar and the azimuthal angles of the magnetization vector **M**, respectively; θ_m, φ_m – values of these angles in domains; A, exchange parameter; K_u, K_1 – constants of the uniaxial and cubic anisotropies, respectively; M_s – saturation magnetization; H – external magnetic field strength, H‖[111]; L_x – the sample width in OX direction ($L_x \to \infty$); D – the slab thickness. Here, the system of coordinates is selected in such a way that OZ‖[111], and OY is perpendicular to the DW plane and makes the angle φ_0 with the $[1\bar{1}0]$ axis. It is assumed that the slab, which is considered infinite in the XOY plane, is sufficiently thick, and the contribution of demagnetizing fields of surface charges to (1) is neglected (an idealized model).

It follows [3] from the analysis of the Euler-Lagrange equations that are written as

$$\frac{\delta E_0}{\delta\theta} = 0, \ \frac{\delta E_0}{\delta\varphi} = 0, \ \frac{\delta E_0}{\delta\varphi_0} = 0 \tag{2}$$

subject to

$$\delta^2 E_0(\theta, \varphi, \varphi_0) > 0 \tag{3}$$

that at H=0 within the range 4/3 <æ<3/2, K_u> 0 (æ =K1/|K_u|), the following solutions arise:

$$tg\theta = (\pm a \cdot ch(\xi) + c)^{-1}, \quad \varphi = 0, \pi, \quad \varphi_0 = \pi k/3, k \in Z$$

$$\xi = by/\Delta_0, \quad a = \sqrt{3\,æ - 4}/2b^2, \quad b = \sqrt{1 - 2æ/3}, \quad c = \sqrt{2}æ\cos 3(\varphi - \varphi_0)/6b^2 \tag{4}$$

where $\Delta_0 = \sqrt{A/K_u}$. Those are consistent with magnetic inhomogeneities of the 0°-DW type, which represent large-scale fluctuations of the vector ***M*** arising in the vicinity of the spin reorientation phase transition of the first order. Those may be of two types: the large-amplitude soliton (LAS) and the small-amplitude soliton (SAS) that differ in energy E_s, width Δ_s and maximum deflection angle of the vector ***M*** from the homogeneous state θ_s (in amplitude). The expressions for the latter are presented in [3]. Those are unstable in the idealized model [3, 5]. However, if inhomogeneity of material parameters and finiteness of the sample are taken into account, 0°-DWs become stable formations.

In a nonzero field, solutions of the Euler-Lagrange equations [3] cannot be obtained using the known functions. However, an analysis of the phase portrait of these equations for $\varphi = 0, \pi$ shows (Fig. 1) that there exist phase trajectories **(**separatrices**)** shaped as closed loops to which 0°-DWs correspond, these DWs having a structure similar to (4). This allows one to investigate the process of magnetization reversal of real crystals, this process being conditioned by the mechanism of non-coherent rotation of magnetic moments. The variation method [3] is used as a basis for modeling the origination processes of reverse-magnetization domains that are fixed at the defects. In this variation method, the function of the kind (4), where *a, b, c* are considered as variation parameters of the problem, is used as the magnetization distribution near a defect. If this approach is used, determination of the stable conditions of 0°-DW in a magnetic field will require considering the influence of the slab demagnetizing fields and the presence of defects therein. Their contribution to (1) will be regarded similarly to [3]. Then, the full energy of the 0°-DW, with both of the factors taken into consideration, will acquire the form

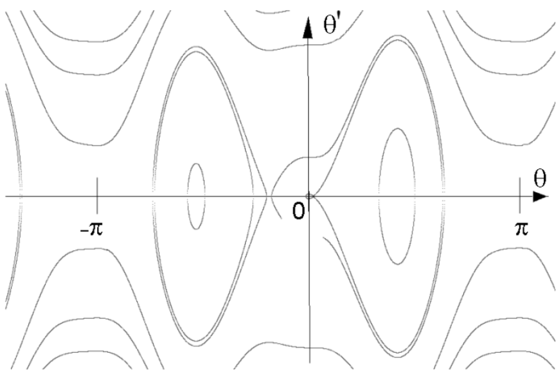

Fig. 1. Phase portrait of Euler-Lagrange equations for æ = 1.4; h = 0.3.

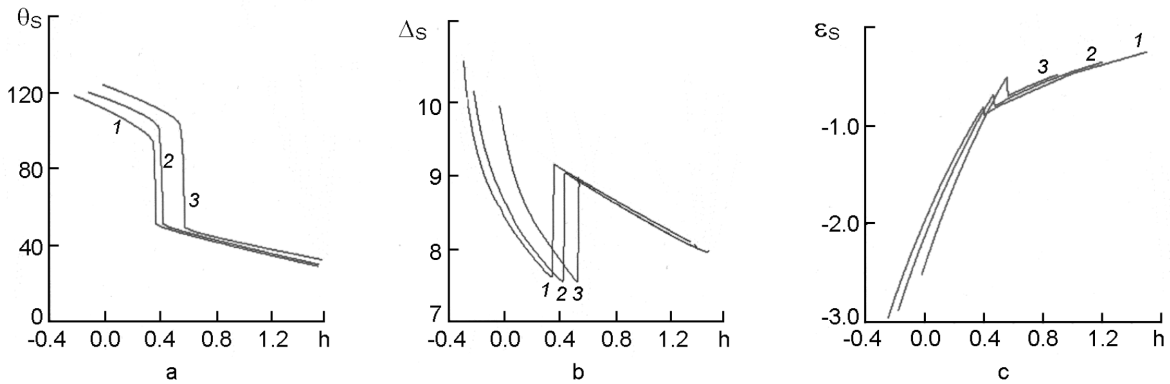

Fig. 2. Graphs showing the dependence of the 0°-DW parameters on the value of external field h at various values D for the case ΔA = 0.2, ΔK_1 = 0.5, L = 5, Q = 5, ΔK_u = −1.5, ΔM_s = 0.5, æ=1.42. Here, curve *1* corresponds to D=3, curve 2 – D=7, curve *3* – D=30.

$$E = E_0 + E_{ms} + E_d, \qquad (5)$$

where E_{ms} is the magneto-static energy of "surface" charges [3] that is determined by the expression

$$E_{ms} = M_s^2 L_x \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(y,y')dydy'\,;\; f(y,y') = [\cos\theta(y)\cos\theta(y') - 1]\ln\left(1 + \frac{D^2}{(y-y')^2}\right), \qquad (6)$$

whereas E_d represents the contribution to (5), brought about by inhomogeneity of material parameters $R = \{A, K_1, K_u, M_s\}$, which in the vicinity of the defect ($|y| \le L/2$, where L is the defect size) experience a jump by the value of $\Delta R = \{\Delta A, \Delta K_1, \Delta K_u, \Delta M_s\}$. Accordingly, E_d is written as:

$$E_d = L_x D \int_{-L/2}^{L/2} \left\{ \Delta A \left(\frac{\partial\theta}{\partial y}\right)^2 + \Delta K_u \sin^2\theta + \Delta K_1 \left[\frac{\sin^4\theta}{4} + \frac{\cos^4\theta}{3} + \right.\right.$$
$$\left.\left. + \frac{\sqrt{2}}{3}\sin^3\theta\cos\theta\right] - \Delta M_s H(\cos\theta - 1)\right\} dy \; +$$
$$+ 2M_s \Delta M_s L_x \int_{-\infty}^{+\infty}\int_{-L/2}^{L/2} f(y,y')dydy' + \Delta M_s^2 L_x \int_{-L/2}^{L/2}\int_{-L/2}^{L/2} f(y,y')dydy'. \qquad (7)$$

The corresponding variational problem to find the stable conditions of 0°-DW is solved by numerical minimization of the functional (5) with respect to the parameters a, b, c. It should be stressed here that, as shown in [3], this approach is acceptable if the following conditions are met:

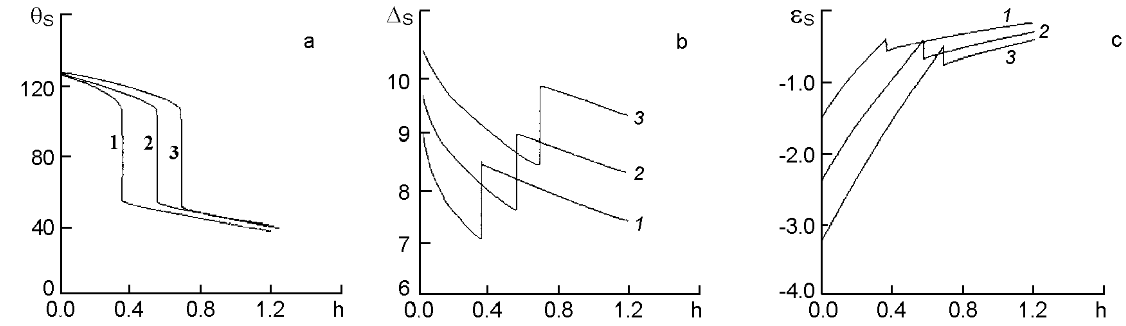

Fig. 3. Graphs showing the dependence of the 0^0 DW parameters on the value of external field h at various values L for the case ΔA = 0.2, ΔK_1 = 0.5, Q = 5, ΔK_u = -1.5, ΔM_s = 0.5,æ=1,42. Here, Curve 1 corresponds to L=4, 2 – L=5, 3 – L=7.

$D >> \Delta_0$, $Q > 1$, where $Q = K_u / 2\pi M_s^2$ is the factor of the material quality.

The numerical minimization results of the expression for the reduced energy of the 0°-DW ε_s ($\varepsilon_s = E/(2K_u L_x D\Delta_0)$) are presented in Fig. 2-5 where all the parameters having the length dimensionality are reduced to Δ_0. Besides, the jump values of material parameters ΔR are also reduced: $dR = \Delta R / K_u$. Here, the output parameters of the problem defining the structure and stability of the 0°-DW include its energy ε_s, width Δ_0 and amplitude θ_s. It follows from the calculations [3] that in the absence of an external field, 0°-DWs as stable formations exist within the certain ranges of defect and material parameters. The stability area thereof is limited by two limiting values: at some values, 0°-DWs collapse, at others, dissipate. Another important result of [3], which agrees with the experimental data [2], is that magnetic inhomogeneities originating at defects adapt to the defect profile.

The presence of an external field with $\boldsymbol{H} \parallel [111]$ affects essentially the structure and stability area of 0°-DW and, in particular, results in a situation where these inhomogeneities, differing in the direction of $\boldsymbol{M}$ in domains and having identical energy in the zero field, behave differently. Thus, it can be seen from Fig. 2, that when the field is switched on, the dimensions of the 0°-DWs with $\boldsymbol{M}$ | | [111] in domains decrease, whereas those of 0°-DWs with $\boldsymbol{M} \parallel [\bar{1}\bar{1}\bar{1}]$ in domains increase. This is explained by the trend of magnetic moments in 0°-DW to "rotate" towards the field direction. As the field increases further, the 0°-DWs with $\boldsymbol{M} \parallel [\bar{1}\bar{1}\bar{1}]$ in domains dissipate ($\Delta_s \to \infty$, $\theta_s \to \pi$), which is connected with unfavourableness of such a structure (here, magnetic moments are mainly directed against the field). In case of the 0°-DW with $\boldsymbol{M} \parallel [111]$ in domains (i.e. magnetic moments in domains are directed along the field), an increasing $\boldsymbol{H}$ may result in a jump-like transition of the 0°-DW from the structure, corresponding the LAS, to that characteristic of SAS [3]. And, as calculations show, the SAS width in the transition point exceeds that of LAS. Moreover, it is a more energy-favorable formation. The field value h_t at which the transition from LAS to SAS takes place depends essentially on the material parameters, on the defect characteristics (to the greatest degree) and the slab thickness. In particular, it is seen from Fig. 3 that as the defect size increases, the transition critical field h_t is shifted towards larger h values. This is explained by the fact that defect type under consideration, for which ΔR >0, is the factor that stabilizes the 0°-DW structure of the Bloch type. In this case, the larger is L, the more energy-favorable is the 0°-DW in the state characteristic of LAS.

The 0°-DW shows a similar behavior as a function of the slab thickness D (Fig. 2). Moreover, as D increases, the h_t values also shift towards stronger fields. Here, the critical factor is a reduction

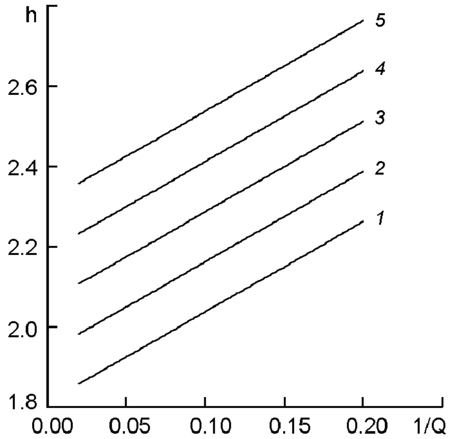

Fig. 4. Graph showing the dependence of the critical field on the value of $1/Q$ at various ΔK_1 for the case $\Delta A = 0.2$, $L = 5$, $Q = 5$, $\Delta K_u = -1.5$, $\Delta M_S = 0.5$, æ=1.42. Here, curve *1* corresponds to $\Delta K_1 = 0.3$, *2* – $\Delta K_1 = 0.4$, *3* – $\Delta K_1 = 0.5$, *4* – $\Delta K_1 = 0.6$, *5* – $\Delta K_1 = 0.7$.

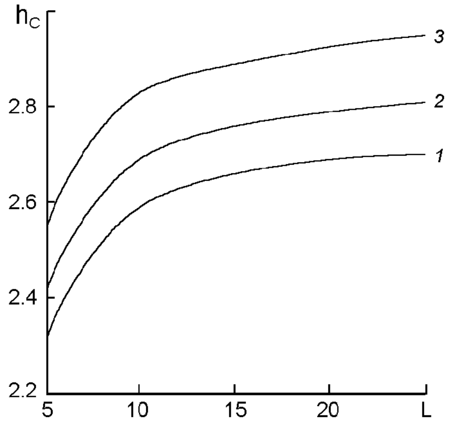

Fig. 5. Graphs showing the dependence of the critical field on the value of L at various Q for the case of $\Delta A = 0.1$, D=35; $\Delta K_1 = 0.5$, $\Delta K_u = -1.0$, $\Delta M_S = 0.1$, æ=1.42. Curve *1* corresponds to $Q = 5$, *2* – $Q = 7$, *3* – $Q = 10$.

in the influence of the slab demagnetizing fields on the stable conditions for the 0°-DW. Quite useful (from the practical point of view) information on this dependence can be obtained by plotting the dependence of the critical field h_c, at which the inhomogeneity under consideration disappears, on the Q^{-1} value. This critical field corresponds in essence to the field at which the reverse magnetization nuclei originate at the defect and is associated with the sample coercive force H_c [5]. It is seen from the calculations that the dependence obtained (Fig. 4) is a linear function of the Q^{-1} value, i.e.

$$h_c = \alpha + \beta Q^{-1} \tag{8}$$

where α, β are empirical constants. After substituting explicit expressions h_c and Q into that function, we get

$$H_c = \alpha \cdot 2K_u / M_s + 4\pi\beta M_s \tag{9}$$

which in its form (without considering the field sign) coincides with the expressions for H_c, obtained under various approximations [5-7]. In particular, in the theory of the magnetization reversal conditioned by the coherent rotation mechanism of the magnetic moments [5], $\alpha = 1$, $\beta = 1$. At the same time, in the linear theory of inhomogeneous magnetization reversal [6], $\beta = 1$, while the parameter α (the "microstructural" factor) depends on the defect characteristics. Moreover, $\alpha \sim \Delta R$ and the dependence of the "microstructural" factor on the defect dimensions is more complex: at small L ($L \le \Delta_0$), $\alpha \sim L^{-1}$, at greater L ($L >> \Delta_0$), α is practically independent of L. The dependences thus obtained provided to explain to some extent the "Brown's paradox" consisting in a significant divergence (by two or three orders) between the coercive force values obtained theoretically in the coherent rotation model of magnetic moments, and the experimental data for H_c. However, an essential drawback of these calculations (and others, too, for example [7]) consists in disregarding of the slab demagnetizing fields (to be more exact, taking them into account in the Winter approximation [8] which results only in renormalization of the induced uniaxial anisotropy constant).

The method developed in this work allows to consider the magnetostatic factor more fully and expand the applicability range of the expression (4) for H_c. Thus, it follows from the analysis of the 0°-DW stability area with respect to field that the β value still differs from 1 and depends on some material parameters (though weakly), in particular, on thickness D. It follows from Fig. 4 that

$\alpha \sim \Delta K_1$, which agrees with [6], while the dependence $\alpha(L)$ (Fig. 5) has a more complex nature: as L increases, the "microstructural" factor increases, asymptotically tending to a certain constant value. The latter does not altogether agree with the results [6] and indicates certain limitations of the model under consideration, in particular for small L. In this range of L values, the 0°-DW does not exist at all: it has collapsed long before these values. At the same time, the presence, in the zero field, of the stability limit of the 0°-DW with respect to the collapse on parameter L [3] does not fully agree with the experimental data, since at small L ($L < \Delta_0$) there is nevertheless a magnetic inhomogeneity of the 0°-DW type on the defect, but having a different structure. In further investigations, this circumstance can be taken into account and the model can be generalized, considering the 0°-DW with the quasi-Bloch structure, where $\theta = \theta(y)$ and $\varphi = \varphi(y)$ [7]. Such a model is a more challenging problem and requires a special research.

Thus, it follows from the above calculations that the considered model of reverse magnetization domains based on the concepts of 0°-DW agrees well enough with the models known before. At the same time, it has certain advantages over them. First, the presented model can describe the magnetization reversal processes in a wide class of materials, including the samples of finite dimensions and containing the structure defects. Second, it allows to find not only specific characteristics of the material that are important from the practical point of view (for example, coercive force $\boldsymbol{H}_c$) but also to study the structure and properties of magnetic inhomogeneities localized near a defect, i.e. to investigate the nucleation process on the defect when the crystal is re-magnetized from one state to another. Finally, the model possesses certain "resources" for its improvement. In particular, having considered the 0°-DW with the quasi-Bloch structure, it is possible to remove restrictions on the model application not only with respect to the defect width, but also with respect to the sample thickness, and to expand it to arbitrary values of the quality factor, including $Q \leq 1$.

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Процеси неоднорідного перемагнічування у реальних кристалах

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Розглядається модельне представлення магнітних неоднорідностей, що зароджуються в області дефектів кристала, за допомогою розподілу намагніченості, відповідного 0-градусній доменній межі. Показано, що дане рішення, що виникає у моделі необмеженого кубічного феромагнетика (без урахування дефектів у кристалі), що ідеалізується, в області переорієнтації спину, дозволяє вивчити не тільки кінетику фазового переходу, але і процеси перемагнічування реальних кристалів.