REVIEW OF FORMULAS TO DESCRIBE THE FATIGUE CRACK GROWTH RATE

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The paper presents a review of formulas of fatigue crack growth rate. The equations are divided into three groups according to the fatigue damage parameters used, i.e. stress, strain or displacement and energy. The parameter K or its ΔK range corresponds to brittle materials and to the initial stage of the crack of elastic-plastic materials. The parameter ε or CTOD is used in elastic-plastic materials and plastic materials to describe the yield strength. The energy approach is based on J parameter or the strain energy density W and corresponds to the whole range of the crack growth rate curve.

Keywords: *fatigue crack growth rate, stress intensity factor range, CTOD, ΔJ parameter.*

The fatigue crack growth rate is applied to describe the increasing slots in the material. Because of the applied stress, strain, displacement or energy approach, the fatigue crack growth rate can be presented versus one of the mentioned parameters. Description of the rate da/dN versus the stress parameter K is one of the most often applied descriptions. The energy approach using the J parameter in description of the curve $da/dN = f(\Delta J)$ was proposed by the authors [1] as the equation discussed in this paper. Relation between the range of ΔJ and the fatigue crack growth rate da/dN (da – crack length increment for dN of load cycles) was shown as the curve of crack growth kinetics [2] (see figure). The graph in figure is presented in a double logarithmic system and its shape is the reverse S. At low values of the range ΔJ for the given material and a constant stress ratio we have the threshold value of ΔJ_{th}. The fatigue crack does not develop below that value. In the threshold value range and in the case of short fatigue cracks, plasticity does not occur and equations of linear-elastic fracture mechanics can be applied (the range of the stress intensity factor ΔK is used for description). From figure it appears that instability occurs under higher values of ΔJ thus causing a quick increase of the crack growth rate to the critical value of J_{Ic} (just before total failure of the tested material). There are two possible reasons of such behaviour. First of all, the increasing crack length under constant loading causes the fact that the stress reaches the critical value. Then the observed unstable course of the crack growth curve is connected with the previous stages of brittle cracking [1]. Such behaviour takes place in brittle materials where the stress is dominating and the test results are related mainly to the linear crack mechanics. The other reason is connected with the crack growth causing reduction of the non-cracked area of the specimen that influences the total plasticization of the material under limited loading. That aspect concerns elastic-plastic and plastic materials where a visible yield point occurs and the materials are subjected to large plastic deformations. Then application of the parameter K or its range ΔK for the results description is senseless because the limitations of the linear-elastic fracture mechanics are exceeded.

The test results can be interpreted by means of the idea of strain energy density W or the parameter range ΔJ, including both stresses and strains at the crack tip [3].

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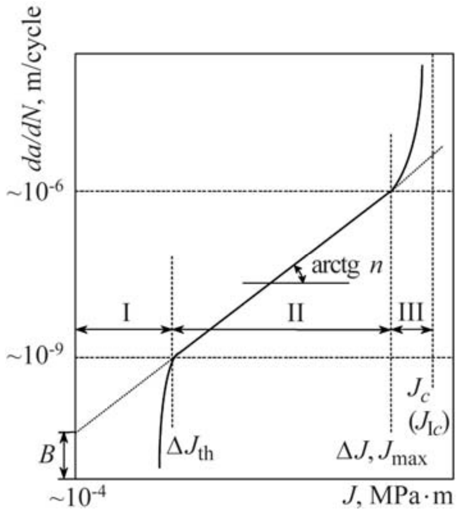
The aim of this paper is to review the formulas of fatigue crack growth rate.

The stress models. At the initial stage of fracture mechanics development the rate was described versus stress. In 1952 Stanley [4] presented the following equation for the crack growth rate description

$$\frac{da}{dN} = B\sigma_a^n a\,, \qquad (1)$$

where B and n are coefficients determined from experiments, σ_a is the stress amplitude and a – the crack length.



Fatigue crack growth rate curve in the energy approach.

Many other authors presented the relations including the stress amplitude or its range, and their descriptions can be found in [4, 5]. In those papers the authors gave various ideas of determination of da/dN versus stress, the mean value σ_m, the stress ratio R, material properties and so on depending on the needs resulting from the conducted experiments.

The idea formulated by Paris [6] was very important. He related the crack growth rate to the stress intensity factor K. According to that idea, the crack development is strongly influenced by the change of local stresses at its front, and parameter K describes the effect of loading and stress field in the front area. Thus, the crack increment is a function of the stress intensity factor

$$\frac{da}{dN} = B\,(\Delta K)^n\,. \qquad (2)$$

According to the Paris proposal, in this equation the coefficient B and the exponent n should be material constants. They could be understood in such a way though they depend on some other factors too (for example, loading). Eq. (2) is known as the Paris law, and it is widely used in many papers [7–10] concerning fatigue crack growth. Eq. (2) is valid for the next linear range of the crack kinetics curve (see figure) [11], and because of a big divergence of the results it is not valid in the first period when reaching the threshold value K_{th} (J_{th}) and in the third period when reaching the crack resistance K_c (J_c). The authors of [7] propose to extend the Paris equation and include the threshold stress intensity factor or its range

$$\frac{da}{dN} = B\left(\Delta K^n - \Delta K_{th}^n\right). \qquad (3)$$

The crack growth from the threshold value was known in the past. However, it was only in the seventies when the authors of [7] included those values in formulas expressing the propagating crack rate. The threshold values ΔK_{th} seemed to be interesting for the practical reasons because they concerned the crack growth rates of the order of 10^{-10} m/cycle. Such low rates can be expected in various structures or machine elements where the fatigue process is typical of a large number of cycles.

Next modification of Eq. (2) was done by Elber [12]. He included the crack closing and opening as a result of compressing and tensile residual stresses at the crack tip zone. He also introduced a new parameter, so-called the effective stress intensity factor

$$\Delta K_{eff} = U_{cl}\left(\Delta K\right), \tag{4}$$

where

$$U_{cl} = \frac{\sigma_{max} - \sigma_{op}}{\sigma_{max} - \sigma_{min}}, \tag{5}$$

where σ_{op} is the opening stress at the crack front.

Thus, Eq. (2) can be written as

$$\frac{da}{dN} = B\left(\Delta K_{eff}\right)^n \quad \text{or} \quad \frac{da}{dN} = B\left(U_{cl}\Delta K\right)^n. \tag{6}$$

Elber [12] tested different materials and found that the crack closure for steel took place under the stress from 0.15 to $0.3\sigma_{max}$, and for the aluminium alloy at $0.5\sigma_{max}$. He defined the relationship for the aluminium alloy AlCu4Mg1 that is equal to $U_{cl} = 0.5 + 0.4R$ (R – the stress ratio). The parameter K_{eff} depends on many factors, such as: a type of material, cycle asymmetry, mean stress, loading conditions. Eq. (6) is applied under overloading [13] and it has been used for description of the crack closure by many authors in many papers, e.g. in [14].

Priddle [15] was one of the first who proposed a description of the crack kinetics curve using the stress approach, $da/dN = f(\Delta K)$

$$\frac{da}{dN} = B\left(\frac{\Delta K - \Delta K_{th}}{K_c - K_{max}}\right)^n. \tag{7}$$

McEvily was another author who described the total crack kinetics curve [16]. There are many other relationships for determination of the total kinetics curve of the fatigue crack growth, some of them are presented in Table 1. Special attention should be paid to the formula proposed by Yarema [17]. Here the range ΔK from Eq. (7) is replaced by K_{max}, and the threshold range ΔK_{th} is replaced by K_{th}. Yarema proposed also the crack growth rate graphs in dimensionless quantities with the determined constant factor, so determination of K_{th} is not required. This quantity is introduced as the ratio K_c / K_{th}.

Some relations of the crack growth rates da/dN versus the parameter K (ΔK) are shown in Table 1.

Table 1. Formulae describing fatigue crack growth rate in the stress approach

№	Author, year	Formula	Source
1	2	3	4
1	Krafft, 1964	$\frac{da}{dN} = B\frac{\left(1-\gamma\right)^4}{1-\left(1+\gamma\right)^2}\left(\frac{K_{max}}{K_c}\right)^2$	[18]
2	Markocev, 1966	$\frac{da}{dN} = B_1 + \exp\left[B_2 K_{max}\right]$	[4]
3	Forman, Kearney, Engle, 1967	$\frac{da}{dN} = B\frac{\Delta K^n}{\left(1-R\right)K_c - \Delta K}$	[19]
4	Roberts, Erdogan, 1967	$\frac{da}{dN} = BK_{max}^2 \Delta K^n$	[20]

1	2	3	4
5	Czerepanov, 1968	$\frac{da}{dN}=B\left[\frac{K_{\max}^2-K_{\min}^2}{K_c^2}+\ln\frac{K_c^2-K_{\max}^2}{K_c^2-K_{\min}^2}\right]$	[21]
6	Lander, 1968	$\frac{da}{dN}=\frac{1-\nu^2}{2ER_m}\Delta K^2$	[22]
7	Pavlenko, 1969	$\frac{da}{dN}=B\left(1-R\right)^2 K_{\max}^n$	[4]
8	Erdogan, Ratwani, 1970	$\frac{da}{dN}=B\frac{\left(\frac{2}{1+R}\right)^n\left(K_c-K_{th}\right)^n}{K_c-\left(\frac{2}{1+R}\right)\Delta K}$	[23]
9	Morozov, 1971	$\frac{da}{dN}=B\left(1-R\right)\frac{K_{\max}^2\sqrt{a}}{R_m K_c}$	[24]
10	Smith, 1972	$\frac{da}{dN}=B\left[K_{\max}\left(1-R\right)^{0.5}\right]^n$	[4]
11	Nordberg, 1972	$\frac{da}{dN}=\frac{(\Delta K)^n}{(1-R)^m[(1-R)K_c-\Delta K]}$	[25]
12	Pearson, 1972	$\frac{da}{dN}=\frac{B\left(1-\beta\right)^{\alpha}\left(\Delta K-\Delta K_{th}\right)^n}{K_c-\left(1-\beta\right)\Delta K},\ \beta=\frac{1+R}{1-R}$	[26]
13	Pook, Forst, 1973	$\frac{da}{dN}=\frac{9}{\pi}\frac{(1-\nu^2)}{E}\Delta K^2$	[27]
14	Yarema, 1975	$\frac{da}{dN}=B\left(\frac{K_{\max}-K_{th}}{K_c-K_{\max}}\right)^n$	[17]
15	Branco et al., 1975	$\frac{da}{dN}=B\left[\frac{2K_m\left(\Delta K-\Delta K_{th}\right)}{K_{Ic}^2-K_{\max}^2}\right]^n$	[28]
16	McEvily, 1977	$\frac{da}{dN}=\frac{B}{R_{0.2}\cdot E}\left[\Delta K-\Delta K_{th}\cdot R\right]^2\cdot\left(1+\frac{\Delta K}{K_c-K_{\max}}\right)$	[29]
17	Miller, 1993	$\frac{da}{dN}=B\left[\left(\Delta\gamma\sqrt{\pi a}\right)-\Delta K_{th}\right]^n$	[30]

The strain and displacement models. The mixed cracks (brittle and plastic) are often met, so the researchers try to join the plastic strain and the crack growth rate. Manson [31] was one of the first who presented the following experimental relationship

$$\frac{da}{dN}=B\left(\Delta\varepsilon_p\sqrt{a}\right)^n, \qquad (8)$$

where $\Delta\varepsilon_p$ is the range of plastic strains at the crack front, a is the crack length.

In that paper, the notch influence on the material behaviour in the fatigue and creep processes under loading gradation was considered. Damages were added up by means of the Palmgren–Miner linear hypothesis.

Tomkins [32] presented a model of the rate increase for the plastic crack developing perpendicular to the loading direction

$$\frac{da}{dN} = \left(\Delta\varepsilon_p + b\Delta\varepsilon_s\right) r_p \,, \tag{9}$$

where $b \cong 1/6$, $\Delta\varepsilon_s$ is the range of elastic strains, $\Delta\varepsilon_p$ is the range of plastic strains, r_p is the size of the plastic strain zone at the crack front.

It has been found that the equation provides good results for small and big strains. Usability of the model has been discussed in [33].

Serensen and Makhutov [34] analyzed Eq. (2) while testing notched elements under a small number of cycles. In their opinion, in the case of higher plastic strains the stress range can be replaced by the strain range. Then the following relationship for determination of the crack growth rate should be valid

$$\frac{da}{dN} = B_\varepsilon \left(\Delta K_\varepsilon\right)^m . \tag{10}$$

In equation (10) we have a range of the strain intensity factor equal to

$$\Delta K_\varepsilon = \Delta\varepsilon\sqrt{a} \,. \tag{11}$$

It was proved that the exponent m in Eq. (10) is about two times lower than the exponent n in Eq. (2), and it usually equals 2.

In [35] the following equation for the crack growth rate was proposed for the short cracks

$$\frac{da}{dN} = \Delta\gamma_p r_{\max} \,, \tag{12}$$

where $\Delta\gamma_p$ is the plastic strain range under shearing, $r_{\max} = c - a$ is the maximum length of the plastic zone, a is a half of the crack length.

The tests presented in [35] were done with the analysis of the crack front development along the grain boundary and the passage of the slip bands was shown. In the proposed approach the crack growth rate is proportional to the shear strain range and the maximum size of the plastic zone. The equation was verified on the basis of the experimental results obtained in the tests under uniaxial tension, at different stress amplitudes and the mean stress values.

Other authors proposed some more models for the fatigue crack growth rate in the strain approach (see Table 2).

Eq. (5) in Table 2 seems to be interesting. The author [33] defines the semi-elliptic fatigue crack growth rate on the specimen surface versus the plastic strain range $\Delta\varepsilon_p$. This relation allows describing the crack growth rate for various levels of tensile stress and the stress ratio R. The value of the directional coefficient k_α applied in the equation should correspond to the directions favorable for the slip bands start. For an infinitely large disk with the central crack loaded by uniaxial tensile stress there are two perpendicular directions of the maximum shear stresses inclined at 45° to the crack plane.

Donahue et al. [39] proposed one of the first displacement models for description of the fatigue crack growth rate using the crack tip opening displacement (δ, CTOD)

$$\frac{da}{dN} = B\left(\sqrt{\delta_{\max}} - \sqrt{\delta_{\min}}\right)^n . \tag{13}$$

Li [40] uses the range of the crack tip opening displacement for description of the mixed cracking mode I and II as

$$\frac{da}{dN} = B\left(\Delta \mathrm{CTD}_{\mathrm{I+II}}\right)^{n}, \tag{14}$$

where $\Delta\overline{\mathrm{CTD}} = \Delta\overline{\mathrm{CTOD}}_{\mathrm{I}} + \Delta\overline{\mathrm{CTSD}}_{\mathrm{II}}$ and $\mathrm{CTOD}_{\mathrm{I}} = \frac{4K_{\mathrm{I}}}{\pi E R_e}\sqrt{K_{\mathrm{I}}^2 + 3K_{\mathrm{II}}^2}$ are the vectors of crack tip opening displacement acting in the direction of mode I crack growth, $\mathrm{CTSD}_{\mathrm{II}} = \frac{4K_{\mathrm{II}}}{\pi E R_e}\sqrt{K_{\mathrm{I}}^2 + 3K_{\mathrm{II}}^2}$ is the effective vector of crack tip sliding displacement acting in the tensile growth direction of a mode II fatigue crack, i.e. 45° from the extension line of original crack.

Table 2. Formulae describing fatigue crack growth rate in the strain approach

№	Author, year	Formula	Source
1	Manson, 1966	$\frac{da}{dN} = \left[C_1\Delta\varepsilon_p + C_2\left(\Delta\varepsilon_p\right)^2\right]a$	[31]
2	McEvily, Johnston, 1965	$\frac{da}{dN} = \frac{2\sigma_{\max}^4 a^2}{E(R_{0.2} + R_m)\varepsilon_2 R_{0.2}^2}$	[36]
3	Gillemot, 1971	$\frac{da}{dN} = \frac{C}{W_c}\Delta\varepsilon_p^m$	[37]
4	Taira, Tanaka, 1971	$\frac{da}{dN} = C_1 r_p^{m_1}, \quad r_p = aC_2\left[\sec\frac{\pi\sigma}{2C_3R_{0.2}} - 1\right]$	[38]
5	Werner, 2000	$\frac{da}{dN} = \frac{1}{8}Ck_\alpha\sqrt{1+\frac{2a}{h}}\left(1+\frac{b}{g}\right)^{n\left(\frac{b}{g}\right)^2}\left[\frac{(1-R)\sigma_{\max}}{R_e}\right]^2\frac{b^2}{\phi^2 a}\Delta\varepsilon_p,$ $k_\alpha = \cos^2\frac{\alpha}{2}\left(1+3\sin^2\frac{\alpha}{2}\right), \ \phi = \int_0^{\pi/2}\left(1-\frac{a^2-b^2}{a^2}\right)^{1/2}d\varphi$	[33]

The energy models. Dowling and Begley [1] proposed the energy equation for description of the second (linear) range of the crack growth kinetics curve versus the range of the ΔJ parameter

$$\frac{da}{dN} = B(\Delta J)^n. \tag{15}$$

The above equation was verified for the elastic-plastic material under loading controlled by displacement and force. A good conformity of the experimental results and those obtained according to Eq. (15) was found in the case of displacement control. In the case of the loading control many differences were found. In the authors' opinion the proposed relationship does not give satisfactory results, so another expression including the mean loading level should be looked for.

Lu and Kobayashi [41] introduced an experimental parameter $J_{\max}$ in order to predict the fatigue crack growth da/dN

$$\frac{da}{dN} = (1-R)^{n_1} B J_{\max}{}^{n_2} , \tag{16}$$

where n_1, n_2 and B are coefficients determined from experiments.

Specimens of CT type (Compact Tension) were tested under tension for various stress ratios R = 0.05, 0.6 and 0.7. It has been shown that for the increasing ΔK the parameter $J_{\max}$ can be used as an important index for predicting characteristics of the fatigue crack growth in elastic-plastic materials.

Rozumek and Gasiak [42] proposed a nonlinear formula (modification of the Forman equation) for description of the second and third range of the crack growth kinetics curve in the energy approach (see figure)

$$\frac{da}{dN} = \frac{B(\Delta J)^n}{(1-R)^m J_{\mathrm{I}c} - \Delta J} . \tag{17}$$

This relationship includes not only experimental coefficients B and n, but also the mean level of loading by the stress ratio R and the critical value of the parameter $J_{\mathrm{I}c}$. The presented Eq. (17) was verified during tests under loading controlled by force for three steels [43, 44] under cyclic tension and bending, and for the aluminium alloy [2] under cyclic bending.

In [45], the author proposed the equation for description of the total curve of the crack growth kinetics versus the range of parameter ΔJ, i.e. from the range of the threshold value ΔJ_{th} to the critical value of the parameter $J_{\mathrm{I}c}$. In preliminary considerations the presented experimental relation did not include the mean level, and it caused the differences between the experimental and theoretical results for higher stress mean values. After modification the equation for the total crack kinetics curve in the energy approach takes the following form

$$\frac{da}{dN} = B\left[\frac{\Delta J - \Delta J_{th}}{(1-R)^2 J_{\mathrm{I}c} - J_{\max}}\right]^n , \tag{18}$$

where $\Delta J = J_{\max} - J_{\min}$, B and n are experimental coefficients; $J_{\max}$ is the maximum value of parameter J.

In order to include the crack closure in the elastic-plastic materials the authors of [46] proposed the following equation

$$\frac{da}{dN} = B(\Delta J_{eff})^n . \tag{19}$$

That relation was applied for description of short fatigue cracks under non-proportional loadings in cyclically hardened and softened materials. Satisfactory results were obtained according to Eq. (19) for multiaxial non-proportional loadings.

The authors of [47] proposed a relationship for description of the crack growth rate in the energy approach based on the strain energy density factor range

$$\frac{da}{dN} = B(W)^n . \tag{20}$$

The experimental coefficients from Eq. (20) can be calculated in a similar way as in the case of the Paris equation.

The above formulae for description of the fatigue crack growth rate are widely used in literature, and they concern stress, strain (displacement) and energy approaches.

Other parameters for description of the crack growth rate were also looked for. In [48], the influence of the material structure on the crack growth rate was presented

$$\frac{da}{dN} = B_S (S)^{n_S}, \tag{21}$$

where $S = \pi r_i^2 l / d$, l is cylinder length equal to thickness of the specimen, d is mean size of the grain, $r_{i_{\min}} = 0.02(K_\mathrm{I} / \sigma_y)^2$ is cylinder radius, σ_y is yield point.

The influence of the plastic zone size and interphase surface fraction on the crack propagation rate was considered [48]. It was assumed that generation of damages depended on a number of sources of dislocations, distribution of which was determined by the interphase surface.

CONCLUSIONS

The presented relations describe mainly empirical relationships resulting from the tests. The stress approach for determination of the fatigue crack growth rate is the most often applied because of an easy way of this parameter verification. At first, the parameter was based directly on the stress σ and many better or worse equations for its description were proposed. Paris introduced the parameter K for determination of the crack growth rate, and it seemed to be the best for such considerations in the stress range. As compared with the stress criteria applied for the crack growth rate description a number of formulas based on the parameter J (range ΔJ) is low because of the problems with determination of energy (strain) with use of J or W. Similar remark can be related to the strain or displacement approach. New calculation and measuring techniques allow developing the energy parameter. Thus, many authors came to the conclusion that the stress approach to description of the crack growth rate curve was not able to represent the test results for the elastic-plastic or plastic materials, especially in the case of the third period of the crack growth curve. The stress approach represented by Paris [6] and Forman [19] and the energy approach represented by Dowling and Begley [1] are the most known and widely used.

РЕЗЮМЕ. Подано формули для опису швидкості росту тріщини. Вони розділені на три групи відповідно до використаних параметрів руйнування, а саме: напруження, деформація чи переміщення, енергія. Коефіцієнт інтенсивності напружень K та його амплітуда ΔK використані для дослідження крихких матеріалів та початкової стадії росту тріщини у пружно-пластичних матеріалах. Енергетичний підхід ґрунтується на параметрі J або густині енергії деформації W.

РЕЗЮМЕ. Представлены формулы для описания скорости роста трещины. Они разделены на три группы в соответствии с использованными параметрами разрушения, а именно: напряжение, деформация или перемещение, энергия. Коэффициент интенсивности напряжений K и его амплитуда ΔK использованы для исследования хрупких материалов и начальной стадии роста трещины в упругопластических материалах. Энергетический подход основан на параметре J или плотности энергии деформации W.

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