CALCULATIONS OF WAVE FUNCTIONS OF THE *nd*-SYSTEM, PHASES, AND CROSS-SECTIONS OF *nd*-SCATTERING WITH THE USE OF MODIFIED FADDEEV'S EQUATIONS AND THE METHOD OF HYPERSPHERICAL FUNCTIONS

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The neutron-deuteron wave functions, *nd* phases, and *nd* scattering cross-sections have been calculated for neutron energies below the deuteron breakup threshold. Starting from the modified Faddeev's equations which have got in our previous paper, the most complicated part of the full wave function describing the three-nucleon motion in the NN interaction region is separated and expanded into a series in hyperspherical polynomials with $K = 0, 1, 2$. The system of one-dimensional integral equations for the expansion coefficients has been constructed and solved numerically for incident neutron energies of 2.45, 3, and 3.27 MeV. The Malfliet–Tjon and the Hulthén local NN potentials were used in calculations. The calculated *nd* phases and *nd* scattering cross-sections satisfactorily fit the corresponding experimental data.

1. Introduction

The contemporary theory of nuclei has mainly the phenomenological character, and its microscopic substantiation is very poorly developed [1]. Therefore, the important step to a better physical comprehension of multiparticle nucleonic systems and processes with their participation is the study of namely three-nucleon systems. As compared with the binary systems, the description of three-particle states in the continuous spectrum is a nontrivial theoretical problem. This is related, first of all, to the possibility of the processes of redistribution and fission in a three-particle system in addition to the elastic scattering. Second, the mentioned systems have essentially different energy spectra. Whereas the spectrum of a binary system is composed separately from discrete levels and the continuum, a three-particle system has a purely degenerate spectrum, i.e. a certain value of the total energy can correspond to physically different states of the system. At the present time, the well-developed powerful methods of calculation of three-particle wave functions in the continuum are available. Among them, the most known methods are one based on the Faddeev equations [1–3] and the variational method of Kohn–Hulthén with the use of expansions in a hyperspherical basis [4].

The Faddeev method consists in that the Schrödinger equation for a three-particle wave function is transformed and is reduced to a system of three equations [2] with boundary conditions, like the Lippmann–Schwinger equation [1]. Each of the Faddeev equations is a six-dimensional integral equation, whose solution is firstly started by its transformation into an infinite system of coupled integral equations. Then this collection of equations is truncated and is solved by ordinary numerical methods [3]. An essential drawback of the method consists in the impossibility to directly use the potentials with infinite action radius such as, for example, the Coulomb potential [5]. Within this method, the solution of the problem on the scattering of a proton by a deutron is extremely complicated and, in addition, is not strictly mathematically substantiated [5–7], as distinct from the formalism of the Faddeev equations themselves.

Another approach to the problem of *nd*-scattering is the solution of the input three-nucleon Schrödinger equation by the variational method of Kohn–Hulthén [8]. The wave function of the system of nucleons is separated into two parts – short-range and asymptotic ones [9]. The latter can be presented in the ordinary way in terms of the incident and scattered waves and the collision matrix. The short-range parts is expanded in a series in hyperspherical functions. The coefficients of this expansion and the scattering phases can be determined after the application of a variational procedure. Both approaches, the variational one and the Faddeev method, give very close results at the fitting of experimental data on the *nd*- and *pd*-scattering [10, 11]. But, in the latter case, only the variational method can be strictly substantiated at the consideration of the Coulomb interaction of particles.

The aim of our work is to develop and to use a method of calculation of the cross-sections of *nd*-scattering at energies below the deuteron breakup threshold. Such a study was begun else in our previous works. Starting from the well-known Faddeev equations [2] and using the expansion in hyperspherical functions, we reduced the problem on the determination of a neutron-deuteron wave function in the continuum to a system of one-dimensional integral equations, whose solution requires a significantly less computer time than the numerical solution of the Faddeev equations. We used simple model NN potentials with a nonseparable interaction without spin-isospin dependence. These potentials, as test ones, are used in similar tasks by many researchers for a long time [12–18]. Of course, the Faddeev equations containing only spatial variables can be extended on the case where the interaction and the wave function depend on spin and isospin variables [19]. But they will be quite different complicated equations which are not considered here. In our case of spinless wave functions and particles with identical masses, the Faddeev equations themselves can be considered, in this sense, as model equations. Moreover, the specific results obtained with their help for cross-sections indicate that the effect of the spin degrees of freedom is quite small at fitting the experiment at least in some kinematic regions, but in many cases. In calculations, we also neglect the doublet component of the scattering amplitude, since it follows from the phase analysis data [20, 21] that the contribution of the doublet state to the scattering cross-section is about 1%. Despite the mentioned approximations, we managed to obtain a satisfactory agreement with experiments on the cross-sections and on the scattering phases. The present work is a continuation and a natural development of the cycle of our works devoted to the study of the scattering of a particle on a system of two coupled particles which was started in [22].

2. Formalism

We are based on the well-known Faddeev equations [2] written for a system of three strongly interacting particles with identical masses m, in which one particle is free, and two ones are bound:

$$\Psi^{(1)} = \Phi + G_0(Z)T_{23}(Z)(\Psi^{(2)} + \Psi^{(3)});$$

$$\Psi^{(2)} = G_0(Z)T_{31}(Z)(\Psi^{(3)} + \Psi^{(1)});$$

$$\Psi^{(3)} = G_0(Z)T_{12}(Z)(\Psi^{(1)} + \Psi^{(2)}), \tag{1}$$

Here,

$$\Psi = \Psi^{(1)} + \Psi^{(2)} + \Psi^{(3)} \tag{2}$$

is the total three-particle wave function; Φ is the asymptotic wave function which the product of the plane wave with a momentum of the relative motion of the 1-st particle and the coupled system of two other particles and the wave function of a bound state of particles 2 and 3; $G_0(Z) = (Z - H_0)^{-1}$; $Z = E \pm i0$; E is the total energy of the system; H_0 is the operator of kinetic energy; T_{ij} are the two-particle transition operators which are connected with the pairwise potentials V_{ij} $(ij = 12, 23, 31)$ by the equations

$$T_{ij}(Z) = V_{ij} + V_{ij}G_0(Z)T_{ij}(Z)\,. \tag{3}$$

Here and below, we use the systems of units, in which $\hbar = c = 1$; all kinematic quantities are referred to the center-of-mass system (unless otherwise stated). Substituting (3) in system (1) and adding the equations, we obtain a single equation for the total wave function

$$\Psi = \Phi + G_0(Z)(U\Psi - V_{23}\Phi), \quad U = V_{12} + V_{23} + V_{31}\,, \tag{4}$$

which contains, like the Faddeev equation, the same boundary conditions and has also a unique solution, because it was obtained from (1)–(3) with the use of only the addition without the division by operators.

Let us expand the difference $\Psi - \Phi$ in a series in K-harmonics:

$$\Psi - \Phi = \sum_{Kn} B_{Kn}(\rho)u_{Kn}(\Omega). \tag{5}$$

Substituting this expansion in (4) and using the condition of normalization for K-harmonics, we get the system of coupled integral equations for the functions $B_{Kn}(\rho)$ [23]

$$B_{K'n'}(\rho) = \frac{\pi m}{\rho^2}\int\limits_0^\infty d\bar{\rho}\,\bar{\rho}^3 P_\pm^{(K')}\int d\Omega\, u^*_{K'n'}(\Omega)\times$$

$$\times \left\{ U \sum_{Kn} B_{Kn}(\bar\rho) u_{Kn}(\Omega) + (V_{12} + V_{31})\Phi \right\}, \qquad (6)$$

$$P_{\pm}^{(K')} \equiv P_{\pm}^{(K')}(\rho,\bar\rho) = -\frac{2}{\pi}\int_0^\infty dq\, q\, \frac{J_2(q\rho)\, J_2(q\bar\rho)}{q^2 - k_{K'}^2 \mp i0}, \qquad (7)$$

where $k_{K'}^2 = k_0^2 - K'(K'+4)/\rho^2$, $k_0^2 = 4m(E_n - \varepsilon)/3$, m is the nucleon mass, E_n is the kinetic energy of an incident neutron, ε is the binding energy of a deuteron. The potentials V_{ij} and the function Φ on the right-hand side of Eq. (6) depend on the hyperradius $\bar\rho$ and five angular variables Ω. We emphasize one more that the boundary conditions for the Faddeev equations are contained also in Eqs. (6), because they are obtained directly from (1) (see [23]). Since we consider the elastic scattering, we need only the function $P_{+}^{(K')}$ defined in (7) for the subsequent calculations.

In our previous work [24], we restricted ourselves only by the first term in expansion (5) with $K=0$ in calculations of the nd-scattering cross-sections. By using the formalism developed in [23], we now take else the terms with $K=1$ and $K=2$ into account. The calculation of these terms will give us the possibility to establish, first, that series (5) converges, indeed, rapidly, and, second, that the corrections to the scattering amplitude, which are related to the harmonics $K=1,2$, are relatively small.

Let $\mathbf{r}_1$, $\mathbf{r}_2$, $\mathbf{r}_3$ be the radius-vectors of particles 1,2,3. For each K, we introduce the collection of quantum numbers $n \equiv \{\ell_x, \ell_y, L, M\}$ [23,25], where ℓ_x is the orbital moment of the coupled pair of particles 2 and 3, ℓ_y is the orbital moment of the 1-st particle relative to the center of masses of pair (23), x and y are the corresponding Jacobi coordinates: $\mathbf{x} = (\mathbf{r}_2 - \mathbf{r}_3)/\sqrt{2}$ and $\mathbf{y} = \sqrt{2/3}(\mathbf{r}_1 - (\mathbf{r}_2 + \mathbf{r}_3)/2)$. The quantum numbers L and M are, respectively, the total orbital moment and its projection.

Retaining only the terms with $K = 0,1,2$ in expansion (5), we have, in the general case, 27 coupled integral equations for the unknown functions $B_{Kn}(\rho)$ (by the number of collections Kn [26]). We denote each of the K-harmonics in the following way [23]:

$$u_K^{\ell_x \ell_y LM} \equiv \Phi_j(\Omega), \quad j = 1,2,...,27. \qquad (8)$$

These K-harmonics have a rather simple form [23]. Moreover, what is very important, the system of integral equations (6) is essentially simplified with their use. Namely, it becomes a collection of two systems of equations with two unknowns and 23 uncoupled equations, among which 22 equations are homogeneous. Since these homogeneous equations are Volterra equations of the second kind, it follows from the results in [27, 28] that such equations have only trivial (zero) solutions (which is supported, by the way, by direct calculations) under the condition of square integrability of the kernel (it holds in our case).

Thus, all five inhomogeneous equations, which remain in (6) and should be solved, are related to the following K-harmonics [23, 26]:

$$\Phi_1 \equiv u_0^{0000} = \frac{1}{\sqrt{\pi^3}}, \quad \Phi_7 \equiv u_1^{0110} = \sqrt{\frac{6}{\pi^3}} \sin\theta \cos\theta_y,$$

$$\Phi_{12} \equiv u_2^{2020} = \frac{2}{\sqrt{\pi^3}} \cos^2\theta (3\cos^2\theta_x - 1),$$

$$\Phi_{17} \equiv u_2^{0220} = \frac{2}{\sqrt{\pi^3}} \sin^2\theta (3\cos^2\theta_y - 1),$$

$$\Phi_{26} \equiv u_2^{0000} = \frac{2}{\sqrt{\pi^3}} \cos 2\theta. \qquad (9)$$

The corresponding coefficients $B_1(\rho)$, $B_7(\rho)$, $B_{12}(\rho)$, $B_{17}(\rho)$, and $B_{26}(\rho)$ in expansion (5) are the solutions of the integral equations

$$B_1(\rho) = \frac{8m}{\rho^2}\int_0^\infty d\bar\rho\, \bar\rho^3\, P_+^{(0)} \left\{ \int_0^{\pi/2} d\theta\, \sin^2\theta\, \cos^2\theta \left(B_1(\bar\rho) + \right.\right.$$

$$\left. +2B_{26}(\bar\rho)\cos 2\theta \right) V^* + \frac{\sqrt{2/3}\,\pi^{3/2}}{p\,\bar\rho} \int_0^{\pi/2} d\theta\, \sin\theta\, \cos^2\theta$$

$$\left. \phi(\sqrt{2}\,\bar\rho\cos\theta) \sin(\sqrt{3/2}\, p\,\bar\rho \sin\theta)(V^* - 2V_{23}) \right\}; \qquad (10)$$

$$B_{26}(\rho) = \frac{16m}{\rho^2}\int_0^\infty d\bar\rho\, \bar\rho^3\, P_+^{(2)} \left\{ \int_0^{\pi/2} d\theta\, \sin^2\theta\, \cos^2\theta\, \cos 2\theta \times \right.$$

$$\times \left(B_1(\bar\rho) + 2B_{26}(\bar\rho)\cos 2\theta \right) V^* + \frac{\sqrt{2/3}\,\pi^{3/2}}{p\,\bar\rho} \times$$

$$\times \int_0^{\pi/2} d\theta\, \sin\theta \cos^2\theta\, \cos 2\theta \phi(\sqrt{2}\,\bar\rho\cos\theta) \times$$

$$\times \sin(\sqrt{3/2}\, p\,\bar\rho \sin\theta)(V^* - 2V_{23})\Big\}; \qquad (11)$$

$$B_7(\rho) = \frac{16m}{\rho^2}\int\limits_0^\infty d\bar\rho\,\bar\rho^3\, P_+^{(1)}\Big\{\int\limits_0^{\pi/2} d\theta\, \sin^4\theta\, \cos^2\theta\, B_7(\bar\rho)\, V^* +$$

$$+\, i\,\frac{\pi^{3/2}}{p\,\bar\rho}\int\limits_0^{\pi/2} d\theta\, \sin^2\theta\, \cos^2\theta\, \phi(\sqrt{2}\,\bar\rho\cos\theta)(V^* - 2V_{23})\times$$

$$\times\left[\frac{\sin(\sqrt{3/2}\, p\,\bar\rho\sin\theta)}{\sqrt{3/2}\, p\,\bar\rho\sin\theta} - \cos(\sqrt{3/2}\, p\,\bar\rho\sin\theta)\right]\Big\}; \qquad (12)$$

$$B_{12}(\rho) = \frac{128m}{5\rho^2}\int\limits_0^\infty d\bar\rho\,\bar\rho^3\, P_+^{(2)}\Big\{\int\limits_0^{\pi/2} d\theta\, \sin^2\theta\times$$

$$\times\cos^6\theta\, B_{12}(\bar\rho)\, V^* + \frac{5\sqrt{2/3}\,\pi^{3/2}}{8p\,\bar\rho}\int\limits_0^{\pi/2} d\theta\, \sin\theta\, \cos^4\theta\times$$

$$\times\phi(\sqrt{2}\,\bar\rho\cos\theta)\sin(\sqrt{3/2}\, p\,\bar\rho\sin\theta)\times$$

$$\times\int\limits_0^\pi d\theta_x\, \sin\theta_x(3\cos^2\theta_x - 1)(V_{12} + V_{31})\Big\}; \qquad (13)$$

$$B_{17}(\rho) = \frac{128m}{5\rho^2}\int\limits_0^\infty d\bar\rho\,\bar\rho^3\, P_+^{(2)}\Big\{\int\limits_0^{\pi/2} d\theta\, \sin^6\theta\times +$$

$$\times\cos^2\theta\, B_{17}(\bar\rho)\, V^* + \frac{5\,\pi^{3/2}}{6p^2\,\bar\rho^2}\int\limits_0^{\pi/2} d\theta\, \sin^2\theta\times$$

$$\times\cos^2\theta\,\phi(\sqrt{2}\,\bar\rho\cos\theta)(V^* - 2V_{23})\Big[\Big(\sqrt{3/2}\, p\,\bar\rho\sin\theta -$$

$$-\frac{3}{\sqrt{3/2}\, p\,\bar\rho\sin\theta}\Big)\sin(\sqrt{3/2}\, p\,\bar\rho\sin\theta) +$$

$$+3\cos(\sqrt{3/2}\, p\,\bar\rho\sin\theta)\Big]\Big\}. \qquad (14)$$

In (10)–(14), the function $\phi(\sqrt{2}\bar\rho\cos\theta)$ is the wave function of a deuteron, and the quantities V^*, V_{23}, V_{12}, and V_{31} are defined as follows:

$$V^* = 2V_{23} + \int\limits_0^\pi d\theta_x \sin\theta_x(V_{12} + V_{31}),$$

$$V_{23} \equiv V(\sqrt{2}\bar\rho\cos\theta),$$

$$V_{12} \equiv V\left(\frac{\bar\rho}{\sqrt{2}}\sqrt{1 + 2\sin^2\theta - \sqrt{3}\sin 2\theta\cos\theta_x}\right),$$

$$V_{31} \equiv V\left(\frac{\bar\rho}{\sqrt{2}}\sqrt{1 + 2\sin^2\theta + \sqrt{3}\sin 2\theta\cos\theta_x}\right),$$

where V is the nucleon-nucleon potential.

We note that the used approach possesses, in addition, the advantage allowing us to numerically solve a system of integral equations for a small number of functions depending on a single continuous variable.

3. Analysis of the Results of Calculations and Conclusions

Equations (10)–(14) were solved numerically for several values of energies of incident neutrons E_n (2.45, 3, and 3.27 MeV in the laboratory reference system). In calculations, we used the following NN interaction potentials:

1) Hulthén potential [12]

$$V(r) = -\frac{\lambda_{\rm H}}{\exp(\mu_{\rm H} r) - 1}\,, \quad \lambda_{\rm H} = 0.177\ \ {\rm fm}^{-1},$$

$$\mu_{\rm H} = 1.145\ \ {\rm fm}^{-1}; \qquad (15)$$

2) Malfliet–Tjon triplet potential [13] with a repulsive soft core

$$V(r) = -\lambda_A\exp(-\mu_A r)/r + \lambda_R\exp(-\mu_R r)/r\,,$$

$$\lambda_A = 3.22\ \ {\rm fm}^{-1},\quad \mu_A = 1.55\ \ {\rm fm}^{-1},$$

$$\lambda_R = 7.39\ \ {\rm fm}^{-1},\quad \mu_R = 3.11\ \ {\rm fm}^{-1}. \qquad (16)$$

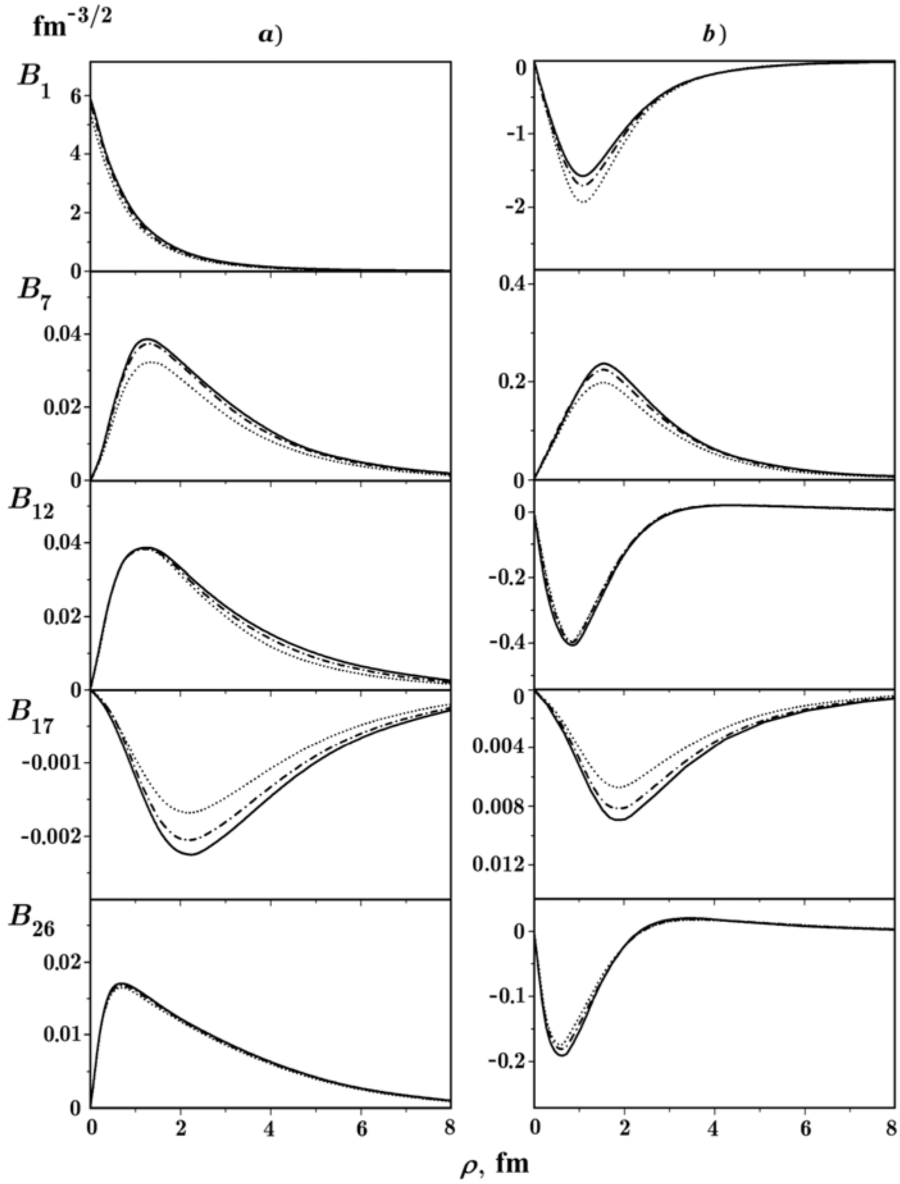

Fig. 1. Functions $B_j(\rho)$ $(j = 1, 7, 12, 17, 26)$ calculated for the potentials of Hulthén *a*) and Malfliet–Tjon *b*) at energies of an incident neutron of 2.45 MeV (dotted curves), 3 MeV (dash-dotted), and 3.27 MeV (continuous)

The wave function of a deuteron was chosen in the form [1]

$$\varphi(r) = \sqrt{\frac{\alpha\beta(\alpha+\beta)}{2\pi(\beta-\alpha)^2}}\,\frac{\exp(-\alpha r)-\exp(-\beta r)}{r} \qquad (17)$$

with the parameters $\alpha = \sqrt{m\varepsilon}$ and $\beta \simeq 7\alpha$.

The calculated functions $B_j(\rho)$ $(j = 1, 7, 12, 17, 26)$ are shown in Fig. 1. The analysis of the curves presented in this figure implies that, first, $B_j(\rho)$ depend weakly on the energy of a neutron in the interval $E_n = 2.45 \div 3.27$ MeV. Second, the maximum values of $B_1(\rho)$ (for the basic K-harmonic with $K = 0$) exceed, in modulus, the maximum values of the functions $B_j(\rho)$ by at least one order of magnitude for K-harmonics with $K = 1$ and $K = 2$.

In Table 1, we present separate contributions of each of five nonzero partial amplitudes A_j $(j = 1, 7, 12, 17, 26)$ to the nd-scattering amplitude A calculated for the same interaction potentials and values of E_n. In the last col-

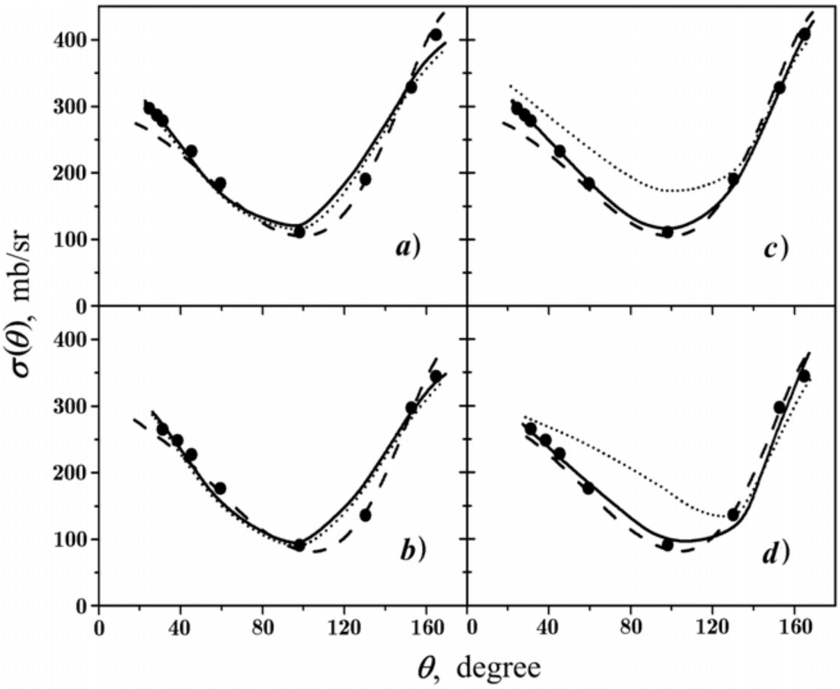

Fig. 2. Differential nd-scattering cross-sections calculated with the use of the potentials of Hulthén (a, b) and Malfliet–Tjon (c, d) at the energies of a neutron $E_n = 2.45$ MeV (a, c) and 3.27 MeV (b, d). The explanation of the curves is given in the text. The experimental data are taken from [29]

T a b l e 1. Contributions of A_j to the nd-scattering amplitude from each of five nonzero partial amplitudes ($j = 1, 7, 12, 17, 26$)

Potential	E_n, MeV		A_1	A_7	A_{12}	A_{17}	A_{26}	δA, %
		K	0	1	1	2	2	
Hulthén	2.45		−7.019	−0.141	−0.158	−0.003	−0.074	5.1
	3		−7.616	−0.174	−0.173	−0.004	−0.076	5.3
	3.27		−8.011	−0.190	−0.179	−0.005	−0.077	5.3
Malfliet–Tjon	2.45		25.061	−1.569	-0.025	−0.013	−0.738	8.6
	3		23.098	−1.991	−0.018	−0.020	−0.745	13.6
	3.27		22.122	−2.212	−0.014	−0.024	−0.749	15.7

umn of Table 1, we give the relative contributions δA (in percents) for higher K-harmonics with $K = 1$ and $K = 2$,

$$\delta A = \frac{A_7 + A_{12} + A_{17} + A_{26}}{A_1 + A_7 + A_{12} + A_{17} + A_{26}}\,. \tag{18}$$

Table 1 indicates that the quantity δA for the Hulthén potential almost does not depend on E_n, by reaching 5.3%, whereas δA for the Malfliet–Tjon potential increases significantly even within a short interval in energies (from 8.6% for $E_n = 2.45$ MeV to 15.7% for $E_n = 3.27$ MeV).

In Table 2, we show the calculated nd-scattering phases for the relative orbital moments $\ell = 0, 1, 2$ for potentials (15) and (16) and two energies $E_n = 2.45$ and 3.27 MeV, for which the experimental data on the elastic scattering of neutrons on deuterons are available [29]. For comparison, Table 2 presents also the phases obtained by other authors in [18, 20, 30–32] for an analogous problem on the nd-scattering. The quantities η are the dimensionless normalizing coefficients [24] for the internal part of the total wave function of the three-particle scattering problem.

In Fig. 2,(a–d), we give the angular distributions of the nd elastic scattering cross-sections which are calculated with the use of potentials (15) and (16). The continuous curves in Fig. 2 are calculated with regard for all five functions $B_j(\rho)$, i.e. three K-harmonics with $K = 0, 1, 2$ were taken in expansion (5). The dotted curves correspond to the case where only $B_1(\rho)$ (the term with $K = 0$) was taken in (5). The analysis of the curves presented in Fig. 2 indicates that the contribution of the higher K-harmonics ($K = 1, 2$) to the cross-section for the Hulthén potential is at most $\sim 5\%$. But such a contribution for the Malfliet–Tjon potential can be significant (especially at the minima of angular distributions). The continuous curves describe the experimental data satisfactorily in all the cases. For comparison, we give the results of the Pisa group [33] in Fig. 2 (dashed curves) which fit the experimental data in [29]. Despite the fact that the calculations of the cross-sections in [33] involved the realistic NN potential AV18 (moreover, the three-particle interaction was also taken into account), the results of our work and work [33] are quite close. Some difference of the S, P, and D phases calculated by us from the results of other authors (see Table 2) can be explained by the approximation used in the solution of the problem: we considered only those factors which make the main contribution to the scattering amplitude.

Thus, the method developed here for the calculation of the elastic nd-scattering cross-sections within simple models of the NN interaction allows one to satisfactorily describe the relevant experiments at energies of a neutron below the deuteron breakup threshold. By representing the total wave function of the problem Ψ as a sum of its asymptotic part and a part which describes the three-nucleon system in the interaction region, we reduced the problem of the determination of Ψ to the numerical solution of a system of one-dimensional integral equations. This solution does not require a significant computer time, as distinct from the traditional methods of solution of three-nucleon problems in the continuum, which are based on the direct numerical solution of two-dimensional integral equations in the momentum representation. We have established, first, that the consideration of three first K-harmonics in the expansion of the internal part of the wave function of a nd-system is sufficient for the satisfactory description of the relevant experiments on nd-scattering at subthreshold energies of a neutron. Second, for such energies and the NN potentials used in calculations, the contribution of the basic K-harmonic with $K = 0$ to the reaction amplitude is dominant. All this confirms the efficiency of the proposed approach.

Table 2. Phases δ_ℓ (in degrees)

E_n, MeV	ℓ	Potential type and calculated δ_ℓ				Other data
		Hulthén	η	Malfliet–Tjon	η	
2.45	0	−60.8	0.62	−53.3	0.78	−66.7 [18,30]
	1	15.4		16.3		23.1 [31]
	2	−4.0		−4.1		−4.2 [31]
3.27	1	−71.5	0.47	−64.3	0.80	−73.6 [20,32]
	2	15.8		17.8		25.6 [32]
	0	−7.5		−7.9		−4.6 [20]

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РОЗРАХУНКИ ХВИЛЬОВИХ ФУНКЦІЙ nd-СИСТЕМИ, ФАЗ І ПЕРЕРІЗІВ nd-РОЗСІЯННЯ З ВИКОРИСТАННЯМ МОДИФІКОВАНИХ РІВНЯНЬ ФАДДЄЄВА І МЕТОДУ ГІПЕРСФЕРИЧНИХ ФУНКЦІЙ

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Р е з ю м е

Розраховано хвильові функції nd-системи, а також фази і перерізи nd-розсіяння при енергіях, нижчих за поріг розвалу дейтрона. Виходячи з модифікованих рівнянь Фаддєєва, що одержані у нашій попередній роботі для повної хвильової функції, ми виділили найбільш складну її частину, яка описує рух трьох нуклонів в області взаємодії, і розклали її у ряд по гіперсферичним поліномам з $K = 0, 1, 2$. Для коефіцієнтів розкладу (радіальних функцій від колективної змінної) складено систему одновимірних інтегральних рівнянь, яку потім було чисельно розв'язано для енергій нейтрона 2,45, 3 і 3,27 МеВ. У розрахунках використовували локальні NN-потенціали Малфлі–Тьона і Хюльтена. Результати обчислень фаз і перерізів nd-розсіяння задовільно узгоджуються з відповідними експериментальними даними.