A GENERALIZED BOSE–EINSTEIN CONDENSATION THEORY OF SUPERCONDUCTIVITY INSPIRED BY BOGOLYUBOV

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We survey the unification of the Bardeen, Cooper, Schrieffer (BCS) and the Bose–Einstein condensation (BEC) theories via a generalized BEC (GBEC) formalism. The GBEC describes a *ternary* boson-fermion gas mixture consisting of fermion-particle- as well as fermion-hole-Cooper-pairs (CPs) that are bosons in thermal and chemical equilibrium with unpaired electrons. One then switches on an interaction Hamiltonian ($H_{\rm int}$) that is reminiscent of the single-vertex Fröhlich "two-fermion/one-boson" interaction. In contrast with the well-known BCS "four-fermion" two-vertex $H_{\rm int}$, the full GBEC $H \equiv H_0 + H_{\rm int}$ is *exactly* diagonalized with a Bogolyubov–Valatin transformation provided only that one ignores nonzero-total-momenta CPs in the interaction $H_{\rm int}$ although *not* in the unperturbed H_0 that describes an *ideal* ternary gas. Nonzero-total-momenta CPs are completely ignored in the full BCS H. Exact diagonalization is possible since the reduced GBEC H becomes *bilinear* in the fermion creation/annihilation operators on applying the Bogolyubov "recipe" of replacing the remaining zero-total-momenta boson hole- and particle-CP operators by the square root of their respective temperature- and coupling-dependent boson c-numbers. The resulting GBEC theory subsumes all five statistical theories of superconductors, including the Friedberg–T.D. Lee (1989) BEC theory, and yields hundredfold enhancements in predicted T_cs when compared with BCS predictions with the same two-electron BCS model phonon interaction producing the CPs.

1. Introduction

Boson-fermion (BF) models of superconductivity (SC) as a Bose–Einstein condensation (BEC) go back to the mid-1950s [1–4], pre-dating even the BCS–Bogoliubov theory [5–7]. Although BCS theory only envisions the presence of "Cooper correlations" of single-electron states, BF models [1–4, 8–19] posit the existence of actual bosonic Cooper pairs (CPs). With two [18, 19] exceptions, however, all BF models neglect the effect of *hole* CPs included on an equal footing with electron CPs to give the "complete" BF model (CBFM) that constitutes the generalized Bose–Einstein condensation (GBEC) formalism to be surveyed.

2. The GBEC Hamiltonian

The GBEC [18, 19] formalism is described by the Hamiltonian $H = H_0 + H_{\rm int}$, where

$$H_0 = \sum_{\mathbf{k_1},s_1} \epsilon_{\mathbf{k_1}} a^+_{\mathbf{k_1},s_1} a_{\mathbf{k_1},s_1} + \sum_{\mathbf{K}} E_+(K) b^+_{\mathbf{K}} b_{\mathbf{K}} -$$

$$- \sum_{\mathbf{K}} E_-(K) c^+_{\mathbf{K}} c_{\mathbf{K}}, \qquad (1)$$

and $\mathbf{K} \equiv \mathbf{k_1} + \mathbf{k_2}$ is the CP total or center-of-mass-momentum (CMM) wavevector, while $\epsilon_{\mathbf{k_1}} \equiv \hbar^2 k_1^2/2m$ are the single-electron and $E_\pm(K)$ the 2e-/2h-CP *phenomenological,* energies. Here, $a^+_{\mathbf{k_1},s_1}$ ($a_{\mathbf{k_1},s_1}$) are the creation (annihilation) operators for electrons and similarly $b^+_{\mathbf{K}}$ ($b_{\mathbf{K}}$) and $c^+_{\mathbf{K}}$ ($c_{\mathbf{K}}$) for 2e- and 2h-CP bosons, respectively. As originally suggested by the work of Cooper [20], the b and c CP operators depend only on $\mathbf{K}$ and so are *distinct* from the BCS pair operators depending on both $\mathbf{K}$ *and* the relative wavevector $\mathbf{k} \equiv \frac{1}{2}(\mathbf{k_1} - \mathbf{k_2})$ discussed in [5] [Eqs. (2.9) to (2.13)] for the particular case of $\mathbf{K} = 0$ and shown there *not* to satisfy the ordinary Bose commutation relations. Nonetheless, CPs are objects easily seen to obey Bose–Einstein statistics as, in the thermodynamic limit, an indefinitely large number of distinct $\mathbf{k}$ values correspond to a given $\mathbf{K}$ value defining the energy levels $E_+(K)$ or $E_-(K)$. This is all that is needed to ensure a BEC (or the macroscopic occupation of a given state that appears below a certain fixed $T = T_c$). This was found [18, 19] numerically *a posteriori* in the GBEC theory. Also, the BCS gap equation is recovered for equal numbers of both kinds of pairs, both in the $\mathbf{K}=0$ state and in all $\mathbf{K}\neq 0$ states taken collectively, and in weak coupling, regardless of CP overlaps. The precise familiar BEC T_c formula emerges [18] when i) 2h-CPs are ignored (whereupon the Friedberg–T.D. Lee model [13]-[16] equations are recovered) and ii) one switches off the BF interaction but under a strong interelectron coupling, whereby no unpaired electrons survive in the remaining binary mixture. The interaction Hamiltonian $H_{\rm int}$ consists of four distinct BF interaction *single* vertices each with two-fermion/one-boson creation or annihilation operators. Each vertex is reminiscent of the Fröhlich electron-phonon interaction with CPs replacing phonons. Here, $H_{\rm int}$ depicts how unpaired electrons (or holes) combine to form the 2e- (and 2h-CPs), and vice versa, assumed in a d-dimensional system of size L, namely

$$H_{\rm int}=L^{-d/2}\sum_{\mathbf{k},\mathbf{K}}f_{+}(k)\times$$

$$\times[a^{+}_{\mathbf{k}+\frac{1}{2}\mathbf{K},\uparrow}a^{+}_{-\mathbf{k}+\frac{1}{2}\mathbf{K},\downarrow}b_{\mathbf{K}}+a_{-\mathbf{k}}+\frac{1}{2}\mathbf{K},\downarrow a_{\mathbf{k}+\frac{1}{2}\mathbf{K}},\uparrow b^{+}_{\mathbf{K}}]+$$

$$+L^{-d/2}\sum_{\mathbf{k},\mathbf{K}}f_{-}(k)\times$$

$$\times[a^{+}_{\mathbf{k}+\frac{1}{2}\mathbf{K},\uparrow}a^{+}_{-\mathbf{k}+\frac{1}{2}\mathbf{K},\downarrow}c^{+}_{\mathbf{K}}+a_{-\mathbf{k}+\frac{1}{2}\mathbf{K},\downarrow}a_{\mathbf{k}+\frac{1}{2}\mathbf{K},\uparrow}c_{\mathbf{K}}]. \qquad (2)$$

The energy form factors $f_{\pm}(k)$ in (2) are taken as those in [18, 19], where the associated quantities E_f and $\delta\varepsilon$ are *new* phenomenological dynamical energy parameters (in addition to the positive BF vertex coupling parameter f introduced in [18, 19]) that replace the previous such $E_{\pm}(0)$, through the relations $E_f\equiv\frac{1}{4}[E_{+}(0)+E_{-}(0)]$ and $\delta\varepsilon\equiv\frac{1}{2}[E_{+}(0)-E_{-}(0)]\geq 0$, where $E_{\pm}(0)$ are the (empirically *un*known) zero-CMM energies of the 2e- and 2h-CPs, respectively.

We refer to E_f as the "pseudo-Fermi" energy. It serves as a convenient energy scale and is not to be confused with the usual Fermi energy $E_F=\frac{1}{2}mv_{\rm F}^2\equiv k_{\rm B}T_{\rm F}$, where T_F is the Fermi temperature. If n is the total number-density of charge-carrier electrons of effective mass m, the Fermi energy E_F equals $\pi\hbar^2 n/m$ in 2D and $(\hbar^2/2m)(3\pi^2 n)^{2/3}$ in 3D, while E_f is similarly related to another density n_f which serves to scale the ordinary density n. The two quantities E_f and E_F, and consequently also n and n_f, coincide *only* when the perfect 2e/2h-CP symmetry holds as in the BCS instance.

3. Diagonalization of GBEC Hamiltonian

The interaction Hamiltonian (2) can be further reduced by keeping only the $\mathbf{K}=0$ terms, so that

$$H_{\rm int}\simeq L^{-d/2}\sum_{\mathbf{k}}f_{+}(k)[a^{+}_{\mathbf{k}\uparrow}a^{+}_{-\mathbf{k}\downarrow}b_{\mathbf{0}}+a_{-\mathbf{k}\downarrow}a_{\mathbf{k}\uparrow}b^{+}_{\mathbf{0}}]+$$

$$+L^{-d/2}\sum_{\mathbf{k}}f_{-}(k)[a^{+}_{\mathbf{k}\uparrow}a^{+}_{-\mathbf{k}\downarrow}c^{+}_{\mathbf{0}}+a_{-\mathbf{k}\downarrow}a_{\mathbf{k}\uparrow}c_{\mathbf{0}}] \qquad (3)$$

which allows the *exact* diagonalization as follows. One applies the Bogoliubov "recipe" [21] (see also [22] p. 199 ff.) of replacing all zero-CMM 2e- and 2h-CP boson creation and annihilation operators in the full Hamiltonian $\hat{H}=H_0+H_{\rm int}$ by their respective c-numbers, namely $b_{\mathbf{0}},b^{+}_{\mathbf{0}}\to\sqrt{N_0(T)}$ and $c_{\mathbf{0}},c^{+}_{\mathbf{0}}\to\sqrt{M_0(T)}$, where $N_0(T)$ and $M_0(T)$ are the as yet to be determined T-dependent thermodynamically equilibrated number of zero-CMM 2e- and 2h-CPs, respectively. One eventually seeks, numerically at worst, the highest temperature, say T_c, above which $N_0(T)$ or $M_0(T_c)$ vanishes and below which one or the other is nonzero. Note that T_c calculated thusly can, in principle, turn out to be zero, in which case there is no BEC, but this will not turn out to be for the BCS model interaction to be employed here. If the number operator is

$$\hat{N}\equiv\sum_{\mathbf{k}_1,s_1}a^{+}_{\mathbf{k}_1,s_1}a_{\mathbf{k}_1,s_1}+2\sum_{\mathbf{K}}b^{+}_{\mathbf{K}}b_{\mathbf{K}}-2\sum_{\mathbf{K}}c^{+}_{\mathbf{K}}c_{\mathbf{K}}, \qquad (4)$$

the reduced $\hat{H}-\mu\hat{N}$ with (1) plus (3) is now entirely *bilinear* in the a^{+} and a operators. It can thus be diagonalized exactly via a Bogoliubov–Valatin transformation [23, 24]

$$a_{\mathbf{k},s}\equiv u_k\alpha_{\mathbf{k},s}+2sv_k\alpha^{\dagger}_{-\mathbf{k},-s}, \qquad (5)$$

where $s=\pm\frac{1}{2}$. Transformation (5) simplifies (1) plus (3) to the fully bilinear form

$$\hat{H}-\mu\hat{N}\simeq\sum_{k,s}\underbrace{[\xi_k\left(u_k^2-v_k^2\right)+2\Delta_k u_k v_k]}_{\equiv E_k}\alpha^{\dagger}_{\mathbf{k},s}\alpha_{\mathbf{k},\mathbf{s}}+$$

$$+\sum_{k,s}2s\overbrace{[\xi_k u_k v_k-\Delta_k\left(u_k^2-v_k^2\right)]}^{\equiv 0}\times$$

$$\times\left(\alpha^{\dagger}_{\mathbf{k},s}\alpha^{\dagger}_{-\mathbf{k},-\mathbf{s}}+\alpha_{\mathbf{k},s}\alpha_{-\mathbf{k},-\mathbf{s}}\right)+\sum_{\mathbf{k},s}2\left[\xi_k v_k^2+\Delta_k u_k v_k\right]+$$

$$+[E_+(0)-2\mu]\,N_0+\sum_{\mathbf{K}\neq 0}[E_+(K)-2\mu]\,b^\dagger_{\mathbf{K}}b_{\mathbf{K}}+$$

$$+[2\mu-E_-(0)]\,M_0+\sum_{\mathbf{K}\neq 0}[2\mu-E_-(K)]\,c^\dagger_{\mathbf{K}}c_{\mathbf{K}}\quad \mathbf{GBEC} \tag{6}$$

with $\xi_k\equiv\epsilon_k-\mu$. A little algebra shows that $v_k^2\equiv\frac{1}{2}[1-\xi_k/E_k]$ and $E_k=\sqrt{\xi_k^2+\Delta_k^2}$, precisely as in the BCS theory [5] as reformulated [6] by Bogoliubov. The term set equal to zero in (6) is justified, as this merely fixes the coefficient, say v_k, that was restricted only by $u_k^2+v_k^2=1$ which follows, in turn, from the requirement that both the a and α operators obey Fermi anticommutation relations. There are no products such as $\alpha^\dagger_{\mathbf{k},s}\alpha^\dagger_{-\mathbf{k},-\mathbf{s}}$ remaining, nor any other nonbilinear terms, as with [25] the BCS two-vertex, *four*-fermion Hamiltonian [5] that neglects other than $K=0$ pairings

$$H\equiv H_0+H_{\rm int}=\sum_{\mathbf{k},s}\epsilon_k a^+_{\mathbf{k},s}a_{\mathbf{k},s}-$$

$$-V\sum_{\mathbf{k},\mathbf{k}',s}a^+_{\mathbf{k}',s}a^+_{-\mathbf{k}',-s}a_{-\mathbf{k},-s}a_{\mathbf{k},s}\qquad \mathbf{BCS}, \tag{7}$$

where $-V\leqslant 0$, and the last summation is restricted by $E_{\rm F}-\hbar\omega_D\leqslant\hbar^2k^2/2m\equiv\epsilon_k,\epsilon_{k'}\leqslant E_{\rm F}-\hbar\omega_D$.

Eigenstates of the now fully diagonalized reduced GBEC $\hat{H}-\mu\hat{N}$ (6) are

$$|\,...n_{\mathbf{k},s}...N_{\mathbf{K}}...M_{\mathbf{K}}...\rangle=\prod_{\mathbf{k},s}\left(\alpha^\dagger_{\mathbf{k},s}\right)^{n_{\mathbf{k},s}}\prod_{\mathbf{K}\neq 0}\frac{1}{\sqrt{N_K}}\left(b^\dagger_{\mathbf{K}}\right)^{N_{\mathbf{K}}}\times$$

$$\times\prod_{\mathbf{K}\neq 0}\frac{1}{\sqrt{M_K!}}\left(c^\dagger_{\mathbf{K}}\right)^{M_{\mathbf{K}}}\;|\mathbf{O}\rangle,$$

where the three exponents $n_{\mathbf{k},s}=0,1$ and $N_{\mathbf{K}}$ and $M_{\mathbf{K}}=0,1,2\cdots$ are occupation numbers. Here, $|\,\mathbf{O}\rangle$ is the vacuum state for a fermionic "bogolon" quasiparticle with the gapped dispersion energy E_k appearing in (6) and rewritten below in (10) as $E(\epsilon)$. It is simultaneously a vacuum state for 2e-CP and 2h-CP boson creation and annihilation operators which is to say that $|\,\mathbf{O}\rangle$ is defined by $\alpha_{\mathbf{k},s}\,|\,\mathbf{O}\rangle\equiv b_{\mathbf{K}}\,|\,\mathbf{O}\rangle\equiv c_{\mathbf{K}}\,|\,\mathbf{O}\rangle\equiv 0$.

With the Hamiltonian explicitly diagonalized, one can now straightforwardly construct the thermodynamic potential $\Omega\equiv-PL^d$ for the GBEC, with L^d the system "volume" and P its pressure, which is defined as ([22], p. 228)

$$\Omega(T,L^d,\mu,N_0,M_0)\;=$$

$$=-k_{\rm B}T\ln\left[{\rm Tr}\exp\{-\beta(H-\mu\hat{N})\}\right], \tag{8}$$

where "Tr" stands for "trace." Inserting (1) plus (2) into (8) [18], one obtains, after some algebra, an explicit expression for $\Omega(T,\,L^d,\,\mu,\,N_0,\,M_0)/L^d$ (see [26], Eq. 10). In $d=3$, one usually has

$$N(\epsilon)\equiv\frac{m^{3/2}}{2^{1/2}\pi^2\hbar^3}\sqrt{\epsilon}\quad\text{and}\quad M(\varepsilon)\equiv\frac{2m^{3/2}}{\pi^2\hbar^3}\sqrt{\varepsilon} \tag{9}$$

for the (one-spin) fermion density-of-states (DOS) at energies $\epsilon=\hbar^2k^2/2m$ and the boson DOS for an **assumed** *quadratic* [1] boson dispersion $\varepsilon=\hbar^2K^2/2(2m)$, respectively. The latter assumption is to be lifted later so as to include Fermi-sea effects which change the boson dispersion relation from quadratic to *linear*, as mentioned before. Finally, the relation between the resulting fermion spectrum $E(\epsilon)$, which is as before, and the fermion energy gap $\Delta(\epsilon)$, are of the form

$$E(\epsilon)=\sqrt{(\epsilon-\mu)^2+\Delta^2(\epsilon)}, \tag{10}$$

$$\Delta(\epsilon)\equiv\sqrt{n_0}f_+(\epsilon)+\sqrt{m_0}f_-(\epsilon). \tag{11}$$

This last expression for the gap $\Delta(\epsilon)$ implies a simple T-dependence rooted in the 2e-CP $n_0(T)\equiv N_0(T)/L^d$ and 2h-CP $m_0(T)\equiv M_0(T)/L^d$ number densities of BE-condensed bosons, i.e., $\Delta(T)=\sqrt{n_0(T)}f_+(\epsilon)+\sqrt{m_0(T)}f_-(\epsilon)$.

4. Minimizing the Helmholtz Free Energy

By definition, the Helmholtz free energy is

$$F(T,L^d,\mu,N_0,M_0)\equiv\Omega(T,L^d,\mu,N_0,M_0)+\mu N.$$

Minimizing it with respect to N_0 and M_0, and simultaneously fixing the total number N of electrons by introducing the electron chemical potential μ in the usual way, specifies an *equilibrium state* of the system at fixed volume L^d and temperature T. The necessary conditions for an equilibrium thermodynamic state are thus

$$\partial F/\partial N_0=0,\quad \partial F/\partial M_0=0,\quad\text{and}\quad \partial\Omega/\partial\mu=-N, \tag{12}$$

where N evidently includes both paired and unpaired CP fermions. The second partial derivatives of F have been examined in [27]. After some algebra, Eqs. (12) then lead to the three coupled transcendental Eqs. (7)–(9) of [18]. These can be rewritten somewhat more transparently as: a) two *"gap-like equations"*

$$[2E_f+\delta\varepsilon-2\mu(T)]=\frac{1}{2}f^2\int_{E_f}^{E_f+\delta\varepsilon}d\epsilon N(\epsilon)\times$$

$$\times \frac{\tanh \frac{1}{2}\beta\sqrt{[\epsilon-\mu(T)]^2+f^2 n_0(T)}}{\sqrt{[\epsilon-\mu(T)]^2+f^2 n_0(T)}} \tag{13}$$

and

$$[2\mu(T)-2E_f+\delta\varepsilon] = \frac{1}{2}f^2 \int\limits_{E_f-\delta\varepsilon}^{E_f} d\epsilon N(\epsilon)\times$$

$$\times \frac{\tanh \frac{1}{2}\beta\sqrt{[\epsilon-\mu(T)]^2+f^2 m_0(T)}}{\sqrt{[\epsilon-\mu(T)]^2+f^2 m_0(T)}} \tag{14}$$

with $\beta \equiv 1/k_{\rm B}T$, as well as b) a single *"number equation"*

$$2n_{\rm B}(T) - 2m_{\rm B}(T) + n_f(T) = n. \tag{15}$$

This last relation ensures the charge conservation in a ternary mixture. In general, $n \equiv N/L^d$ is the total number density of electrons, $n_f(T)$ that of the *unpaired* electrons, while $n_{\rm B}(T)$ and $m_{\rm B}(T)$ are, respectively, those of 2e- and 2h-CPs in *all* bosonic states, ground plus excited, i.e., condensed and noncondensed. These turn out to be

$$n_{\rm B}(T) \equiv n_0(T) + \int\limits_{0+}^{\infty} d\varepsilon M(\varepsilon)\times$$

$$\times \left(\exp\beta[2E_f+\delta\varepsilon-2\mu+\varepsilon]-1\right)^{-1}, \tag{16}$$

$$m_{\rm B}(T) \equiv m_0(T) + \int\limits_{0+}^{\infty} d\varepsilon M(\varepsilon)\times$$

$$\times \left(\exp\beta[2\mu+\varepsilon-2E_f+\delta\varepsilon]-1\right)^{-1}, \tag{17}$$

which are clear manifestations of the bosonic nature of both kinds of CPs. For the number density of unpaired electrons at any T, one also obtains

$$n_f(T) \equiv \int\limits_0^{\infty} d\epsilon N(\epsilon)[1-\frac{\epsilon-\mu}{E(\epsilon)}\tanh\frac{1}{2}\beta E(\epsilon)] = 2\sum_{\mathbf{k}} v_k^2(T), \tag{18}$$

where $v_k^2(T) \equiv \frac{1}{2}[1-(\epsilon_k-\mu)/E_k] \underset{T\to 0}{\longrightarrow} v_k^2$ with E_k being given by (10) is precisely the BCS–Bogoliubov T-dependent coefficient that is linked with $u_k(0) \equiv u_k$ through $v_k^2+u_k^2=1$ of the normalized BCS trial wavefunction

$$|\,\mathbf{O}\rangle \equiv \prod_{\mathbf{k}}(u_{\mathbf{k}}+v_{\mathbf{k}}a^+_{\mathbf{k}\uparrow}a^+_{-\mathbf{k}\downarrow})\,|\,O\rangle \quad \text{with} \quad \langle\mathbf{O}\,|\,\mathbf{O}\rangle = 1, \tag{19}$$

where $|\,O\rangle$ is the ordinary vacuum. The zero-T version of the two amplitude coefficients v_k and u_k originally appeared in (19) and shortly afterwards in the Bogoliubov–Valatin canonical transformation. Next, one picks $\delta\varepsilon = \hbar\omega_D$ and identifies [18, 19] nonzero $f_+(\epsilon)$ and nonzero $f_-(\epsilon)$ with $f \equiv \sqrt{2\hbar\omega_D V}$ but such that $f_+(\epsilon)f_-(\epsilon) \equiv 0$. In the very special case where $n_0(T) = m_0(T)$, adding together (13) and (14) gives the precise BCS gap *provided* one identifies the pseudo-Fermi energy E_f with μ. This is guaranteed, in turn, if $n_{\rm B}(T) = m_{\rm B}(T)$, namely, if (16) and (17) are set equal to each other so that the arguments of the two exponentials become identical.

The self-consistent (at worst, numerical) solution of the *three coupled equations* (13) to (15) yields the three thermodynamic variables of the GBEC formalism

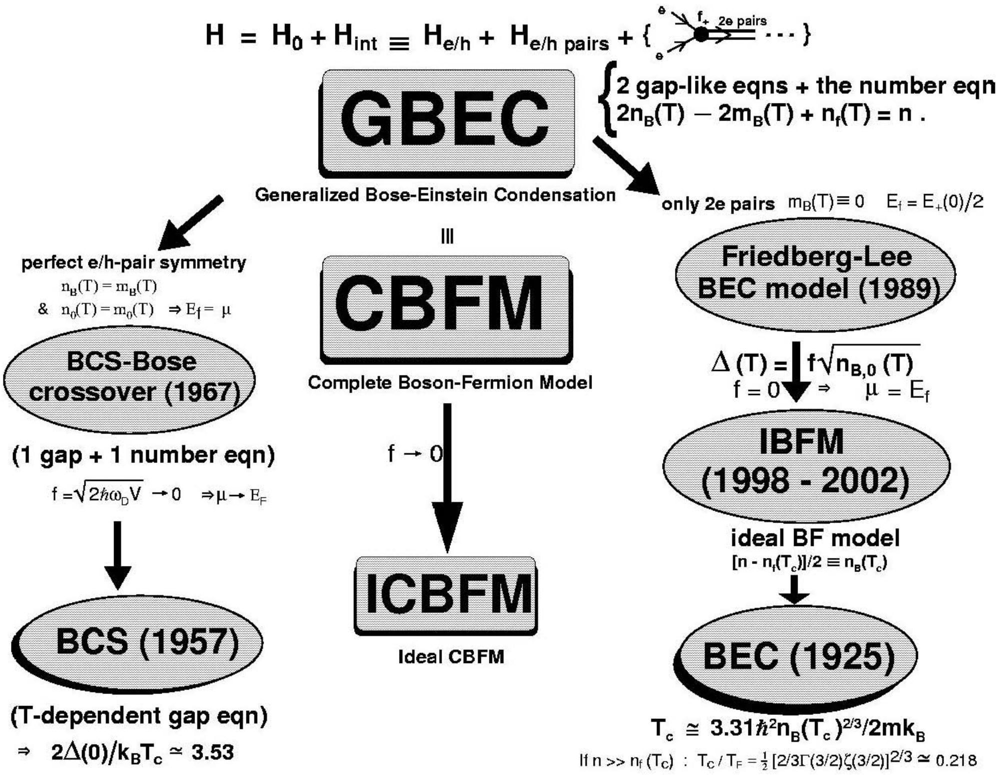
$$n_0(T,n,\mu), \quad m_0(T,n,\mu) \quad \text{and} \quad \mu(T,n). \tag{20}$$

The existence of a *nonzero* T_c associated with these expressions vindicates the GBEC theory. The numerical elimination of $\mu(T,n)$ shows [19] that, in addition to the normal phase at high temperatures defined by $n_0(T,n) = m_0(T,n) = 0$, *three* condensed phases appear at lower temperatures: two pure phases of 2e-CP- and 2h-CP-BE-condensed states and one mixed phase with arbitrary proportions of both kinds of BE-condensed states.

If hole pairs are ignored, the relation $\Delta(T) = f\sqrt{n_0(T)}$ resulting from (11) has recently been generalized [28] to include nonzero-$\mathbf{K}$ pairs beyond expression (3) with the help of two-time Green functions [29, 30]. This leads to a generalized gap $E_g(\lambda,T)$ defined as

$$E_g(\lambda,T) = \sqrt{2\hbar\omega_D V n_{\rm B}(\lambda,T)} \equiv f\sqrt{n_{\rm B}(\lambda,T)}, \tag{21}$$

where $n_{\rm B}(\lambda,T)$ is the *net* number density of CPs, both in and above the BE-condensate, in the BF mixture which was taken in [28] for simplicity as a binary mixture instead of a ternary one. The generalized gap $E_g(\lambda,T)$ accommodates recently discovered pseudogap phenomena [31], whereby the so-called "depairing" or pseudogap critical temperature $T^* \geqslant T_c$ arises. The pseudogap T^* is the solution of $E_g(\lambda,T^*) = 0$, whereas the superconducting T_c is that of $\Delta(T_c) = 0$.



Flowchart outlining conditions, under which the GBEC formalism reduces to all five statistical theories of superconductivity (ovals). The GBEC formalism has alternately been called the "complete boson-fermion model" (CBFM) in that it does not neglect hole CPs

5. Five Statistical Theories Subsumed

All told, the three GBEC equations (13) to (15) subsume *five* different theories as special cases, see a flowchart in Figure. The vastly more general GBEC formalism has been applied and gives sizeable enhancements in T_cs over the BCS theory that emerge [32] by admitting, apparently for the first time, departures from the very special case of the perfect 2e/2h-pair symmetry in the mixed phase.

6. Conclusions

In conclusion, five statistical continuum theories of superconductivity, including both the BCS and BEC theories, are contained as special limiting cases within a single generalized Bose–Einstein condensation (GBEC) model. This model includes, for the first time, along with unpaired electrons, both two-electron and two-hole pair-condensates in freely variable proportions. The BCS and BEC theories are thus completely *unified* within the GBEC. The BCS condensate emerges directly from the GBEC *as a BE condensate* through the condition for phase equilibria when both total 2e- and 2h-pair number, as well as their condensate, densities are *equal* at the given T and coupling provided the coupling is weak enough so that the electron chemical potential μ roughly equals the Fermi energy E_F. The ordinary BEC T_c-formula, on the other hand, is recovered from the GBEC when hole pairs are completely neglected, the BF coupling f is made to vanish, and the limit of zero unpaired electrons is taken, this implying a very strong interelectron coupling. The practical outcome of the BCS-BEC unification via the GBEC is an *enhancement* in T_c by more than two orders-of-magnitude in 3D. This enhancement in T_c falls within empirical ranges for 2D and 3D "exotic" SCs, whereas BCS T_c values remain low and within the empirical ranges for conventional, elemental SCs using standard interaction-parameter values. Lastly, room temperature superconductivity is possible for a material with a Fermi temperature $T_F \lesssim 10^3 K$, with the *same* interaction parameters used in BCS theory for conventional SCs.

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УЗАГАЛЬНЕНА ТЕОРІЯ
НАДПРОВІДНОСТІ З БОЗЕ-ЕЙНШТЕЙНІВСЬКОЮ
КОНДЕНСАЦІЄЮ, СТИМУЛЬОВАНА БОГОЛЮБОВИМ

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Р е з ю м е

Дано огляд об'єднання теорій Бардіна–Купера–Шріффера (БКШ) і бозе-ейнштейнівської конденсації (БЕК) в узагальненому формалізмі БЕК, який описує трикомпонентний бозе-ферміонний газ, що містить куперівські пари (КП) частинок-ферміонів і дірок-ферміонів, які є бозонами в тепловій і хімічній рівновазі з неспареними електронами. Введено гамільтоніан взаємодії H_{int}, що нагадує одновершинну “двоферміонну/однобозонну” взаємодію Фреліха. На відміну від добре відомого БКШ “чотириферміонного” двовершинного H_{int}, повний узагальнений гамільтоніан БЕК $H \equiv H_0 + H_{\text{int}}$ точно діагоналізується перетворенням Боголюбова–Валатина, якщо знехтувати КП з ненульовим повним імпульсом в H_{int}, а не в незбурюваному H_0, який описує ідеальний трикомпонентний газ. Куперівські пари з ненульовим повним імпульсом повністю ігноруються в повному БКШ гамільтоніані H. Точна діагоналізація можлива, оскільки узагальнений редукований гамільтоніан БЕК H стає білінійним у термінах ферміонних операторів народження/знищення при застосуванні “процедури” Боголюбова із заміною інших операторів куперівських пар із дірок-бозонів та частинок-бозонів із нульовим імпульсом коренем квадратним із відповідних бозонних c-чисел, що залежать від температури і взаємодії. Результуюча узагальнена теорія БЕК підсумовує всі п'ять статистичних теорій надпровідності, також теорію Фрідберга–Т.Д. Лі, та дає на два порядки більшу T_c, ніж за теорією БКШ з тією ж електрон-фононною взаємодією, яка породжує КП.